Excited Superdeformed Bands in ^{133}Pr: Identification of the $g_{9/2}$ Proton Orbital

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Four superdeformed bands, consisting of γ-ray cascades of up to 15 transitions, have been found in the light rare earth nucleus ^{133}Pr. This is the first observation of multiple superdeformed bands in the $A \approx 130$ mass region. These bands are populated weakly at 0.2%–1% of the reaction channel intensity. The single particle configurations have been identified as $\pi g_{9/2}[404]9/2^+$ for bands 1 and 2 and $\pi h_{11/2}[532]5/2^-$ for bands 3 and 4. Dipole linking transitions between two superdeformed bands have been observed, branching ratios have been measured, and $B(M1)/B(E2)$ ratios deduced. A $B(M1)$ value for the $\pi g_{9/2}[404]9/2^+$ orbital of $(1.7 \pm 0.2)\mu_N^2$ (where μ_N is the nuclear magneton) has been extracted, based on the measured quadrupole moment for the ^{132}Ce core.

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The study of superdeformed bands at high angular momentum has been one of the most important topics in nuclear structure in recent years. These bands occur in nuclei where there is a second minimum in the potential energy surface at a high deformation. The bands studied to date lie in three regions of the chart of nuclides. A group of nuclei occur around $A \approx 150$ with a deformation corresponding to a prolate ellipsoid with a 2:1 axis ratio ($\beta_2 \approx 0.6$). Lighter nuclei near $A \approx 130$ have a lower deformation with a 3:2 axis ratio ($\beta_2 \approx 0.4$), while heavier nuclei near $A \approx 190$ have an intermediate deformation ($\beta_2 \approx 0.45$–0.5). In all regions the nuclei have similar properties. The nuclei ^{132}Ce [1], ^{152}Dy [2], and ^{192}Hg [3] are "magic nuclei" with proton and neutron numbers corresponding to large shell gaps and hence have deep second minima. All three nuclei have a characteristic cascade of strongly enhanced $E2$ transitions (up to $\approx$20 transitions), a complex (as yet undetermined) decay path from the second to the first minimum, and a variation in the dynamic moment of inertia with frequency that is characteristic of a few particles in high N orbitals. These bands have all been previously described as superdeformed. The physics addressed in each region is similar, and interesting comparisons can be made. In the region around ^{132}Ce, single superdeformed bands are known to exist in a number of nuclei, e.g., ^{130}La [4], ^{131}Ce [5], ^{133}Nd [6], etc. Prior to the implementation of the Eurogam array there were no examples of excited bands within the second minimum or of identical bands where transitions with very similar gamma-ray energies are observed in neighboring nuclei, phenomena common in the other two superdeformed regions. Hence, near $A \approx 130$ very little is known about the proton and neutron single particle orbitals that are near the Fermi surface at high deformations. Excited bands are predicted to occur, so it is important that they are found and, hence, the single particle configurations to which they belong are experimentally identified.

In this Letter we have searched for superdeformed bands in the nucleus ^{133}Pr. This nucleus has one more proton than its even-even neighbor ^{132}Ce, and so should provide interesting information on proton energy levels within the second minimum.

The experiment was performed at the Daresbury Laboratory Nuclear Structure Facility (NSF), using the ^{100}Mo(^{37}Cl,4n)^{133}Pr reaction. The ^{37}Cl beam, at an energy of 155 MeV, was incident on a self-supporting, 500 μg cm^{-2} ^{100}Mo foil. The Eurogam [7] γ-ray spectrometer, equipped with 41 escape suppressed, high efficiency, HPGe detectors, was used to collect a large data set of 4.5×10^9, 3.4×10^9, 2.5×10^9, double, triple, and quadruple γ suppressed coincidence events, respectively.

Four weakly populated bands have been observed (see Fig. 1), extending over 10–15 transitions from around 800 to 1600 keV, with a roughly constant transition spacing of $\approx$70 keV. This spacing is similar to that found in other superdeformed bands in this region (e.g., ^{132}Ce [1]), and it is therefore reasonable to assume that these states are also superdeformed. An analysis of where the four bands feed into the normal deformed states has been made using the previously known level scheme [8] and the results of this work. It is found that bands 1 and 2 feed into a positive parity, negative signature, band at a spin of $\frac{41}{2}\hbar$, while bands 3 and 4 both feed into the negative parity yrast band at a spin of $\frac{43}{2}\hbar$. Making the same assumption for all four bands that the unseen linking transitions contribute $\approx 6\hbar$ to the total spin, it is estimated that the bands cover a range in spin of $(\frac{53}{2}-\frac{93}{2})\hbar$, $(\frac{51}{2}-\frac{95}{2})\hbar$, $(\frac{53}{2}-\frac{113}{2})\hbar$, and $(\frac{55}{2}-\frac{103}{2})\hbar$ for bands 1–4, respectively.

