## Are Multifragment Emission Probabilities Reducible to an Elementary Binary Emission Probability

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Experimental intermediate-mass-fragment multiplicity distributions for the E/A = 80 and 110 MeV  $^{36}$ Ar +  $^{197}$ Au reactions are shown to be binomial at all excitation energies. From these distributions, a single binary event probability p can be extracted that has a thermal dependence. Thus, it is inferred that multifragmentation is reducible to a combination of nearly independent emission processes. If sequential decay is assumed, the increase of p with excitation energy implies a contraction of the time scale that is qualitatively consistent with recent fragment-fragment correlation data.

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At low excitation energies, complex fragments are emitted with low probability by a compound nucleus mechanism [1,2]. At increasingly larger energies, the probability of complex fragment emission increases dramatically, until several fragments are observed within a single event [3–5]. The nature of this multifragmentation process is at the center of much current attention. For example, the time scale of fragment emission and the associated issue of sequentiality vs simultaneity are the objects of intense theoretical [3–8] and experimental [9–17] study. Recent experimental work [18,19] has shown that the excitation functions for the production of two, three, four, etc. fragments give a characteristically linear Arrhenius plot [20], suggesting a statistical energy dependence.

A fundamental issue, connected in part to those mentioned above, is that of reducibility: Can multifragmentation be reduced to a combination of (nearly) independent emissions of fragments? More to the point, can the probability for the emission of n fragments be reduced to the emission probability of just one fragment?

In what follows, we show evidence that the *n*-fragment emission probabilities are indeed reducible to an elementary binary emission probability. Furthermore, we shall show that the energy dependence of the extracted elementary probabilities gives a linear Arrhenius plot. Thus, these probabilities are likely to be thermal. While reducibility does not strictly imply time sequentiality, we point out in the following the time implications associated with a temporal reading of a reducible thermal theory.

The partial decay width  $\Gamma$  associated with a given binary channel can be approximated by

$$\Gamma = \hbar \omega_0 e^{-B/T},\tag{1}$$

where  $\omega_0$  is a frequency characteristic of the channel under consideration, *B* is the barrier associated with the channel, and *T* is the temperature. For instance, in fission,

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 $\omega_0$  is the collective frequency of assault on the barrier and *B* is the fission barrier.

The elementary probability p for a binary decay to occur at any given "try" defined by the channel period  $\tau_0 = 1/\omega_0$  is

$$p = \frac{\Gamma}{\hbar\omega_0} = e^{-B/T}.$$
 (2)

The corresponding time  $\tau$  is given by

$$\tau = \tau_0 e^{B/T}.$$
 (3)

In the case of a compound nucleus, the total decay width is the sum of the widths of all channels, and the lifetime is calculated accordingly. For the case of sequential multifragmentation, only the decay width and lifetime for binary fragment formation need be considered, while the abundant light particle decay can be treated as a background that may progressively modify the temperature and possibly the barrier.

Now, we note that the elementary binary probability p can be directly related to the experimental branching ratios for binary, ternary, quaternary, etc. decay. For simplicity, let us assume that the system has the opportunity to try m times to emit an "inert" fragment with constant probability p. The probability  $P_n^m$  of emitting exactly n fragments is given by the binomial distribution

$$P_n^m = \frac{m!}{n! (m-n)!} p^n (1-p)^{m-n}.$$
 (4)

The average multiplicity and variance are then

$$\langle n \rangle = mp \text{ and } \sigma_n^2 = \langle n \rangle (1-p).$$
 (5)

It should be pointed out that this is a rather special way to build multifragment probabilities from binary probabilities. It has been chosen *a posteriori* because it happens to work extremely well. Other ways associated

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with different decay branchings (e.g., each produced fragment can, in turn, decay into two fragments with probability p) yield nonbinomial distributions.

From the experimental values of  $\langle n \rangle$  and  $\sigma_n^2$  one can extract values for p and m at any excitation energy. Alternatively, one can extract p from the ratio of any pair of excitation functions  $P_n^m(T)$ :

$$\frac{1}{p} = \frac{\tau}{\tau_0} = \frac{P_n^m}{P_{n+1}^m} \frac{m-n}{n+1} + 1.$$
 (6)

We now proceed to examine the experimental data for signatures of reducibility. References [21] and [22] report values of  $\langle n \rangle$  and  $\sigma_n^2$  for the reaction  ${}^{36}\text{Ar} + {}^{197}\text{Au}$  at E/A = 80 and 110 MeV (available center-of-mass energy of 2.4 and 3.3 GeV, respectively) as a function of the transversal energy  $E_t$ . It is defined as  $E_t = \sum \varepsilon_i \sin^2 \theta_i$ , where  $\varepsilon_i$  is the kinetic energy of each detected particle in an event and  $\theta_i$  is the angle between the particle and the beam direction. We choose the transversal energy as our observable and assume that it is proportional to the excitation energy E of the source [23,24], where  $E_t \cong K(E_{\text{beam}}, A_p, A_T) E$ .

