Low-Temperature Spin Diffusion in a Spin-Polarized Fermi Gas

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We present a finite temperature calculation of the transverse spin-diffusion coefficient $D_\perp$ in a dilute degenerate Fermi gas, in the presence of a small external magnetic field H. While the longitudinal diffusion coefficient displays the conventional Fermi-liquid dependence, $D_\parallel \propto T^{-2}$, the behavior of $D_\perp$ shows three separate regimes: (a) $D_\perp \sim H^{-2}$ for $T \ll H$, (b) $D_\perp \sim T^{-2}$, $D_\perp/D_\parallel \neq 1$ for $T \gg H$ and large spin-rotation parameter $\xi \gg 1$, and (c) $D_\perp = D_\parallel \propto T^{-2}$ for $T \gg H$ and $\xi \ll 1$. Our results are qualitatively consistent with the experimental data in weakly spin-polarized ^{3}He and ^{3}He-^{4}He mixtures.

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The unusual features of spin dynamics in spin-polarized quantum systems have attracted considerable interest, principally motivated by the pioneering work of Leggett and Rice [1] on spin diffusion in normal liquid ^{3}He. The main effect arises from the observation that the presence of a molecular field (induced by the applied magnetic field) leads to an additional precession of the spin current which, in steady state, acquires a component perpendicular to the magnetization gradient; through the continuity equation this results in an anomalous reactive component (damped spin wave) to spin transport. This "spin-rotation" effect is also present in the case of spin-polarized Boltzmann gases [2].

In addition, in the degenerate limit a finite polarization leads to highly anisotropic spin diffusion, as proposed by Meyerovich [3]. This suggestion was recently confirmed by measurements of the transverse spin-diffusion coefficient in weakly polarized liquid ^{3}He [4]. Theoretically, $D_\perp$ was calculated in the dilute gas limit at $T = 0$ [5,6], and only an approximate estimate based on a variational solution of the Boltzmann equation is available for $T \neq 0$ [7,8].

In this Letter, we present an exact low-temperature solution of the kinetic equation for a dilute, weakly polarized Fermi gas, in the s-wave approximation. This allows us to extract analytically (up to a summation which must be done numerically) the finite temperature behavior of $D_\perp$. To the extent to which, in the limit of small polarization, strong interactions only lead to constant renormalizations of the weakly interacting result, we expect that our findings should also apply to the case of weakly polarized ^{3}He. Indeed, as explained below, the detailed low (H,T) behavior of our expression for $D_\perp$ appears to be consistent with the small systematic deviations (lying within the error bars) in the results of Ref. [4] from previous theoretical estimates.

We note that the exact solution of the kinetic equations does not merely lead to quantitative renormalizations of the transport coefficients; rather, it brings out new qualitative effects. In particular, due to the existence of two dimensionless parameters, the low-temperature behavior of transverse diffusion displays two crossovers, the first at $T \sim H$ and the other at $T \propto \sqrt{H\epsilon_F/ap_F}$ (a is the s-wave scattering length, ϵ_F is the Fermi energy, and $p_F = \sqrt{2m\epsilon_F}$). Spin diffusion becomes isotropic (i.e., $D_\perp = D_\parallel$) only for T above the *second* crossover. This latter behavior had already been found in the context of Fermi-liquid kinetic theory in the limit of low polarization ($T \gg H$) [9]. However, the collision integral of Ref. [9] cannot be used to describe spin diffusion in the opposite case, $T \lesssim H$ [10]. The correct high-field limit of $D_\perp$ was not known until the results of Jeon and Mullin [5]. On the other hand, this and subsequent work [7,8] did not contain the correct low-field behavior. The present work gives the exact solution valid in the entire region, $T, H \ll \epsilon_F$.

Our starting point is the kinetic equation for the Wigner transform of the transverse component of the density matrix (in the frame rotating at the bare Larmor frequency), $n_{\uparrow\downarrow}(t,\vec{r},\vec{p})$ in the s-wave approximation, where $t,\vec{r}$ are the center of mass time and space coordinates and $\vec{p}$ is the relative momentum vector:

