High Frequency Synchronization of Chaos

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Chaotic synchronization involving a yttrium iron garnet film in ferromagnetic resonance at 1.2 GHz has been achieved through small perturbations. The perturbations were derived from the difference between the current system's chaotic signal and a prerecorded chaotic signal from the same attractor, and were applied continuously. A model of the system's dynamics predicts the observed synchronization.

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Synchronization of coupled chaotic systems is a surprising nonlinear phenomenon, given that nearby trajectories of uncoupled chaotic systems diverge. This effect has potential applications in secure communications [1,2]. This report presents the first experimental results of the synchronization of two chaotic signals in the 0.5–10 MHz frequency range in high power ferromagnetic resonance (FMR). The unstable oscillations synchronized in this experiment involved megahertz frequencies, an order of magnitude higher than any previously reported results in the synchronization of chaotic systems.

The ability to synchronize coupled chaotic systems was first demonstrated by Pecora and Carroll [1]. Decomposing a chaotic system into two coupled subsystems called the drive and response, they showed that synchronization could occur if the response system had no positive Lyapunov exponents. Pecora and Carroll verified this in an experimental circuit. Lai and Grebogi [3] later introduced a method of synchronization that did not require decomposition of the chaotic system. They showed that, if two nearly identical chaotic systems displayed similar trajectories, a perturbation could be calculated and applied to one of the systems, enabling the two systems to synchronize. The perturbation is only applied when the two systems have close neighboring trajectories, and exploits the unstable and stable directions about each trajectory point. Pyragas [4] also introduced a method of synchronization that did not require the decomposition of the chaotic system. In Pyragas's scheme, the output of a chaotic system was recorded into a memory. The system could be forced to repeat the recorded motion by taking the difference between the current state of the system and the recorded motion and applying a perturbation to a single variable of the system proportional to this difference.

These studies motivated many experimental results. Largely inspired by Carroll and Pecora's work, Cuomo and Oppenheim [2] were also able to demonstrate synchronization and its potential for communications in chaotic circuits. Newell *et al.* successfully synchronized pairs of diode resonators [5,6] using modifications of Lai and Grebogi's scheme. The synchronization of the chaotic output of coupled lasers has been reported by Roy and Thornburg [7] as well as Sugawara *et al.* [8]. Yu, Kwak, and Lim [9] were able to successfully implement a synchronization scheme applied to coupled chaotic circuits based on the method proposed by Pyragas. Since high power FMR studies involving yttrium iron garnet (YIG) have motivated much experimental [10,11] and theoretical [12,13] activity in nonlinear dynamics, this system would seem to be a promising one for applying chaotic synchronization techniques at much higher frequencies.

Here, we report the first results of synchronization of the chaotic response of a yttrium iron garnet film in high power FMR. The experiments were preformed on a circular disk of YIG 2 mm in diameter and 1 μm thick. A static magnetic field of about 2000 Oe is applied perpendicular to the film plane. A rf pumping field at 1.2 GHz was generated by a slotline structure [14] to excite the magnetostatic modes in the sample. Absorption of the rf power is detected by a diode and is proportional to the size of the precession angle of the magnetic spins. As the rf pumping power is increased above some threshold, the absorption of the rf power develops a periodic ac component (auto-oscillation) superimposed on a dc component. The ac component arises from nonlinear dipole interactions between the magnetostatic modes in the sample [11]. Further increasing the rf power above the auto-oscillation threshold often produces a bifurcation route to chaos in the ac component in the absorbed rf power. The frequencies associated with this chaotic signal are in the 0.5–10 MHz range.

To achieve synchronization of this chaotic ac component, the chaotic time series voltage of the diode detector was recorded into a memory. This stored signal is referred to as the master signal, while the real-time output signal of the YIG film is referred to as the slave signal. The output of the YIG film was synchronized with its prerecorded output signal through a small perturbation to the static bias field. Such perturbations to the static bias field modify the torque acting on the magnetic vector, which in turn determines the angle of precession. The perturbation to the static bias field was achieved by a small ten-turn coil 1.5 cm in diameter suspended 2 mm above the film plane. Given the high frequency dynamics of the sample, synchronization methods which require extensive calculations before applying the perturbation would be impractical. Thus, the perturbation to the static bias field required to maintain the magnetic vector in the desired trajectory

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is proportional to the difference between the master and slave signal:

$$H_{\text{pert}} = K(V_{\text{master}} - V_{\text{slave}}). \tag{1}$$

In Eq. (1), V_{master} refers to the prerecorded voltage, V_{slave} refers to the current output, and K designates the proportionality constant. Perturbing the magnetic field in this manner corrects deviations between the master and slave by changing the torque on the magnetic spins in the YIG sample (the slave system). This synchronization scheme is similar to the one proposed by Pyragas [4]. Newell and collaborators have used perturbations expressed in Eq. (1) to synchronize two diode resonators [5], calling it proportional feedback.