From Eq. (5), we extract the elementary probability pand m from the mean and variance of the experimental fragment multiplicity distributions [21,22]. At this point we need to consider the effect of the device efficiency  $\varepsilon$  on the fold probabilities, the mean multiplicity and its variance, and, finally, on the observed probability  $p_{obs}$ . Disregarding details associated with anisotropies, multiple hits, etc. we can estimate that the true probability p is related to the observed probability  $p_{obs}$  by the relationship  $p_{obs} = \varepsilon p$ . This observed probability  $p_{obs}$  should combine exactly like p in the binomial expressions [Eqs. (4)– (6)]. The geometric efficiency of the Miniball is 0.89 [25] and represents an upper limit for the device efficiency. The derived values of  $p_{obs}$  should be corrected by the device efficiency  $\varepsilon$  to obtain the physical probability p.

In Fig. 1 we plot *m* as a function of  $E_t$  for the intermediate mass fragments (IMF:  $3 \le Z \le 20$ ) multiplicity distributions (circles), and for the total charged particle multiplicity distributions (diamonds). In Fig. 2(a), we plot  $\log(1/p)$  vs  $E_t^{-1/2}$  for the IMF distributions (Arrhenius plot). If the probability *p* is thermal, as given in Eq. (2), this plot should be linear [18], since  $T \propto \sqrt{E}$ . The linearity of this plot over 2 orders of magnitude is stunning, and strongly suggests the "thermal" nature of *p*. The straight lines obtained for the two bombarding energies suggest that the simple proportionality law between  $E_t$  and *E* is satisfied. The difference in slopes suggests that the proportionality constant is bombarding energy dependent.

One can also extract p "differentially" [Eq. (6)] by considering the ratios  $P_n/P_{n+1}$  from the experimental IMF excitation functions. For each bombarding energy, all of the excitation functions ( $n \le 4$ ) tightly collapse onto a



FIG. 1. The extracted values of *m* as a function of the transverse energy  $E_t$  for the reaction  ${}^{36}\text{Ar} + {}^{197}\text{Au}$  at E/A = 80 (open symbols) and 110 MeV (solid symbols). See text.

straight line, as shown in Fig. 2(b), when subjected to the above procedure.

In Fig. 3 we show a comparison between the experimental excitation functions and those calculated using the values of p obtained from the linear fits of Fig. 2 and the associated values of m from Eq. (5). The extraordinary quantitative agreement between the calculations and the



FIG. 2. (a) The reciprocal of the binary decay probability 1/p or the ratio  $\tau/\tau_0$  (calculated from the mean and variance of the IMF distributions) as a function of  $E_t^{-1/2}$ . The solid lines are linear fits to  $\log(1/p)$ . (b) Values of 1/p extracted differentially using Eq. (6). The solid lines are fits to the data shown in the upper panel and the different symbols represent the ratios extracted with different values of n.



FIG. 3. The experimental (symbols) and the calculated (solid lines) probability to emit *n* IMFs as a function of  $E_t$ . For numbers of fragments n = 0-8, P(n) is calculated assuming a binomial distribution [see Eq. (4)] with the values of *p* obtained from the linear fits shown in Fig. 2 and the corresponding values of *m* from Eq. (5).

experimental data confirms the binomiality of the multifragmentation process. Preliminary analysis of additional experiments [26] with different target-projectile combinations and bombarding energies indicates the general applicability of this description.

The more directly interpretable physical parameter contained in this analysis is the binary barrier B (proportional to the slope of the data in Fig. 2). One may wonder why a single binary barrier suffices, since mass asymmetries with many different barriers may be present. Let us consider a barrier distribution as a function of mass asymmetry x of the form  $B = B_0 + ax^s$ , where  $B_0$  is the lowest barrier in the range considered. Then,

$$p = \Gamma/\hbar \omega_0$$
  
=  $\int \exp(-B_0/T) \exp(-ax^s/T) dx$  (7)  
 $\approx (T/a)^{1/s} \exp(-B_0/T).$ 

Thus the simple form of Eq. (2) is retained with a small and renormalizable preexponential modification.