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\frac{\partial}{\partial \vec{r}}\right) n_{\uparrow\downarrow}(t,\vec{r},\vec{p}) = n_{\uparrow\downarrow}(t,\vec{r},\vec{p}) \int \left\{ i\left[\frac{4\pi a}{m} + B(\vec{p},\vec{p}\,')\right]\left(n^{\uparrow}_{\vec{p}\,'} - n^{\downarrow}_{\vec{p}\,'}\right) - A(\vec{p},\vec{p}\,')\right\} \frac{d^3p'}{(2\pi)^3}$$
$$- \int \left\{ i\left[\frac{4\pi a}{m} + B(\vec{p}\,',\vec{p})\right]\left(n^{\uparrow}_{\vec{p}} - n^{\downarrow}_{\vec{p}}\right) - A(\vec{p}\,',\vec{p})\right\} n_{\uparrow\downarrow}(t,\vec{r},\vec{p}\,') \frac{d^3p'}{(2\pi)^3}. \qquad (1)$$

Here $n^{\uparrow,\downarrow}_{\vec{p}} = \{\exp[(\epsilon_p \mp H/2 - \epsilon_F)/T] + 1\}^{-1}$ are the equilibrium distribution functions for up- and down-spin particles, respectively, and $\epsilon_p = p^2/2m$ (we use energy units for the field H and we set $\hbar = 1$). The two terms on the right-hand side of Eq. (1) correspond to direct and exchange two-particle scattering processes. The functions

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$A(\vec{p},\vec{p}')$ and $B(\vec{p},\vec{p}')$ contain all effects of second order in ap_F and, in the limit of low temperature and small polarization, $T, H \ll \epsilon_F$, are given by

$$A(\vec{p},\vec{p}') \approx \frac{\pi a^2}{p}\left(\frac{(p^2+p'^2)}{m} - 4\epsilon_F\right)\left(n^{\uparrow}_{\vec{p}'} + n^{\downarrow}_{\vec{p}'} - \frac{1}{1-\exp[(p^2+p'^2)/2mT - (2\epsilon_F/T)]}\right), \tag{2}$$

$$B(\vec{p},\vec{p}') \approx \frac{2a^2}{m}\left(-4p_F + |\vec{p}-\vec{p}'|\log\left|\frac{|\vec{p}-\vec{p}'|+|\vec{p}+\vec{p}'|}{|\vec{p}-\vec{p}'|-|\vec{p}+\vec{p}'|}\right|\right). \tag{3}$$

Our Eqs. (1)–(3) were derived by using the Keldysh technique [11] and agree with the low temperature and low polarization limit of the corresponding kinetic equations of Ref. [10].

To compute the transverse spin-diffusion coefficient we will solve the kinetic equations (1) and extract the steady state transverse spin current driven by a constant magnetization gradient. For simplicity we will consider the case in which $\vec{M}_\perp(\vec{r}) \equiv \vec{M}_\perp(x)$ with $\partial\vec{M}_\perp(x)/\partial x =$ const.

With these assumptions the solution of the kinetic equation (1) can be taken to be of the form, $n_{\uparrow\downarrow}(t,\vec{r},\vec{p}) = g(t,x,p) + f(t,x,p)\cos\psi$, where $p = |\vec{p}|$ and $\cos\psi = \vec{p}\cdot\hat{\mathbf{x}}/p$. Performing explicit integrations in Eq. (1) then leads to the two coupled kinetic equations,

$$\frac{\partial g}{\partial t} + \frac{p}{2m}\frac{\partial f}{\partial x} + \frac{2a}{\pi m}i\int_0^\infty g(t,x,q)\,q^2\,dq\left(n^{\uparrow}_{\vec{p}} - n^{\downarrow}_{\vec{p}}\right) - \frac{4\pi a}{m}i(N_\uparrow - N_\downarrow)\,g = -I_{\rm rel}[g] - iI_{\rm sr}[g] \tag{4}$$

and

$$\frac{\partial f}{\partial t} + \frac{p}{m}\frac{\partial g}{\partial x} - \frac{4\pi a}{m}i(N_\uparrow - N_\downarrow)\,f = -\frac{I_{\rm rel}[f\cos\psi] + iI_{\rm sr}[f\cos\psi]}{\cos\psi} \equiv -I_{\rm rel1}[f] - iI_{\rm sr1}[f]. \tag{5}$$