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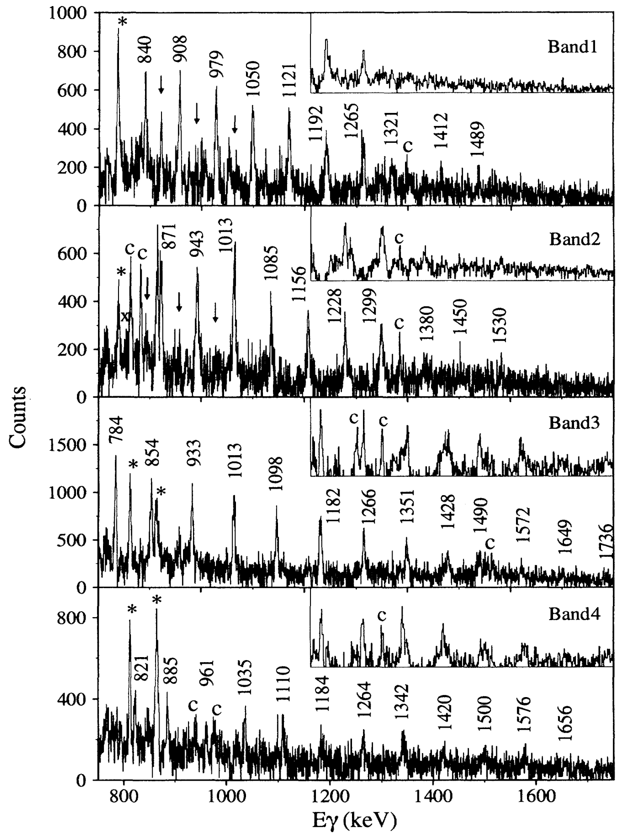

FIG. 1. Coincidence spectra of the superdeformed bands in ^{133}Pr produced from the sum of all combinations of double gates on the energies marked, from triple coincidences. The higher energy transitions produced from double coincidences are inset (using the energy scale on the major spectrum). Peaks marked with an asterisk denote known transitions. Peaks marked with a C are contaminants. Peaks marked with arrows show the first indication of cross talk between bands 1 and 2. The position of the 800 keV transition in band 2 is marked with an $\times$, although it is difficult to see in this spectrum due to contamination.

The intensities of the strongest transitions in all four bands relative to the total population of ^{133}Pr have been measured as 1% for bands 1 and 2, 0.5% for band 3, and 0.2% for band 4, using spectra in coincidence with single transitions in ^{133}Pr. The relative intensities of in-band transitions have also been measured using spectra created by double gating on transitions from the bands. The intensity information has been plotted as a function of spin for bands 1 to 4 in Fig. 2(a). It can be seen that the intensity patterns for bands 3 and 4 are very different from those for bands 1 and 2, with the feeding of bands 3 and 4 occurring over a much broader range and extending to a higher spin. Bands 1 and 2 are fed much more strongly in the region below $\approx \frac{89}{2}\hbar$. This suggests that at the highest spins bands 3 and 4 are lower in energy, with band 3 lying lowest, as it is fed more strongly. At low spins, bands 1 and 2 are fed more strongly, and are, hence, likely to be yrast at these spins. The crossing between bands 1 and 2 and bands 3 and 4 occurs even if the relative spin difference between the bands is increased by $8\hbar$.