One possible interpretation of the reducibility discussed above is sequential decay with constant probability p. Assuming that the (small) fragments, once produced do not generate additional fragments or disappear, the binomial distribution follows directly. In this framework, it is possible to translate the probability p into the mean time separation between fragments. In other words, we can relate the *n*-fragment emission probabilities to the mean time separations between fragments. The validity of this interpretation is testable by experiment.

Equation (3) shows that the decay probability and the associated decay lifetime are dramatically affected even by moderate changes in temperature. Furthermore, as the temperature becomes comparable with the barrier, the binary decay probability approaches unity and the lifetime approaches the characteristic (dynamical) time constant of the channel,  $\tau_0$ . This behavior is indeed shown by the extracted times ( $\tau = \tau_0/p$ ), shown in Fig. 2.

To measure the mean time separation between fragments, groups have utilized the pairwise fragmentfragment correlations introduced by their mutual Coulomb interaction [9-17]. The results suggest rather short upper limits ( $\tau < 100$  fm/c) for the decay time scales for central collisions (large values of  $E_t$ ). A recent experiment [15] has studied the "proximity" effect of the surviving partner, produced in a deep inelasticlike collision, on the angular distribution of the fragments resulting from the breakup of the other partner. This remarkable experiment shows that at small excitation energies, the proximity effects are essentially absent, but become very pronounced at large excitation energies. This onset of proximity effects was taken to signify a transition from slow sequential multifragmentation to fast, nearly simultaneous multifragmentation. However, the observed decrease of the decay lifetime with increasing excitation energy [11,12,14,15] is also consistent with the expected energy dependence of sequential decay, and by itself does not prove a change in mechanism.

The detailed accuracy and the broad applicability of the binomial distribution are somewhat disconcerting. For instance, what is the significance of the parameter m? In the sequential description the system is given m chances to emit a fragment, with fixed probability p, after which the emission is shut off. One might have guessed that the probability p would decrease progressively as a function of time due to evaporative cooling, and that m is just an approximate cutoff made necessary by the constant pin the binomial distribution. This hypothesis, however, may not be correct. A simple evaporation calculation shows that during the time  $t = m\tau_0$  ( $\hbar\omega_0 \approx 1$  MeV) the system has insufficient time to cool completely. Therefore p may be nearly constant, and one is led to attribute a more physical significance to m. What switches the emission off after m tries must remain a speculation here. Let us venture to say that dynamics may be responsible for such an effect. Could it be that the fragments are statistically emitted while the system undergoes an expansion phase [27-30] only to be shut off as it reverts to normal density? If it were so, this would be a significant dynamical feature in an otherwise rather thermal picture.

To see if the light charged particles give any evidence for a longer cooling time, we performed the same analysis on the total charged particles emitted in these reactions. From the means and variances, one obtains values of m almost 4 times larger than those obtained for the fragments (see Fig. 1). In our picture, this could be a reflection of a longer total emission time and/or a shorter intrinsic period  $\tau_0$  for light charged particle emission.

We have tried to find alternative explanations to the sequential description for the binomial distributions with thermal probabilities. An obvious model is a chain of m links with probability p that any of the links is broken. The probability that n links are broken is given by Eq. (4). This result is, of course, strictly dependent on the dimensionality of the model, and its relevance to multifragmentation is unclear. Nevertheless, it stresses again the *fundamental reducibility* of the multifragmentation probability to a binary breakup probability p.

The final proof for or against sequentiality must rest on independent time measurements. The establishment of an agreement between the times inferred from the emission probabilities and from the particle-particle correlations would go a long way toward resolving this issue.

In summary, (1) the multifragment emission probability has been found to be *binomial* and *reducible* to an elementary binary probability. Thus, multifragmentation is empirically reducible to single fragment emission. (2) This binary elementary probability is observed to have a *thermal* energy dependence under the assumption that the excitation energy is proportional to the transversal energy.

Under the assumption of sequentiality, the inferred emission time scale contracts rapidly with increasing excitation energy. Such a contraction could explain the observed rapid onset of the fragment-fragment Coulomb interaction with increasing excitation energy and would obviate the need for "simultaneous" multifragmentation as a distinct process. While for very short time scales the distinction between sequential and simultaneous emission may become blurred, the retention of reducibility still conveys a very interesting message regarding the structure of the multifragmentation event.

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