In (4) and (5) $I_{\rm rel}$ and $I_{\rm sr}$ represent the relaxational and spin-rotational parts of the "collision integral,"

$$I_{\rm rel}[W(\vec{p})] = W(\vec{p})\int A(\vec{p},\vec{p}')\frac{d^3p'}{(2\pi)^3} - \int A(\vec{p}',\vec{p})W(\vec{p}')\frac{d^3p'}{(2\pi)^3}, \tag{6}$$

$$I_{\rm sr}[W(\vec{p})] = -W(\vec{p})\int B(\vec{p},\vec{p}')\left(n^{\uparrow}_{\vec{p}'} - n^{\downarrow}_{\vec{p}'}\right)\frac{d^3p'}{(2\pi)^3} + \left(n^{\uparrow}_{\vec{p}} - n^{\downarrow}_{\vec{p}}\right)\int B(\vec{p},\vec{p}')W(\vec{p}')\frac{d^3p'}{(2\pi)^3}, \tag{7}$$

As can be shown by analyzing the eigenvalue spectrum of the relaxational part of the collision integral $I_{\rm rel1}[f]$, the spin current decays to its steady-state value in a microscopic time scale $\tau_\perp$ beyond which the time derivative terms in Eqs. (4) and (5) can be omitted. In addition, since $\partial\vec{M}_\perp(x)/\partial x =$ const or equivalently $\partial g/\partial x$ is independent of x, Eq. (5) implies $\partial f/\partial x = 0$. The solution of (4) can then be shown to take the form $g_0(x,p) = G(x)(n^{\uparrow}_{\vec{p}} - n^{\downarrow}_{\vec{p}})$, in which case the equation for $f(p)$ becomes

$$v_F\frac{\partial G}{\partial x}\left(n^{\uparrow}_{\vec{p}} - n^{\downarrow}_{\vec{p}}\right) - i\Omega^{(1)}f(p) = -I_{\rm rel1}[f(p)] - iI_{\rm sr1}[f(p)]. \tag{8}$$

Here, v_F is the Fermi velocity and $\Omega^{(1)} = (8\pi a/m)M_\parallel \approx 2ap_FH/\pi$ represents the leading correction to the precession frequency of the spin current, $J^-_\perp \equiv J^x_\perp - iJ^y_\perp = \int_0^\infty f(p)p^3\,dp/12\pi^2m$ ($M_\parallel$ is the longitudinal magnetization induced by the field H). Note that no such correction appears in the equation for the transverse magnetization itself, $M^-_\perp \equiv M^x_\perp - iM^y_\perp = \int_0^\infty g(p)p^2\,dp/4\pi^2$, i.e., there is no renormalization of Larmor precession, as expected from rotational invariance of the interparticle interactions.

To identify the diffusion coefficient we want to match the conventional form [1,12] of the macroscopic constitutive relation, $J^-_\perp = -[D_\perp/(1-i\xi)]\frac{\partial}{\partial x}M^-_\perp$, to the one implied by the microscopic equation (8). Here, $D_\perp$ is the transverse spin-diffusion coefficient, while ξ is referred to as the "spin-rotation" parameter. The nonzero imaginary part reflects the fact that, due to the spin-rotation effect, the spin current is not parallel (in spin space) to the driving magnetization gradient. We recall that in the relaxation-time approximation [1] $\xi \approx \Omega^{(1)}\tau_{\rm relax}$, where $\tau_{\rm relax}$ is the corresponding relaxation time. We note from the outset that, except in the high-field regime, $H \gg T$, the relaxation-time approximation breaks down and the above parametrization of ξ is inapplicable. We are now in position to compute $D_\perp$ and ξ as functions of T and H. As we show below, the calculation is tractable analytically in both "high"- and "low"-field limits:

High-Field Behavior, $H/T \gg (ap_F)(T/\epsilon_F)$.—In this case the relaxational term in (8) is small, $I_{\rm rel1}[f] \ll \Omega^{(1)}f$, and the current $\vec{J}_\perp$ is almost perpendicular to the magnetization gradient. In this limit the solution of Eq. (8), $f_D(p)$, can be obtained iteratively (in powers of $1/\Omega^{(1)}$), leading upon integration to a corresponding iterative solution for $J^-_\perp$. In turn, using the constitutive relation immediately yields,

$$D_\perp = \frac{v_F^2\tau_D}{3} = \frac{3\pi v_F^2}{8ma^2(H^2+4\pi^2T^2)}, \tag{9}$$

$$\xi = (\Omega^{(1)} + \Omega^{(2)})\tau_D = \frac{9Hv_F}{4a(H^2+4\pi^2T^2)}\left[1 + \frac{2}{5\pi}ap_F\left(1 - \frac{4}{3}\log 2\right)\right], \tag{10}$$

where $\Omega^{(2)}$ is the second order contribution to the spin-current precession frequency.