When proportional feedback was applied in our experiment, the correlation between the master and slave signals was enhanced by more than an order of magnitude. To demonstrate the effectiveness of proportional feedback in synchronizing chaos in our experiment, the master and slave signals were sampled in 10 ns intervals and plotted against one another, in the absence of control and with control as shown in Fig. 1. In comparison to the 2000 Oe static bias field, the perturbation to this field to maintain synchronization has an amplitude less than 0.1 Oe. In the initial transient, the perturbation field amplitude is no larger than 1 Oe. A delay coordinate plot of the sample's output signal during the synchronization process is shown in Fig. 2 and indicates the sample was acting chaotically during the controlling process. Since the master signal loop has a discontinuity due to its finite length in time, transient behavior of the control can be observed at this discontinuity, and it was found that the system synchronizes in about 10 μs.

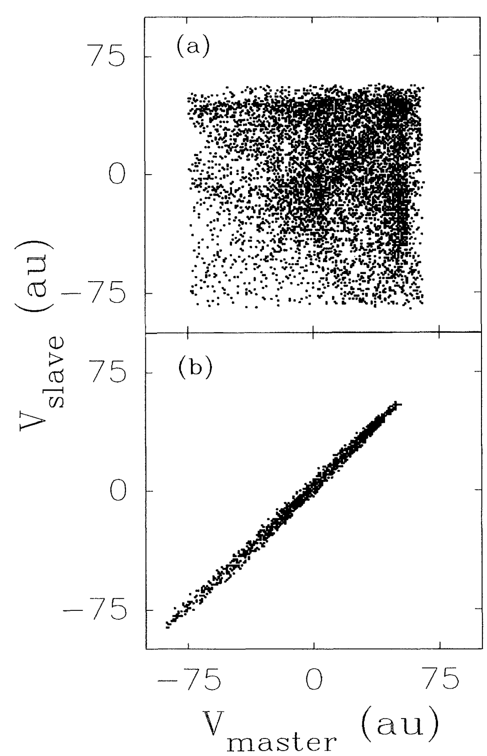

FIG. 1. Values of the master and slave experimental FMR signals plotted against each other without control (a) and in the presence of control (b).

A theoretical model, which has accurately predicted the behavior of YIG films in high power FMR [11] has also been found to predict the observed synchronization through this control technique. The normal modes in thin circular YIG films are the magnetostatic modes, which have the form of Bessel functions. The equations of motion for the complex amplitude c_i of these modes are

$$\frac{dc_i}{dt} = -i\gamma\left[(H_0 - H_i^{\text{res}} - i\Gamma)c_i + \frac{h_p}{2} I_i^* + 2\pi M_s \sum_{jkl} A_{ijkl} c_j^* c_k c_l\right], \tag{2}$$

where H_0 and H_i^{res} are the applied magnetic field and the resonant field of mode i, Γ is the linewidth of the low power absorption, h_p is the amplitude of the rf pumping field, and I_i^* is the dipole moment of mode i. The A_{ijkl} terms describe the nonlinear couplings between the modes and can be calculated from the integral of Bessel functions over the sample volume.

The FMR signal in the model corresponding to the measured signal in the experiments was constructed from the evolution of the mode amplitudes,

$$S(t) = -\sum I_i^* \operatorname{Im}(c_i). \tag{3}$$

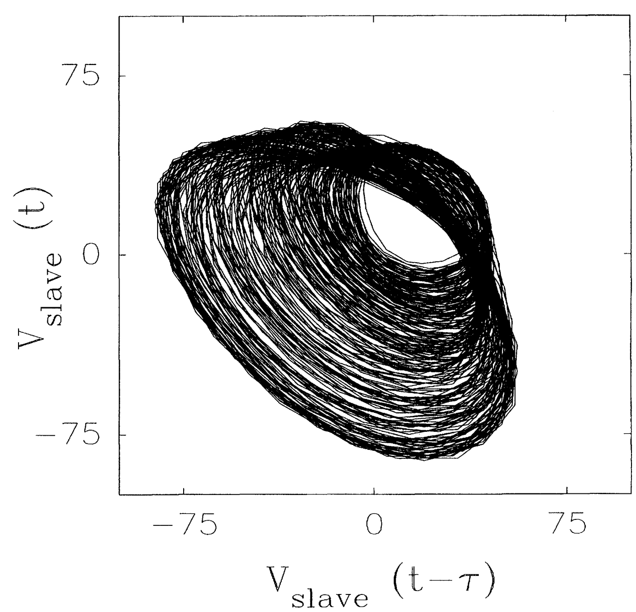

FIG. 2. Delay coordinate plot in arbitrary units of the FMR slave signal's attractor during synchronization (τ = 50 ns).

