Nonlinear Hybrid Simulation of the Toroidicity-Induced Alfvén Eigenmode

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Gyrokinetic-magnetohydrodynamic hybrid simulations have been carried out to study the nonlinear saturation of the torodicity-induced Alfvén eigenmode driven by energetic particles in a tokamak plasma. It is shown that wave-particle trapping is the nonlinear saturation mechanism for the parameters considered. The corresponding density profile flattening of the hot particles is observed. The saturation amplitude is proportional to the square of the linear growth rate. In addition, a new $n = 1$, $m = 0$ global Alfvén eigenmode is shown to be excited by the energetic particles.

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The toroidicity-induced Alfvén eigenmode (TAE) has received a great deal of attention recently in the fusion research community. Experiments have shown that the TAE mode can be strongly destabilized by energetic ions introduced by neutral beam injection heating or ion cyclotron radio frequency wave heating in tokamak plasmas. The instability is driven by the free energy associated with the fast-particle density gradient through the wave-particle resonant interaction. The unstable TAE mode may in turn lead to large losses of energetic particles. This will have important implications for the design of tokamak reactors where the fusion-product alpha particles have velocities comparable to the Alfvén phase velocity and have a density profile peaked near the center of plasma. The lost alpha particles induced by the TAE instability can seriously damage the tokamak first wall. At present, D-T experiments are being carried out in the Tokamak Fusion Test Reactor (TFTR) [1] to study the influence of fusion alpha particles on the TAE modes. Thus, it is of interest to assess the nonlinear saturation of the TAE modes. We note that the general physical model used in this work may also be applied to space plasma phenomena, such as satellite observations of compressional waves in the Earth's magnetosphere [2].

In this work, nonlinear gyrokinetic-magnetohydrodynamic (MHD) hybrid simulations have been carried out using the MH3D-K code [3] to study the TAE instability driven by energetic particles in a tokamak plasma. The aim of this study is to determine the nonlinear saturation mechanism and associated fast-particle transport. In our model, the plasma is divided into two parts, the bulk part, which contains thermal electrons and ions, and the energetic part of the hot ions. The bulk part is described by the ideal MHD equations, whereas the hot ions are described by the gyrokinetic equations [4]. The effects of hot ions are coupled to the motion of the bulk part through the stress tensor term in the momentum equation as follows:

$$\rho_b \frac{d\mathbf{v}_b}{dt} = -\nabla P_b - \nabla \cdot \mathbf{P}_h + \mathbf{J} \times \mathbf{B}, \qquad (1)$$

where the subscript b denotes the bulk part, the subscript h denotes the hot ion component, P_b is the isotropic pressure of the bulk plasma, and $\mathbf{P}_h$ is the pressure-stress tensor of the hot ions and is given by

$$\mathbf{P}_h = P_\perp \mathbf{I} + (P_\parallel - P_\perp)\mathbf{bb} \qquad (2)$$

and

$$\begin{pmatrix} P_\parallel \\ P_\perp \end{pmatrix} = \int d^3v\, F_h \begin{pmatrix} MU^2 \\ \mu B \end{pmatrix}. \qquad (3)$$

Here, $\mathbf{b}$ is the unit vector along the magnetic field line, $F_h = F_h(\mathbf{R}, U, \mu)$ is the distribution function of the hot ions with $\mathbf{R}$ being the position of the guiding centers, U the parallel velocity, and μ the magnetic moment. This distribution function is calculated by following a set of particles using the gyrokinetic equation with the self-consistent electromagnetic field. Equation (1) is closed by the ideal MHD Ohm's law and the Maxwell equations. The system of equations is solved as an initial-value problem. We note here that our model is fully self-consistent, including self-consistent effects of hot particles on the MHD dynamics and the nonlinear MHD mode coupling.

In the simulations, we used the following parameters and profiles for most cases: the aspect ratio $R/a = 6$, the safety factor profile $q = 1.2 + 0.9(r/a)^2$, the plasma density profile $\rho = \rho_0[1 - 0.8(r/a)^2]$, the bulk plasma beta $\beta_b(0) = 0.2\%$, and the hot particle beta $\beta_h(0)$ is of the order of percent. The particle velocity distribution is a Maxwellian in the parallel direction with mean value equal to the Alfvén speed at the center of the plasma. The perpendicular velocity is negligible. The number of particles used ranges from 20 000 to 200 000.