The first thing to stress is that the behavior of $D_\perp$ in (9) differs significantly from that of the longitudinal spin-diffusion coefficient [13], $D_\parallel \approx (v_F^2/8\pi m a^2 T^2)C(-1/3)$, where $C(-1/3) \approx 0.843$ is the Brooker-Sykes coefficient [14]. The origin of this effect is the difference in phase space restrictions associated with the scattering processes leading to transverse and longitudinal spin diffusion in the high-field limit. While collisions responsible for longitudinal spin diffusion are restricted to energies within $k_B T$ of each of the ("up" or "down") Fermi surfaces, those leading to transverse spin diffusion involve spin-flip processes which can also take advantage of the full region enclosed *between* the two surfaces [5,7]. Thus, the expression for $D_\perp$ involves the phase space for scattering generated both by the magnetic field as well as by the thermal smearing of the individual Fermi surfaces. As a result, the transverse diffusion coefficient in (9) remains *finite* at $T = 0$ [3,4].

Also, for $T \gg H$ [but still in the high-field limit, $H \gg (ap_F)T^2/\epsilon_F$] the transverse spin-diffusion coefficient $D_\perp \approx (3v_F^2/32\pi m a^2 T^2) \approx 0.890 D_\parallel$ [9] still differs from the longitudinal one. As we will see below, $D_\perp$ and $D_\parallel$ become equal only at much lower fields, $H \sim (ap_F)T^2/\epsilon_F$. Finally, note that the diffusion time τ_D is not the relaxation time of the spin current usually involved in the phenomenological discussions: The latter describes the relaxation of the distribution function toward the steady state solution, $f_D(p)$, and would arise as the eigenvalue of the relaxational part of the collision operator [11].

Crossover and Low-Field Behavior, $H/T \lesssim (ap_F)T/\epsilon_F$.—In this region, $I_{\rm rel1}[f(p)] \sim \Omega^{(1)} f(p) \gg I_{\rm sr1}[f(p)]$ and, therefore, the spin-rotation terms in the collision integral may be omitted. Furthermore, one may also set $H = 0$ in evaluating the functional $I_{\rm rel1}$. By using methods of Ref. [14], the steady state equation (8) [in terms of reduced variables, $\eta = (\epsilon_p - \epsilon_F)/T$, $h = H/2T \ll 1$, and $\Gamma_0 = 2a^2 m/3\pi$] can be transformed into the differential equation,

$$F''(k) - \pi^2\gamma^2 F(k) - \frac{2}{3}F(k)\mathrm{sech}^2\pi k = \frac{\pi h v_F}{3\Gamma_0 T^2}\frac{\partial G}{\partial x}\frac{\cos kh}{\cosh \pi k}, \tag{11}$$

for the function $F(k) = \int_{-\infty}^{\infty} e^{ik\eta} f(\eta)\cosh(\eta/2)\,d\eta$. Here $\gamma^2 = 1 - i\Omega^{(1)}/3\pi^2\Gamma_0 T^2 \approx 1 - 2iH\epsilon_F/\pi^2 T^2 a p_F$.

The solutions to (11) may be expressed in terms of Gegenbauer polynomials [15] with a complex index, $F(k) = \sum_{n=0}^{\infty} F_n \phi_n(k)$, $\phi_n(k) = (\cosh \pi k)^{-\gamma} C_n^{\gamma+1/2}(\tanh \pi k)$, where

$$F_n = -\frac{v_F h}{\pi^2\Gamma_0 T^2}\frac{\partial G}{\partial x}\frac{g_n}{\frac{2}{3} + (\gamma + n)(\gamma + n + 1)} \tag{12}$$

and

$$g_n = \frac{\pi^{5/2}\left(n + \gamma + \frac{1}{2}\right)\Gamma\left(\gamma + \frac{1}{2}\right)}{3\cos\frac{\pi\gamma}{2}\Gamma\left(\frac{1}{2} - \frac{n}{2}\right)\Gamma\left(1 + \gamma + \frac{n}{2}\right)\Gamma\left(1 + \frac{\gamma}{2} + \frac{n}{2}\right)\Gamma\left(\frac{1}{2} - \frac{\gamma}{2} - \frac{n}{2}\right)}. \tag{13}$$

