^{10}C Superallowed Branching Ratio and the Cabibbo-Kobayashi-Maskawa Matrix Unitarity

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A measurement of the superallowed $0^+ \rightarrow 0^+$ branching ratio in ^{10}C performed with the 8π gamma-ray spectrometer is described. From the measured branching ratio, we extract a new precise value of $\mathcal{F}t(^{10}\mathrm{C}) = 3076.7(6.0)$ s. This does not support previously suggested Z- or Z^2-dependent corrections to the superallowed $0^+ \rightarrow 0^+$ data set, which removed the apparent nonunitarity of the Cabibbo-Kobayashi-Maskawa matrix.

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Superallowed $0^+ \rightarrow 0^+$ nuclear β decay [1] between isobaric analog states provides the most accurate value of G_v, the weak vector coupling constant. According to the conserved vector current (CVC) hypothesis, all such decays should yield the same value of G_v from their measured ft values provided that small isospin-symmetry-breaking (δ_c) and radiative (δ_r) corrections are accounted for. Specifically, for an isospin-1 multiplet,

$$ft(1 + \delta_r)(1 - \delta_c) \equiv \mathcal{F}t = \frac{K}{2G_v^2}. \quad (1)$$

The $\mathcal{F}t$ values for the eight precisely measured such decays are shown as solid points in Fig. 1. This data set is taken from the 1990 survey [1], to which have been added several new lifetimes [2], Q-value measurements [3], and branching ratios [4]. An improved treatment of the radiative corrections [5] is also used. From these data, an average $\mathcal{F}t$ value of 3073.1(3.1) s is obtained, in which the uncertainty incorporates a systematic error attributed to δ_c [1]. With G_v determined from this result and combined with the purely leptonic muon decay data, we obtain the value $V_{ud} = 0.9736(0.0006)$ for the up-down quark matrix element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Finally, taking values for V_{us} and V_{ub} from Ref. [6] we find

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9965(0.0015). \quad (2)$$

Thus, although the data displayed in Fig. 1 show that the agreement among the eight $\mathcal{F}t$ values supports the CVC hypothesis, the value of V_{ud} extracted from the mean $\mathcal{F}t$ value results in a unitarity test of the first row of the CKM matrix which differs from unity by more than two standard deviations. Two- and three-parameter fits [7] have been proposed (see dashed lines in the figure) to take into account possible systematic trends in the $\mathcal{F}t$ values by the arbitrary addition of fitted Z- or Z^2-dependent terms in the analysis. The resulting extrapolated $\mathcal{F}t(Z = 0)$ value would then meet the unitarity requirement. The validity of fits involving such additional free parameters can best be ascertained by adding a new measurement closer to $Z = 0$. This involves precision experiments on the lightest superallowed $0^+ \rightarrow 0^+$ β-decay emitter ^{10}C.

The accuracy of the superallowed $\mathcal{F}t$ value in ^{10}C is presently limited by the branching-ratio measurement. The decay of ^{10}C takes place mainly through a strong Gamow-Teller transition to an excited 1^+ state in ^{10}B while only about 1.5% of the decays go to the isobaric analog 0^+ state [see Fig. 2(a)]. The superallowed branching ratio is simply given by the ratio of the number of gamma rays emitted at 1022 keV over that at 718 keV, i.e.,

$$B(0^+ \rightarrow 0^+) = \frac{R(1022)}{R(718)} = \frac{Y(1022)}{Y(718)} \frac{\epsilon(718)}{\epsilon(1022)}, \quad (3)$$

with R being the emission rate, Y the observed yield, and ϵ the efficiency at a given energy. Any measurement, however, requires excellent statistics to yield precision of a few parts per thousand on such a weak branch. In addition, since the isobaric analog state populated by the superallowed branch is deexcited by the emission of a 1022 keV gamma ray it is necessary to minimize and account for the pileup of 511 keV annihilation radiation which disturbs the measurement.