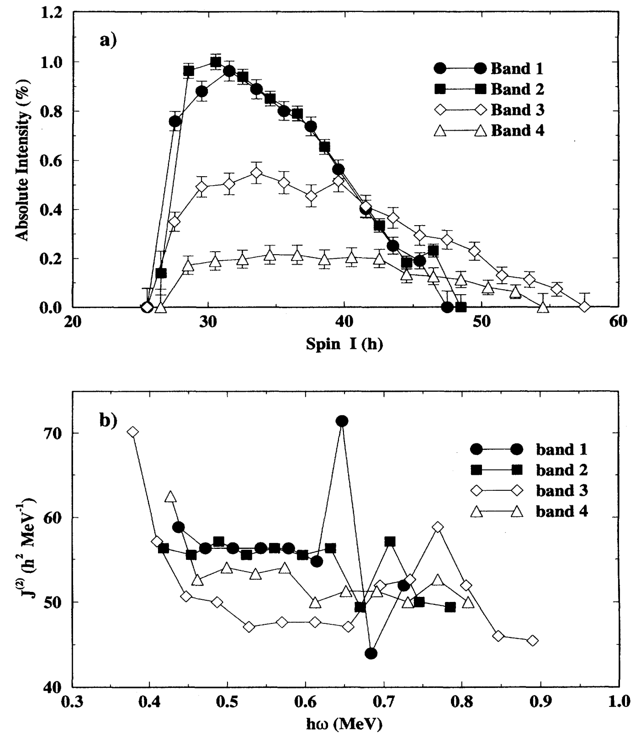

FIG. 2. (a) Absolute intensity variation as a function of spin for transitions in all four SD bands in ^{133}Pr. (b) Dynamic moments of inertia $\mathfrak{J}^{(2)}$ versus rotational frequency ($\hbar\omega$) for the four superdeformed bands in ^{133}Pr.

The $\mathfrak{J}^{(2)}$ dynamic moments of inertia have been extracted from the difference in adjacent γ-ray energies [Fig. 2(b)]. Over a wide frequency range, bands 1 and 2 have the same moment of inertia. Very weak links between bands 1 and 2 have been identified. This is the first observation of dipole links between superdeformed bands in the mass $A \approx 130$ region. Three transitions in band 2 can be seen when gating on band 1 and vice versa (Fig. 1). The dipole transitions can be observed when gating on in-band transitions in a matrix created from quadruple coincidences by setting all combinations of two gates on transitions from either band or any of the dipoles. A typical spectrum is shown in Fig. 3. It has been possible to measure branching ratios for the 430 keV/840 keV, 444 keV/871 keV, and 476 keV/943 keV γ rays, which are found to be 0.20 ± 0.03, 0.17 ± 0.05, and 0.19 ± 0.04, respectively. The corresponding $B(M1)/B(E2)$ ratios are 0.74 ± 0.13, 0.7 ± 0.2, and 0.90 ± 0.16 $(\mu_N/e\,\mathrm{b})^2$ (where μ_N is the nuclear magneton). Bands 1 and 2 are therefore signature partners. One striking feature of the $\mathfrak{J}^{(2)}$ moment of inertia for band 1 is the presence of a large perturbation at a rotational frequency of $\hbar\omega \approx 0.65$ MeV. A smaller perturbation is also visible in band 2, at the same frequency, but in the opposite direction.

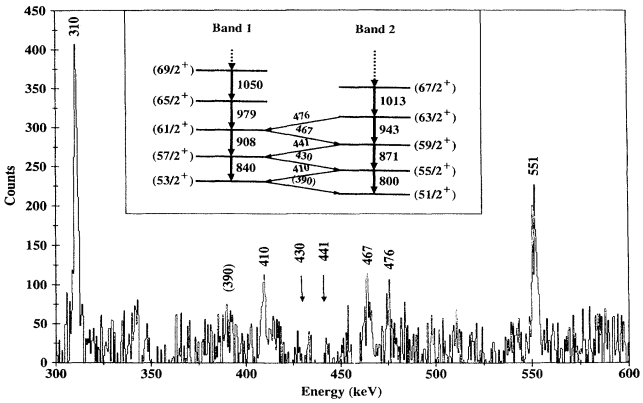

FIG. 3. Coincidence spectrum showing three dipole transitions between superdeformed bands 1 and 2 in ^{133}Pr. This spectrum is in coincidence with the 871 keV transition in band 2, and all other combinations of two gates from all in-band and all cross-band transitions. The 310 and 551 keV γ rays are the two most intense γ rays in the ^{133}Pr level scheme. The arrows denote the positions of the missing 430 and 441 keV γ rays. The partial decay scheme for bands 1 and 2 is inset.

The spacing of γ rays in bands 3 and 4 suggests that they are also signature partners, but now with signature splitting. The moment of inertia of band 3 rises and falls between $\hbar\omega = 0.65$ and 0.85 MeV. This is the result of a band crossing at $\hbar\omega \approx 0.75$–0.8 MeV. No crossing is observed in band 4. All four bands feed out at a rotational frequency of $\hbar\omega \approx 0.4$ MeV, where the ^{132}Ce yrast superdeformed band also feeds out. This provides additional evidence that the bands in ^{133}Pr can be interpreted as the superdeformed core in ^{132}Ce plus a proton. On closer inspection, the feeding out points for the four bands occur at slightly different frequencies. There is a pronounced increase at the lowest frequencies in the dynamic moments of inertia for bands 3 and 4, which is not exhibited in bands 1 and 2 (although in band 1 the lowest point shows a small increase).