The transverse spin current can then be evaluated as $J_\perp^- = (Tp_F^2/12\pi^2)\int_{-\infty}^{\infty} dk\, F(k)/\cosh \pi k = (Tp_F^2/12\pi^2)\sum_{n=0}^{\infty} F_n a_n$, with

$$a_n = \frac{\pi\Gamma(\gamma + 1)\Gamma(2\gamma + n + 1)}{n!\cos\frac{\pi\gamma}{2}\Gamma(2\gamma + 1)\Gamma\left(\frac{1}{2} - \frac{n}{2}\right)\Gamma\left(\gamma + \frac{n}{2} + 1\right)\Gamma\left(\frac{\gamma}{2} + \frac{n}{2} + 1\right)\Gamma\left(\frac{1}{2} - \frac{n}{2} - \frac{\gamma}{2}\right)}. \tag{14}$$

This allows us to write the final expression

$$\frac{D_\perp}{1 - i\xi} = \frac{1}{4\pi}\frac{v_F^2}{m a^2 T^2}\sum_{n=0}^{\infty}\frac{g_n a_n}{\frac{2}{3} + (\gamma + n)(\gamma + n + 1)}, \tag{15}$$

from which we can extract $D_\perp$ and the spin-rotation parameter ξ. The low-field limit, $H \ll (ap_F)T^2/\epsilon_F$ corresponds to $\gamma \to 1$ in which case (15) yields, $D_\perp = D_\parallel = (v_F^2/8\pi m a^2 T^2)C(-1/3)$. In this limit the spin-rotation parameter is given by $\xi \approx 0.139 H\epsilon_F/T^2(ap_F) \ll 1$, reflecting the fact that the spin current becomes parallel (in spin space) to the driving magnetization gradient. The detailed behavior of $D_\perp$ as a function of T/H, which clearly displays the two crossovers discussed above, is shown in Fig. 1. We note that our $T = 0$ results for $D_\perp$ agree with those of the calculations of Ref. [5,6] and are also expected to agree with the finite temperature variational expression of Ref. [7] for $H \gg (ap_F)T^2/\epsilon_F$. However, the exact solution departs from the variational ansatz at $H \sim (ap_F)T^2/\epsilon_F$, leading to $D_\perp/D_\parallel < 1$.

Although the calculations presented above are restricted to low densities, we expect that our qualitative arguments, based on the existence of *two* independent parameters, ξ and H/T (renormalized by the appropriate Fermi-liquid parameters), should also hold in the Fermi-liquid regime, $T, H \ll \epsilon_F$ (see the low-field discussion of Ref. [9]). The s-wave approximation, which made our analytical calculations possible, raises more serious issues,

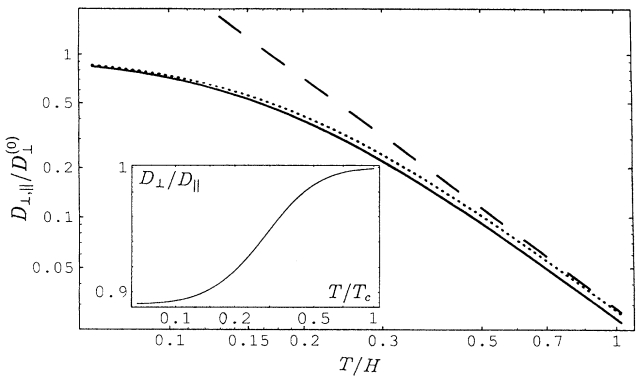

FIG. 1. Spin diffusion coefficient in the high-field region $H \gtrsim T$. The transverse spin diffusion coefficient $D_\perp$ [Eq. (9)], the longitudinal coefficient $D_\parallel$, and the simple fit $(1/D_\perp^{(0)} + 1/D_\parallel)^{-1}$ [4,8] for $D_\perp$ ($D_\perp^{(0)} = 3\pi v_F^2/8ma^2H^2$ being the limiting value of $D_\perp$ at $T \to 0$) are represented by the solid, dashed, and dotted lines, respectively. The inset shows the ratio $D_\perp/D_\parallel$ near the second (low-field) crossover, $T \sim T_c = \sqrt{H\epsilon_F/ap_F}$.

especially concerning the detailed behavior of $D_\perp$ in the crossover regions. Nevertheless, we expect that the presence of two crossovers survives in the presence of higher partial waves.