We first describe our simulation results in the linear regime. Figure 1(a) shows a contour plot of the stream function u of the perturbed fluid velocity for a converged linear $n = 1$ TAE mode. In the figure, the sign of the stream function on the solid lines is opposite to that on the dotted lines. The poloidal structure of the contour clearly shows the presence of $m = 1$ and $m = 2$ modes. The corresponding radial eigenmode structure is shown in Fig. 1(b) for six poloidal modes from $m = 0$ to $m = 5$ with the two most dominating modes marked. The real frequency (normalized to the Alfvén frequency v_A/R) is

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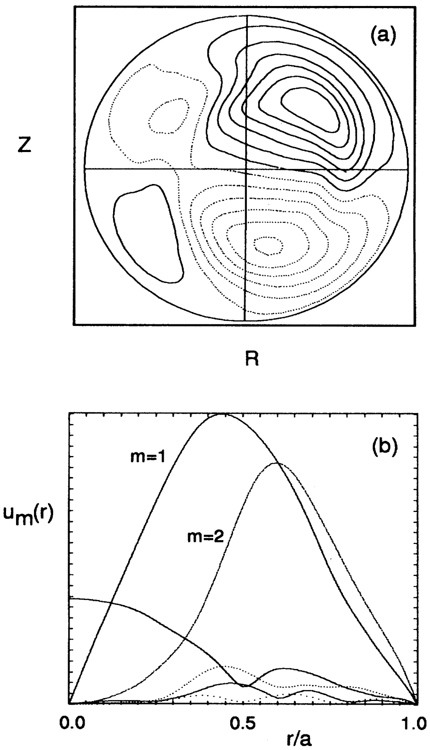

FIG. 1. (a) Contour plot of the fluid velocity stream function u; the solid lines denote equal positive values while the dotted lines denote equal negative values. (b) The fluid velocity stream function $u_m(r)$ versus the normalized radius r/a for six poloidal modes from $m = 0$ to $m = 5$.

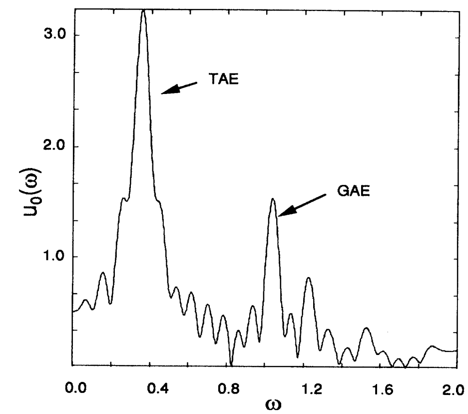

FIG. 2. Frequency spectrum of the fluid velocity function $u_0(\omega)$ for the $n = 1$, $m = 0$ mode.

$\omega = 0.37$. The hot-particle-induced linear growth rate γ_h is $\gamma_h/\omega = 0.14$. The results are obtained with a central hot particle beta of $\beta_h(0) = 3.3\%$ and 200 000 particles. The calculated real frequency, mode structure, and growth rate agree reasonably well with the results of a linear kinetic MHD eigenvalue code NOVA-K [5,6], although our numerical equilibrium with self-consistent anisotropic effects is not identical to that used in the NOVA-K code. Our hybrid code has also been benchmarked using energy conservation. We have derived analytically an energy conservation law from our physical model. Our numerical simulations show that the total energy is indeed well conserved.

In addition to TAE modes, a new global Alfvén eigenmode (GAE) is found to be excited by hot particles in the linear regime. Figure 2 shows the frequency spectrum of the velocity stream function for the $m = 0$ poloidal component. We see that, besides the usual TAE peak at $\omega_{\mathrm{TAE}} = 0.37$, there is another peak at $\omega = 1.05$, which is about 3 times the TAE frequency. This high frequency mode is identified as the $n = 1$, $m = 0$ GAE mode with frequency close to the minimum of the $n = 1$, $m = 0$ shear Alfvén continuum at the plasma center. Our results also indicate that the GAE mode persists in the nonlinear régime. This new GAE mode may be related to the experimental results in TFTR [7], where a high frequency mode was observed with frequency about 3 times the TAE frequency. It should be noted that other smaller peaks beside two main peaks in Fig. 2 are generated by simulation noise and sampling error.

In the nonlinear regime, our results indicate that wave-particle trapping is the nonlinear saturation mechanism of the TAE mode [8]. Figure 3 shows the nonlinear evolution of the kinetic energy of the bulk part for an $n = 1$ TAE mode for two cases, one with a single $n = 1$ perturbation (dashed line), and the other with multiple-n perturbations, including $n = 0$, 1, and 2 (solid line). We see that the saturation level with a single-n case is approximately the same as the multiple-n case. Thus, our results indicate that nonlinear mode coupling is not responsible for the TAE saturation, at least for the parameters studied here. Other possible saturation mechanisms include stochastic particle loss and particle trapping by nonlinear waves. First, large resonant particle losses can be induced when the TAE amplitude exceeds a

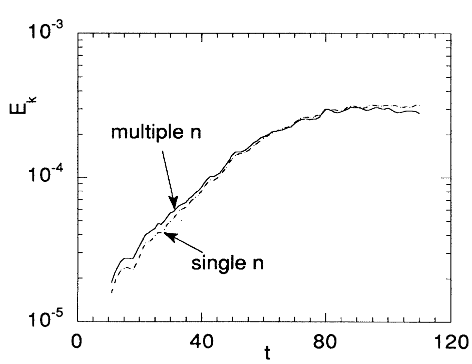

FIG. 3. Nonlinear evolution of E_k for a single $n = 1$ perturbation (dashed line) and for multiple-n perturbation (solid line).