Total Routhian surface (TRS) [9] calculations have been performed to aid the assignment of configurations for the four bands. Interestingly, these calculations predict significantly different deformations for the lowest lying (π, α) combinations. For example, at $\hbar\omega = 0.607$ MeV, these are $\beta_2 = 0.371$ for $(+,+)$ and $(+,-)$, $\beta_2 = 0.406$ for $(-,+)$, and $\beta_2 = 0.350$ for $(-,-)$. This effect can only be confirmed when mean lifetimes for the in-band transitions are measured and quadrupole moments deduced. This type of experiment will only be possible with the next generation of gamma-ray spectrometers (e.g., Eurogam 2).

Unpaired, cranked shell model calculations, based on the universal Woods-Saxon single particle potential [9], have been used to produce the single particle Routhians for protons near the Fermi surface for ^{133}Pr. Calculations were performed covering the range of deformation parameters deduced from the TRS calculations (examples are given in Fig. 4). The effect of hexadecapole and triaxial deformations has been included. It can be seen from these calculations that there are four favored orbitals available for occupation by the 59th proton, $g_{9/2}[404]9/2^+$ and $h_{11/2}[532]5/2^-$, which change positions relative to the Fermi surface as the deformation is increased from $\beta_2 = 0.371$ to $\beta_2 = 0.406$.

Bands 1 and 2 can be associated with the $\pi g_{9/2}[404]9/2^+$ proton orbital. This is consistent with the decay scheme, i.e., very little signature splitting and decay to positive parity states in the normal deformed minimum. The observation of dipole linking transitions between bands 1 and 2 is also consistent with the $\pi g_{9/2}[404]9/2^+$ high K orbital.

For the $B(M1)/B(E2)$ ratios deduced from the branching ratios an average $B(M1)$ value of $(1.7 \pm 0.2)\mu_N^2$ can be extracted, assuming a quadrupole moment of 8.0 ± 0.8 e b (the value measured experimentally [10] for the ^{132}Ce core). This is in good agreement with the value of $1.9\mu_N^2$ expected using a particle-rotor model for an $\Omega = 9/2$ orbital in the asymptotic limit [11]. The alternative

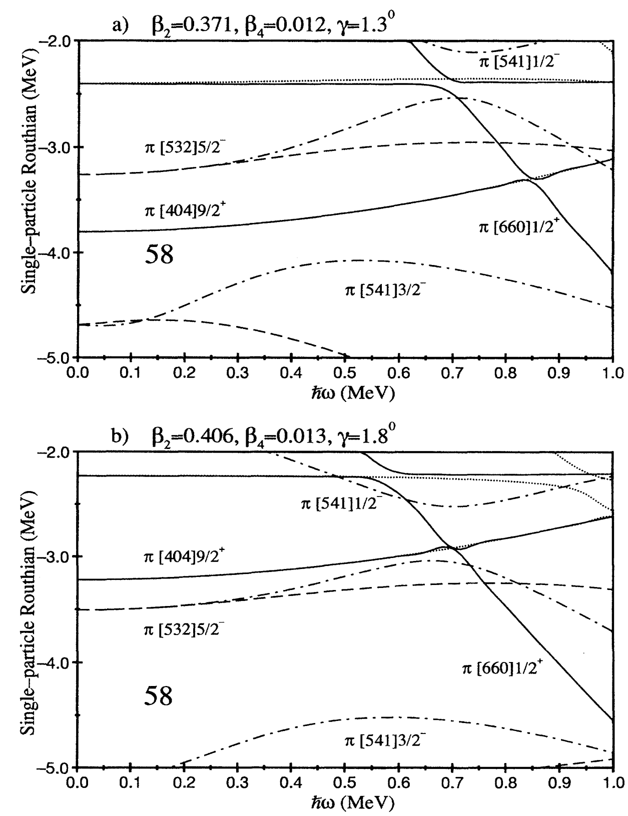

FIG. 4. Proton single particle Routhians calculated using the deformed Woods-Saxon potential, with different deformation parameters. The Routhians are labeled using the ($\hbar\omega = 0$) asymptotic Nilsson quantum numbers $[N, n_z, \Lambda]\Omega$. The parity and signature (π, α) of the orbitals is given by solid curve, $(+, +1/2)$; dotted curve, $(+, -1/2)$; dot-dashed curve, $(-, +1/2)$; dashed curve, $(-, -1/2)$.