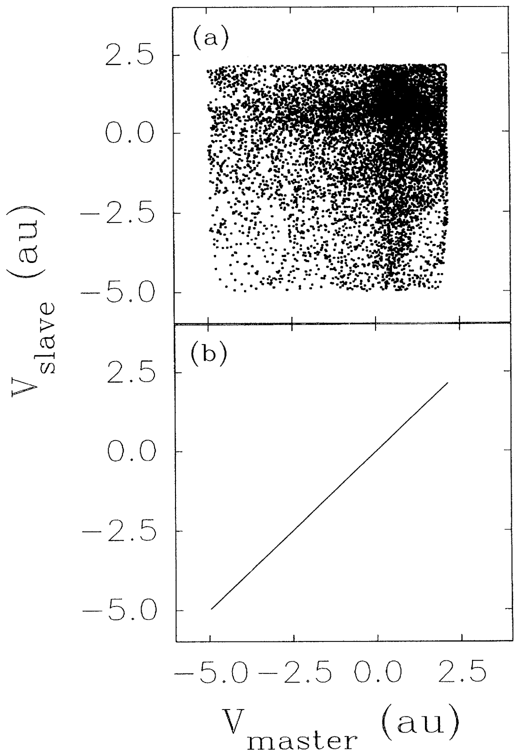

FIG. 3. Value of the master and slave FMR signals in the model plotted against each other without control (a) and in the presence of control (b).

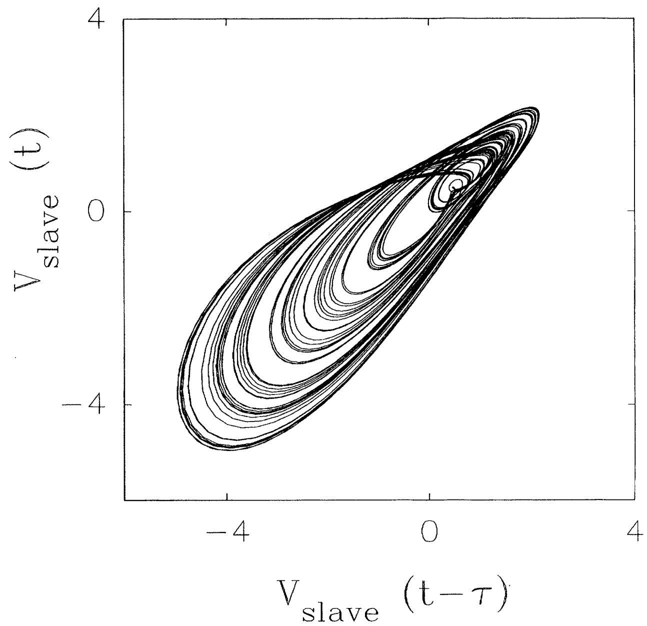

FIG. 4. Delay coordinate plot in arbitrary units of the model slave signal's attractor during synchronization ($\tau = 50$ ns).

To simulate the synchronizing feedback, the model evolved for several hundred microseconds of simulated time, and the output signal of the model was stored. To simulate the application of continuous proportional control in our experiment, the model was restarted, and a perturbation of the form of Eq. (1) was applied, where the master signal was a random segment of the previously stored output and the slave signal was the current state of the simulation. A comparison of the correlation between the master and slave signals in the presence of control is presented in Fig. 3. A delay coordinate plot in Fig. 4 suggests that the dynamics of the model are chaotic during the synchronization. Upon turning on the control, the two systems synchronized within 5–10 μs, in good agreement with the experiment.

In both the experiment and the model, the master and slave signals would still synchronize even if the two signals arose from different experimental conditions. In the experiment, sample heating and drifts in the magnetic field created small differences in the experimental conditions between the time sample output was recorded and when the control was applied. Synchronization occurred to a lesser degree when the experimental conditions were slightly altered from the values that produced optimal synchronization. In the model, master and slave signals calculated from different parameter values in Eq. (2) were able to achieve a significant level of synchronization with the application of control.

To conclude, we have synchronized the chaotic output of a YIG disk in FMR at 1.2 GHz to its prerecorded output. This was achieved through the application of continuous control proportional to the difference between these signals. The chaotic dynamics operate in the megahertz frequency range. The synchronization results in FMR through this technique are consistent with the dynamics of a theoretical model.

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