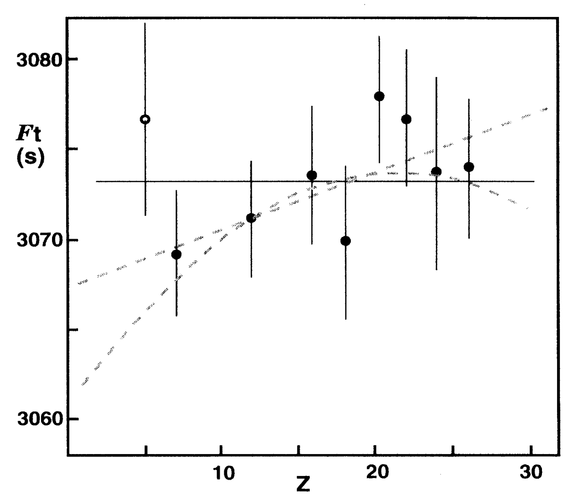

FIG. 1. The solid points are the $\mathcal{F}t$ values for the eight precisely measured superallowed $0^+ \rightarrow 0^+$ β-decay emitters (^{14}O, ^{26}Al^m, ^{34}Cl, ^{38}K^m, ^{42}Sc, ^{46}V, ^{50}Mn, and ^{54}Co) plotted as a function of the Z of the daughter nuclei. The open point is the new ^{10}C $\mathcal{F}t$ value. The full line is the result of the one-parameter fit to the data set before the addition of the ^{10}C point. The dashed lines are the corresponding best two- and three-parameter fits.

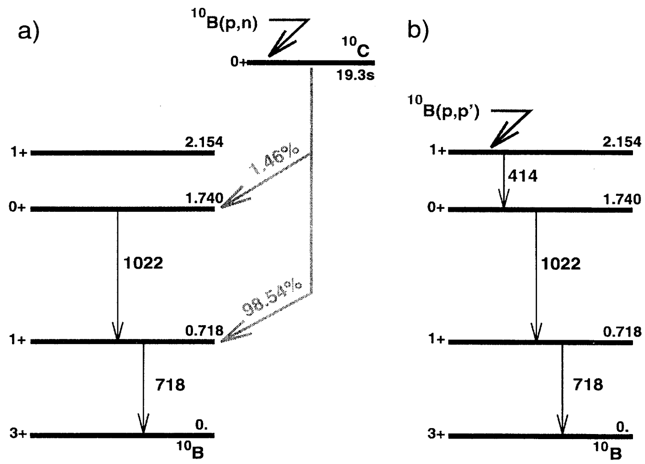

FIG. 2. (a) Decay scheme of ^{10}C observed in the relative yield measurement. The ground state and 2.154 MeV levels both have negligible direct feeding in the decay [9]. (b) Main deexcitation route for the 2.154 MeV level populated in beam for the relative efficiency measurement.

If we look at the 511 keV pileup rate compared to the real 1022 keV gamma-ray rate, we have the simple expression

$$\frac{Y(\text{pileup})}{Y(1022)} \approx \frac{R^2(511)\epsilon^2(511)\Delta\tau}{R(1022)\epsilon(1022)}, \tag{4}$$

with $\Delta\tau$ being the resolving time of the amplifier pileup rejection system. If the same total detector efficiency is now split in N independent detectors, this ratio is decreased by a factor of N:

$$\left(\frac{Y(\text{pileup})}{Y(1022)}\right)_N \approx \frac{NR^2(511)[\epsilon(511)/N]^2\Delta\tau}{NR(1022)\epsilon(1022)/N} \approx \frac{1}{N}\frac{Y(\text{pileup})}{Y(1022)}. \tag{5}$$

The experiment was therefore performed on a large gamma-ray array, the 8π spectrometer [8] at Chalk River. The spectrometer is composed of 20 Compton-suppressed 25% HPGe detectors surrounding a 72-element BGO inner ball. In addition to the twentyfold reduction in the 511 pileup signal obtained because of the geometry of the array itself, a further reduction is obtained via the pileup rejection system on each germanium detector which has a mean resolving time of roughly 420 ns.