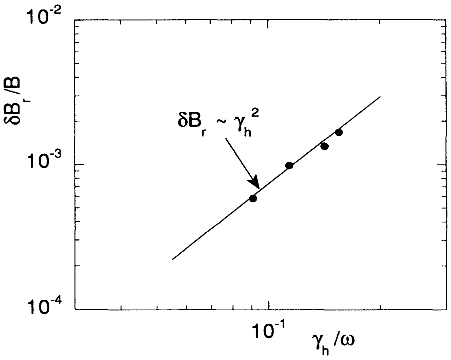

FIG. 4. The saturation amplitude of the normalized perturbed radial magnetic field $\delta B_r/B$ as a function of the normalized linear growth rate γ_h/ω.

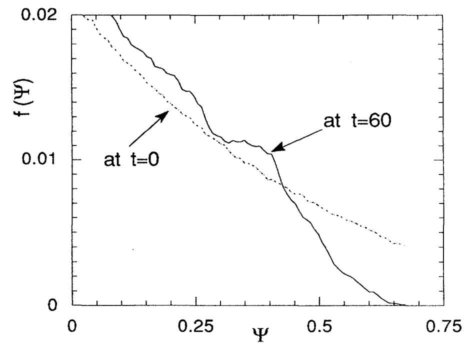

FIG. 5. The particle distribution functions versus radial variable poloidal flux Ψ at $t = 0$ (dashed line) and at $t = 60R/v_A$ (solid line).

stochastic threshold such that the particle orbit becomes chaotic. When this happens, the hot particle drive is diminished and the mode saturates. This scenario is presumed to be applicable when the hot particle drive is sufficiently large [9]. Second, when the hot particle drive is smaller, the TAE mode can still saturate due to particle trapping by nonlinear waves, as suggested in Ref. [10]. In this scenario, the resonant particles can be trapped in the TAE wave. The corresponding nonlinear bounce frequency ω_b is proportional to the square root of the mode amplitude. As a consequence of nonlinear trapping, the hot particle density profile is flattened locally on the $1/\omega_b$ time scale and the hot particle drive decreases to zero. Correspondingly, the mode saturates at $\omega_b \sim \gamma_h$, where γ_h is the linear growth rate induced by the hot particles. This implies that the saturation amplitude scales as the linear growth rate squared. Our numerical results support this wave-particle trapping mechanism. Figure 4 shows the saturation level $\delta B_r/B$ as a function of the normalized linear growth rate γ_h/ω, where δB_r is the perturbed radial magnetic field for the $m = 2$ poloidal mode. We see that our numerical results (solid dots) agree well with the expected scaling $\delta B_r \propto \gamma_h^2$ which is indicated by the solid line. For our parameters, it can be estimated that $\omega_b/\omega \approx 3\sqrt{\delta B_r/B}$. Thus, in terms of ω_b, the saturation level is given by $\omega_b \approx 1.2\gamma_h$. Figure 5 shows the hot particle distribution functions versus the normalized poloidal flux Ψ at $t = 0$ (dashed line) and at $t = 60R/v_A$ (solid line). There is clearly a local flattening near the $\Psi = 0.4$ surface where the wave-particle resonance condition is satisfied. The results of Fig. 5 are obtained by using a fixed saturation level $\delta B_r/B = 2 \times 10^{-3}$ and by using a slice of the total distribution with the parallel velocity ranging from $0.9v_A$ to $1.1v_A$ in order to observe a clear local flattening. A similar result of local distribution flattening has also been obtained in Ref. [11] using a quasilinear kinetic dispersion relation.

In conclusion, we have demonstrated using a fully self-consistent simulation that wave-particle trapping is the dominating mechanism for the TAE saturation. In addition to TAE modes, a new $n = 1$, $m = 0$ global Alfvén eigenmode is shown to be excited by the energetic particles. Future work will assess the possible importance of other TAE saturation mechanisms in a broader range of parameter space.

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[1] R. J. Hawryluk *et al.,* Phys. Rev. Lett. **72**, 3530 (1994).
[2] K. Takahashi, Adv. Space Res. **8**, 427 (1988); L. Chen and A. Hasegawa, J. Geophys. Res. **96**, 1503 (1991).
[3] W. Park *et al.,* Phys. Fluids B **4**, 2033 (1992).
[4] W. W. Lee, J. Comput. Phys. **72**, 243 (1987).
[5] C. Z. Cheng, Phys. Rep. **211**, 1 (1992).
[6] G. Y. Fu, C. Z. Cheng, and K. L. Wong, Phys. Fluids B **5**, 4040 (1993).
[7] K. L. Wong *et al.,* Phys. Fluids B **4**, 2122 (1992).
[8] G. Y. Fu and W. Park, in *Abstracts of the 1994 International Sherwood Fusion Theory Conference, Dallas, TX* (Institute for Fusion Studies, Austin, TX, 1994).
[9] W. W. Heidbrink *et al.,* Phys. Fluids B **5**, 2176 (1993).
[10] H. L. Berk, B. N. Breizman, and H. Ye, Phys. Rev. Lett. **68**, 3563 (1992).
[11] Y. Yu and R. White, Phys. Plasmas **1**, 2733 (1994).