Some comments are in order concerning the possible relevance of our findings to experiment. Even though, strictly speaking, our analysis does not apply to the strongly interacting case, it is worth noting that the available experimental data in weakly polarized ^{3}He [4] deviate systematically from the simple theoretical fit which uses a single adjustable parameter (T_a in [4]) to cover the entire temperature range, including both $\xi > 1$ and $\xi < 1$. Much better agreement is obtained by restricting the fit to the $\xi \leq 1$ region with an overall prefactor smaller than the one implied by fitting to the value of $D_\parallel$ in the low-field, high-temperature regime. This is consistent with our picture, with $D_\perp < D_\parallel$ for $\xi \geq 1$ and $T \gg H$. In addition, although it appears that the region between the two crossovers cannot be clearly identified—most likely due to large Fermi liquid renormalization effects—the isotropic limit is indeed reached in the regime $\xi < 1$ [4]. In principle, our calculations should be more relevant to the measurements in dilute ^{3}He-^{4}He mixtures. Although in the available data (for 0.18% ^{3}He) the crossover to the isotropic limit occurs for $\xi \sim 1$ with $H \ll T$, the temperature is not sufficiently far below ϵ_F, and, moreover, the polarization is somewhat high, ~25%. Nevertheless, for $\xi > 1, D_\perp \propto D_\parallel$ with the ratio $D_\perp/D_\parallel$ slightly less than unity [16,17]. Also, the measured T dependence of the spin-rotation parameter ξ near the crossover to the isotropic (low-field) limit is qualitatively consistent with our results in both the data of Ref. [16] and those obtained in the degenerate regime of more concentrated solutions (2.6% ^{3}He) [18] with lower polarization (~2%). In both situations, the crossover to the isotropic regime can be clearly distinguished. The large discrepancy in the magnitude of the shift of ξT^2 in the latter case can be attributed to Fermi liquid renormalizations anticipated in high concentration solutions. To sharpen the identification of two crossovers the data of Ref. [18] should be extended to lower temperatures (to study the $H/T \sim 1$ behavior). Quantitative comparisons could be made only in the more dilute case of Ref. [16] where lower field and lower temperature experiments should be performed.

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[1] A. J. Leggett and M. J. Rice, Phys. Rev. Lett. **20**, 586 (1968); A. J. Leggett, J. Phys. C **3**, 488 (1970).
[2] See for example, A. G. Aronov, JETP **46**, 301 (1977); A. E. Ruckenstein and L. P. Lévy, Phys. Rev. B **39**, 183 (1989), and references therein.
[3] A. E. Meyerovich, Phys. Lett. **107A**, 177 (1985).
[4] L-J. Wei *et al.*, Phys. Rev. Lett. **71**, 879 (1993).
[5] J. W. Jeon and W. J. Mullin, Phys. Rev. Lett. **62**, 2691 (1989).
[6] A. E. Meyerovich and K. A. Musaelian, Phys. Rev. B **47**, 2897 (1993); J. Low Temp. Phys. **94**, 249 (1994).
[7] W. J. Mullin and J. W. Jeon, J. Low Temp. Phys. **88**, 433 (1992).
[8] A. E. Meyerovich, and K. A. Musaelian, J. Low Temp. Phys. **95**, 789 (1994); Phys. Rev. Lett. **72**, 1710 (1994).
[9] A. Pal and P. Bhattacharyya, J. Low Temp. Phys. **51**, 265 (1983); K. S. Bedell and D. E. Meltzer, Phys. Rev. B **33**, 4543 (1986); **34**, 3475(E) (1986).
[10] J. W. Jeon and W. J. Mullin, J. Phys. (Paris) **49**, 1691 (1988).
[11] D. I. Golosov and A. E. Ruckenstein (unpublished).
[12] A. E. Meyerovich, in *Helium Three*, edited by W. P. Halperin and L. P. Pitaevskii (North-Holland, Amsterdam, 1990), p. 757, and references therein.
[13] W. J. Mullin and K. Miyake, J. Low Temp. Phys. **53**, 313 (1983).
[14] G. A. Brooker and J. Sykes, Phys. Rev. Lett. **21**, 279 (1968); G. A. Brooker and J. Sykes, Ann. Phys. (N.Y.) **56**, 1 (1970).
[15] The functions $\phi_n(k)$ are solutions to the homogeneous equation $F''(k) - [\pi^2\gamma^2 + (\gamma + n)(\gamma + n + 1)\mathrm{sech}^2\pi k]F(k) = 0$.
[16] D. Candela *et al.*, Phys. Rev. B **44**, 7510 (1991).
[17] G. Nunes, Jr. *et al.*, Phys. Rev. B **46**, 9082 (1992).
[18] J. R. Owers-Bradley *et al.*, Physica (Amsterdam) **165&166B**, 729 (1990).