The experiment comprised two interleaved measurements. One, the relative gamma-ray yield measurement, was a repeated cycle in which the activity was first produced by a (p,n) reaction on a gold-backed enriched 550 μg/cm^2 ^{10}B target mounted in the center of the 8π spectrometer, then the beam was turned off and the β-delayed gamma rays from the decay of ^{10}C observed in singles mode. The second measurement, that of the relative gamma-ray efficiency, was performed in beam with γ-γ coincidences recorded from the deexcitation of the 2.154 MeV level in ^{10}B, which was populated by the (p,p') reaction. The feeding of the levels involved in each measurement is shown in Fig. 2 (in which Ref. [9] is cited). The main decay channel of the 2.154 MeV level, populated in beam, involves the sequential emission of three gamma rays of energies 414, 1022, and 718 keV, respectively [Fig. 2(b)]. The ratio of the number of γ_{414}-γ_{718} coincidences over γ_{414}-γ_{1022} coincidences is therefore equal to the efficiency ratio $\epsilon(718)/\epsilon(1022)$ required in Eq. (3).

The experimental technique was tested in a short two-day run and the full-fledged experiment was then performed over two one-week periods, with some small changes made between the two runs to test for possible sources of systematic errors. During the runs, 25 cycles of 30-s-collection/30-s-decay measurements were interleaved with segments of 30 min in-beam efficiency calibration. An 8 MeV proton beam intensity of roughly 100 nA was used in the collection/decay mode while the intensity was reduced to about 15 nA for the in-beam calibration. The total statistical uncertainty for the three runs was about 1.4×10^{-3}. A β-delayed gamma-ray spectrum from ^{10}C decay and a spectrum of in-beam gamma rays coincident with the 414 keV transition obtained over a one-week period are shown in Fig. 3.

Some small corrections must be applied to the raw experimental numbers. They are listed in Table I and elaborated as follows: (1) The 511 keV pileup correction in the decay measurement, which was of the order of 10% in all previous measurements of this branching ratio [10–12], is reduced to a 0.01% correction by the new technique used here. (2) Contamination in the counting sample is a serious concern in the decay measurement, especially for the weaker 1022 keV peak. The decay data were tagged by that time after the collection so that separate analyses could be performed for different decay-time slices. No variation in the gamma-ray yield ratio $Y(1022)/Y(718)$ was observed, setting a limit of 2×10^{-3} on possible contamination. A search was then performed to find β-delayed gamma-ray peaks from other isotopes known to emit gamma rays with energies close to 1022 keV. Finally, background spectra were also recorded to determine the room background and the isotopes produced by the gold backing. No evidence for an unresolved contamination of the peaks of interest was observed. (3) An up-

TABLE I. Experimental corrections to be applied to the branching ratio.

Source of effect	Size	Affects
511 pileup	$-0.9(2) \times 10^{-4}$	Decay data
Background	0	Decay data
1.740 MeV → g.s.	$0(^{+1.1}_{-0}) \times 10^{-4}$	Decay data
Internal conversion	$<10^{-6}$	Both
Angular correlation	0	In-beam data
Kinematics shift	$<10^{-4}$	In-beam data
Random coincidences	$<10^{-4}$	Both
Pileup + suppressors	$6.5(8) \times 10^{-3}$	Both

