Mario Liu Replies: Let me first summarize the pertinent results of the subject Letter [1], before returning to discussing the statements in the preceding Comment. The purpose of the work presented in Ref. [1] is to describe *the dynamic behavior of electromagnetic field in the hydrodynamic regime,* i.e., in the frequency range of $\omega\tau_c \ll 1$, τ_c being the collision time, after which local equilibrium is established. To this end, two variables, D and B, constrained to satisfy $\nabla \cdot \mathbf{D} = \rho^e$, $\nabla \cdot \mathbf{B} = 0$ at all times, are added to the usual set of hydrodynamic variables,

$$d\varepsilon = \mu d\rho + T ds + \mathbf{v} \cdot d\mathbf{g} + \mathbf{H} \cdot d\mathbf{B} + \mathbf{E} \cdot d\mathbf{D}. \quad (1)$$

Note that the conjugate fields are given as $\mathbf{H} \equiv \partial\varepsilon/\partial\mathbf{B}$ and $\mathbf{E} \equiv \partial\varepsilon/\partial\mathbf{D}$; similar to $T \equiv \partial\varepsilon/\partial s$, they contain only equilibrium information. The equations of motion for the two variables are

$$\dot{\mathbf{D}} = \nabla \times \mathbf{H}^M - \mathbf{j}^e, \quad \dot{\mathbf{B}} = -\nabla \times \mathbf{E}^M, \quad (2)$$

where $\mathbf{j}^e = \sigma\mathbf{E}$. Linear response imposes $\mathbf{H}^M = \mathbf{B}(\mu_R^{-1} + i\mu_I^{-1})$, $\mathbf{E}^M = \mathbf{D}(\varepsilon_R^{-1} + i\varepsilon_I^{-1})$; nonlinearly, one usually sets $\mathbf{H}^M = \mathbf{H}$ and $\mathbf{E}^M = \mathbf{E}$ (which neglects damping altogether). So far, these are statements that can be found in textbooks [2]. The new information provided in Ref. [1] is the nonlinear yet dissipative forms of $\mathbf{H}^M$ and $\mathbf{E}^M$, the simplest [3] of which are

$$\mathbf{H}^M = \mathbf{H} - \alpha\nabla \times \mathbf{E}, \quad \mathbf{E}^M = \mathbf{E} + \beta\nabla \times \mathbf{H}. \quad (3)$$

Being of different parity under time reversal than the other terms of the temporal Maxwell equations, Eqs. (2), the terms preceded by α and β (and also σ) are irreversible and account for damping and restoration of equilibrium. For linear constitutive relations, Eqs. (3) reduce to the linear response expressions with $\mu_R = B/H$, $\varepsilon_R = D/E$, $\mu_I = 1/\omega\alpha$, and $\varepsilon_I = 1/\omega\beta$ to lowest order either in the frequency ω or the transport coefficients α, β.

Inspection of Eqs. (2) shows that (irrespective of the form of E^M) B is a true hydrodynamic variable, in the usual sense that the associated frequency $\omega \rightarrow 0$ for $k \rightarrow 0$. Conversely, D is nonhydrodynamic and relaxes (if linear) with the time $\tau_\sigma = \varepsilon_R/\sigma$. More precisely, D is an independent hydrodynamic variable only if $\omega\tau_\sigma \gg 1$, when j^e can be neglected and D is quasiconserved; for $\omega\tau_\sigma \ll 1$, the equilibrium condition $E \equiv 0$ holds instantly, it supersedes the first of Eqs. (2) and hereby eliminates D as an independent variable. (For the frequencies in between, D is independent but nonhydrodynamic.) The Ohmic relaxation time τ_σ varies greatly; it is a few days for amber and 10^{-4} s for distilled water. However, at ambient temperatures, there is a wide frequency window $1/\tau_c \gg \omega \gg 1/\tau_\sigma$ for both systems in which $\alpha\nabla \times \mathbf{E}$, rather than $\mathbf{j}^e = \sigma\mathbf{E}$, acts as the primary dissipative term, and both Maxwell equations, Eqs. (2), are needed. (I am here only laboriously arguing that the familiar concept of *dielectrics* is indeed a useful one. Having assumed its general acceptance, the relevant discussion in Ref. [1], below Eq. (3), was somewhat brief.) In good conductors such a copper, τ_σ tends to exceed τ_c, so these never behave as dielectrics.

Now, since dissipation and nonlinear constitutive relations are generic phenomena, it seems rather worthwhile to formulate a Maxwell theory that can account for both simultaneously, especially in dielectrics. This is what I have done, and I am puzzled why the authors end their comment by stating that barring "very special circumstances, there is no need to—(implying but not quite conveying what I did)—change the classical Maxwell equations of continuous media."

Do Brand and Pleiner have *bona fide* evidence to support this stance? They state that D is nonhydrodynamic, that $\nabla \times \mathbf{B}$ and $\nabla \times \mathbf{D}$ have been introduced as macroscopic variables, and that these two quantities relax on microscopic time scales. I disagree. First, in dielectrics, as discussed, D is hydrodynamic. Second, I have only introduced D and B as additional variables, cf. Eq. (1). The energy ε is certainly not taken as a function of any spatial derivative of the two fields. Third, although D relaxes with τ_σ and B does not at all, both $\nabla \times \mathbf{B}$ and $\nabla \times \mathbf{D}$ are hydrodynamic and do not relax. The simple reason is again given by Eqs. (2), taking the curl (or any spatial derivative) of which clearly renders the associated frequency hydrodynamic: $\omega \rightarrow 0$ for $k \rightarrow 0$. ($\nabla \cdot \mathbf{D} = \rho^e$ is hydrodynamic for exactly the same reason, and charge conservation is a result, as most know, of the Maxwell equations.)

A final point: It is correct that α and β are proportional to the relaxation time of magnetization and polarization, respectively, cf. Eq. (7) of Ref. [1]. But one must not conclude that this is indicative of nonhydrodynamic behavior. All transport coefficients are proportional to some relaxation times: Take the textbook example of second viscosity [4].

Mario Liu
Institut für Theoretische Physik
Universität Hannover
30167 Hannover, Germany

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[1] Mario Liu, Phys. Rev. Lett. **70**, 3580 (1993).
[2] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, Oxford, 1984).
[3] Generally, one has to distinguish between the lab and local rest frame and heed the fact that thermodynamic forces (such as the temperature gradient or the shear flow) enter the expressions for $\mathbf{H}^M$ and $\mathbf{E}^M$, cf. also Mario Liu, Phys. Rev. E **50**, 2925 (1994).
[4] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, Oxford, 1987): Sec. 81, Eq. (81-7).

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