orbital has $\Omega = 5/2$ and is expected to have a $B(M1)$ value of $0.8\mu_N^2$. The $B(M1)$ value can now be used to deduce a value of g factor $g_K = 1.30 \pm 0.05$, the single particle contribution to the magnetic moment (assuming the rotational g factor $g_R = Z/A$). This agrees with the value expected for the $g_{9/2}$ orbital. Calculations of the type described by Dönau and Frauendorf [12,13] can also be used to predict the $B(M1)/B(E2)$ ratio. Assuming full alignment of the particles involved, and converting the deformation parameters derived from the TRS calculations into deformation parameters describing the distribution of the nuclear charge, a theoretical ratio of $B(M1)/B(E2) = 0.9$ is calculated for the $\nu i_{13/2}^2 \pi h_{11/2}^2 g_{9/2}$ configuration, i.e., ^{132}Ce superdeformed configuration plus a $g_{9/2}$ proton. This is in agreement with the experimental value. The calculated value decreases by $\approx$10%–15% when different neutron configurations are used (e.g., $h_{9/2}^2$), instead of $i_{13/2}^2$. There is a larger uncertainty arising from the assumptions used in the calculations (amount of alignment, etc.). There is, therefore, strong experimental evidence for the population of the $\pi g_{9/2}[404]9/2^+$ orbital, but it is not possible to discriminate between possible neutron configurations.

There is strong evidence to suggest that bands 3 and 4 arise from the population of the $\pi h_{11/2}[532]5/2^-$ orbital. There is an experimentally observed band crossing seen in band 3 at $\hbar\omega \approx 0.8$ MeV. From the calculated single particle Routhians it can be seen that there is a predicted band crossing in the positive signature of the $\pi h_{11/2}[532]5/2^-$ orbital at $\hbar\omega \approx 0.7$ MeV. The calculated crossing frequency is deformation dependent and is due to the interaction of the $\pi h_{9/2}[541]1/2^-$, $\alpha = +1/2$ orbital. There is no observed crossing in band 4 below $\hbar\omega = 0.8$ MeV in agreement with the expected behavior of the $\pi h_{11/2}[532]5/2^-$ negative signature orbital. Therefore, the experimental data are consistent with the calculations. However, there remains a problem. The calculations at a fixed deformation predict that the positive signature orbital (assigned to band 3) is higher in energy than the negative signature orbital (assigned to band 4), over the frequency range in which the states are populated. This is in apparent conflict with the experimentally observed intensities where band 3 is twice as intense as band 4.

The unpaired cranking calculations show that for frequencies above 0.7 MeV the $\pi i_{13/2}[660]1/2^-$ orbital comes rapidly to the Fermi surface. The only possible experimental evidence for this is the perturbation in band 1 at a frequency of $\hbar\omega = 0.65$ MeV. However, there should not be a perturbation in band 2 due to the $i_{13/2}$ proton orbital, although there is some experimental evidence for a perturbation in the opposite direction to band 1. It is inconclusive as to whether this orbital has been seen experimentally.

To summarize, four weakly populated, superdeformed bands based on states in the second minimum have been discovered, with configurations based on the $\pi g_{9/2}[404]9/2^+$ and $\pi h_{11/2}[532]5/2^-$ orbitals. This is the first observation of excited superdeformed bands in the mass $A \approx 130$ region. Dipole transitions have been seen between states in the bands based on the $\pi g_{9/2}[404]9/2^+$. The measured $B(M1)/B(E2)$ ratios agree with those expected theoretically and thus allow the $g_{9/2}$ proton orbital to be experimentally identified. There is limited evidence for the $\pi i_{13/2}[660]1/2^-$ which is expected to be near the Fermi surface at $\hbar\omega \approx 0.7$ MeV. The experimentally observed population intensities of the bands cannot easily be understood. The explanation may lie in the different deformations for the bands that are predicted by the TRS calculations. Confirmation of this effect requires the quadrupole moments of the bands to be deduced from lifetime measurements. This experiment is not possible until the next generation of arrays (e.g., Eurogam 2) is available.

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