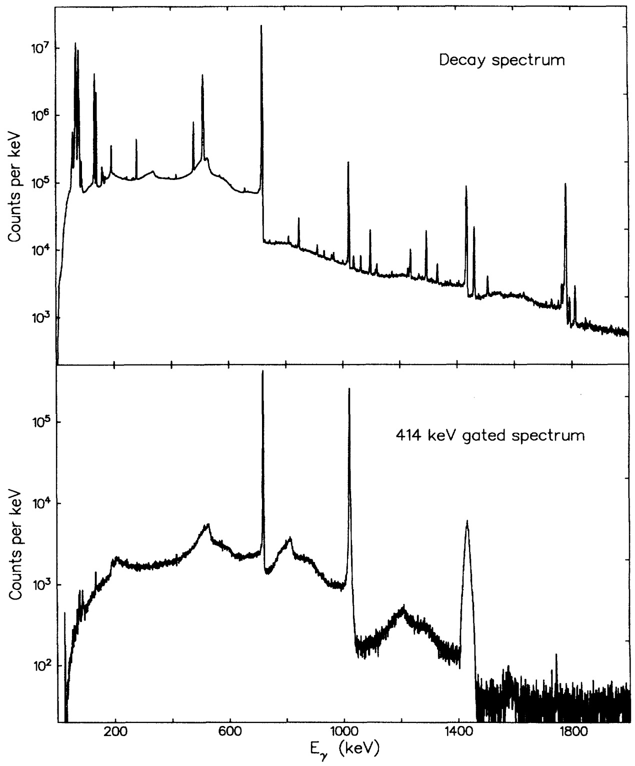

FIG. 3. The top spectrum was obtained in the decay measurement. This spectrum contains peaks from room background, from activity created by the beam (mostly ^{10}C), and from activity induced by neutrons within the 8π spectrometer. The BGO inner ball shields the HPGe detectors from the annihilation gamma rays originating from positrons stopping in the vacuum chamber wall, significantly decreasing the observed 511 keV gamma-ray rate. The lower spectrum shows gamma rays in coincidence with a 414 keV gamma-ray in beam. The two strong peaks are the 718 and 1022 keV peaks of interest. The broad peak at 1433 keV is from a gamma ray feeding the 2.154 MeV state.

per limit of 9×10^{-5} is obtained from this experiment for the deexcitation of the 1.740 MeV state directly to the ground state. This results in a correction of up to 1.1×10^{-4}. (4) Internal conversion is a negligible correction and, in any case, cancels out in the ratio. (5) No angular-correlation corrections are necessary in the γ_{414}-γ data, since the 414 keV gamma ray populates a 0^+ state. (6) For the in-beam data, the 414 and 1022 keV gamma rays are emitted during the slowing-down process, while the 718 keV gamma rays are emitted essentially at rest. A small correction must therefore be applied for the kinematic change in solid angle and efficiency (from the energy shift) for the different detectors. In the 8π spectrometer, the HPGe detectors are placed in four rings at 37°, 79°, 101°, and 143° with respect to the beam axis. The maximum kinematic correction is $+(-)0.8 \times 10^{-3}$ for the most forward (backward) ring, which cancels out when the spectra from all rings of detector are summed. (7) Effects of random coincidences on the γ_{414}-γ data have been removed by subtraction of counts obtained in a noncoincident time gate. Time gates of different duration were also used to verify that no systematic errors were introduced by a possible energy dependence of the coincidence time width.

The final correction consider is the effect of pileup. Pileup due to randoms does not affect the ratios measured. The main concern comes from pileup losses due to the many gamma rays emitted in each event. For example, when the 2.154 MeV state in ^{10}B is populated in beam and valid 414 and 718 keV gamma-ray events are registered in two detectors, the 1022 keV gamma ray which is also emitted in this event could invalidate the event if it deposits any energy in one of these two detectors. Even more significantly, since each HPGe detector has associated with it a large BGO Compton suppressor, the additional gamma ray could also veto the event by depositing a small amount of energy in one of the two suppressors. The effect of this type of event has been investigated with standard sources that emit two gamma rays, the gamma-ray spectrum being recorded with acquisition electronics modified so that half of the HPGe detectors required a suppressor signal for the event to be valid. A valid photopeak event in one of the modified detectors then implied that the second gamma ray deposited energy in the corresponding suppressor but none of the detector itself. These measurements yield a correction of $+6.5(8) \times 10^{-3}$ for the branching ratio. The largest contribution to this correction comes from the decay process, in which emission of a 1022 keV gamma ray is always accompanied by a 718 keV gamma ray, while a 718 keV gamma ray only has a 1022 keV gamma ray emitted with it 1.5% of the time.

With all corrections applied, we determine the total branch to the isobaric analog state to be

$$B(0^+ \to 0^+) = [1.4625 \pm 0.0020(\text{stat}) \pm 0.0015(\text{syst})]\%, \tag{6}$$

where the systematic uncertainty is the one attributed to the sum of all experimental corrections; it should be added quadratically to the statistical uncertainty. As shown in Table II, this result agrees with, but is substantially more precise than, previous measurements. When combined with Q_{EC}(for the allowed transition) = 1907.77(9) keV [13] and $t_{1/2} = 19.209(12)$ s [14], and corrected for electron capture (0.296%), $\delta_c = 0.18(4)\%$ and $\delta_r = 1.30(4)\%$ [5], we extract

$$\mathcal{F}t(^{10}\text{C}) = 3076.7(6.0)\ \text{s}. \tag{7}$$

TABLE II. Precise ^{10}C superallowed branching-ratio measurements.

Branching ratio	Reference
(1.465 ± 0.014)%	[10]
(1.473 ± 0.007)%	[11]
(1.465 ± 0.009)%	[12]
(1.4625 ± 0.0025)%	This work

TABLE III. Fitted value of $\mathcal{F}t(Z=0)$ and other parameters from one-, two-, and three-parameter fits. Also given are the χ^2 values and the resulting unitarity tests. The first column is for the data set without ^{10}C, the second column includes the new ^{10}C value.

	Data set without ^{10}C	Data set with ^{10}C
One-parameter fit		
$\mathcal{F}t(0)$	3073.1 ± 3.1 s	3073.2 ± 3.0 s
χ^2/N	1.49	1.35
V_{ud}	0.9736 ± 0.0006	0.9736 ± 0.0006
$\sum_i V_{ui}^2$	0.9965 ± 0.0015	0.9965 ± 0.0015
Two-parameter fit		
$\mathcal{F}t(0)$	3067.1 ± 3.4 s	3068.4 ± 3.3 s
a_1	$(1.1 \pm 0.5) \times 10^{-4}$	$(0.9 \pm 0.5) \times 10^{-4}$
χ^2/N	1.01	1.09
V_{ud}	0.9745 ± 0.0007	0.9743 ± 0.0007
$\sum_i V_{ui}^2$	0.9983 ± 0.0016	0.9979 ± 0.0015
Three-parameter fit		
$\mathcal{F}t(0)$	3060.8 ± 8.3 s	3066.4 ± 8.0 s
a_1	$(3.9 \pm 3.1) \times 10^{-4}$	$(1.9 \pm 2.8) \times 10^{-4}$
a_2	$(-0.9 \pm 0.9) \times 10^{-5}$	$(-0.3 \pm 0.8) \times 10^{-5}$
χ^2/N	1.03	1.25
V_{ud}	0.9755 ± 0.0014	0.9746 ± 0.0013
$\sum_i V_{ui}^2$	1.0003 ± 0.0028	0.9986 ± 0.0027

This result favors the standard analysis compared to the two- and three-parameter fits to the complete set of $\mathcal{F}t$ values (Fig. 1). Table III shows the parameters obtained for the three analysis procedures with and without the ^{10}C data. The reduced χ^2 for the standard one-parameter analysis is significantly reduced and the extracted $\mathcal{F}t(Z=0)$ is not affected by the new data point; the nonunitarity therefore remains. For the two-parameter approach, the quality of the fit is slightly worse as is the agreement with unitarity. The three-parameter fit suffers from a large increase in the reduced χ^2 and both a_i parameters are consistent with null values. An F-test statistical analysis to determine the need for additional free parameters rejects the three-parameter fit.

The addition of ^{10}C to the set of precisely measured superallowed $0^+ \rightarrow 0^+$ β-decay emitters therefore does not support the existence of Z-dependent corrections unaccounted for by δ_r and δ_c. Consequently, it also leaves the CKM unitarity condition unsatisfied. Possible explanations of nonunitarity include the "trivial" (possibly an inadequate evaluation of V_{us} [15], for example) as well as more profound extensions to the standard model.

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