Missing and Quenched Gamow-Teller Strength

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Gamow-Teller strength functions in the resonance region are calculated in the full $(pf)^8$ space. The observed profile is very sensitive to the level density and may become so diluted as to be confused with background. A model independent proof is given that standard quenching originates in nuclear correlations, and that some 30% of the total strength must be due to states outside the $(pf)^8$ space. By combining this argument with the results of shell model calculations, comparison with the ^{48}Ca$(p,n)^{48}$Sc experimental data strongly suggest that most of the strength that is currently thought to be missing is actually observed.

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Since the time of the pioneering (p,n) [1,2] and (n,p) [3,4] experiments, it has been possible to explore the Gamow-Teller strength function of many nuclei in the resonance region and beyond. The most striking result is that a large fraction of the theoretically expected sum rules for $\sigma\tau$ operators, S_+ and S_-, seems to be missing. The precise amount may be difficult to assess because of calibration and background problems [5,6], but a reduction by a factor of 0.6 of S_+ and S_- is currently accepted as standard [7]. This number is obtained through two different channels. One is the sum rule $S_- - S_+ = 3(N-Z)$, which is model independent. Therefore the strength difference cannot be quenched, i.e., suppressed. It is *missing*, but it must be somewhere [8]. The other indication comes from the well defined, isolated peaks seen in β decays, which are about a factor of 0.6 weaker than predicted by the most accurate shell model calculations available [9,10]. Here we can speak of *quenching* because the data demand it, in the same sense they demand effective charges, though the mechanisms at play are very different in the two cases. Throughout this Letter we shall distinguish the *missing* factor, taken to be unity if no strength is missing, from the *quenching* factor set at the standard 0.6 value.

Below we calculate exactly the strength function for the reaction ^{48}Ca$(p,n)^{48}$Sc, and show the influence of the density of levels on the observed profile. Then we decompose the model independent sum rule in a way that makes it apparent that quenching is a norm effect that originates in deep nuclear correlations. Finally, we compare with the experimental results of Anderson *et al.* [11] and explain why most of the strength that is thought to be missing is in fact observed.

Model strength functions.—To understand how the strength distributes among daughter states we rely on a method proposed by Whitehead [12] and is now quite popular [13–15]. We work in the full pf shell with the KB3 interaction [10,16,17] and obtain the exact target eigenstate $|k\rangle$ in this model space. Following [12] we define states $|s_\pm\rangle = \sigma\tau_\pm|k\rangle$, whose norms are the sum rules $S_\pm$, and we use them as "pivots" (i.e., starting states) to construct a Lanczos tridiagonal matrix in which each new state is obtained by acting on the preceding one with the Hamiltonian. After I iterations we obtain I eigensolutions, and the amplitude of the pivot in each of them determines its contribution to a strength function whose first $2I$ moments are those of the exact distribution. To guard against numerical errors, each new Lanczos vector must be spin and isospin projected, and orthogonalized to the preceding ones. We are mostly interested in the 8590 $J=1$, $T=3$ states of ^{48}Sc, embedded in a total of 1.4×10^5 m-scheme partners. An IBM-3090 (now retired), ANTOINE [18], could cope with about 100 iterations per hour for this problem. In Fig. 1 we show the ^{48}Ca$(p,n)^{48}$Sc, Gamow-Teller (GT) strength after $I=50$ and 700, $|k\rangle$ is the $(pf)^8 T=4$ ground state, and the pivot is projected to keep only the $T=3$ states. At $I=50$ only the lowest four spikes correspond to converged eigenstates. The others should be viewed as doorways that will split as we evolve. At $I=700$, all spikes below 10.5 MeV are eigenstates. Therefore, although the strength functions for $I=50$ and 700 look very different,

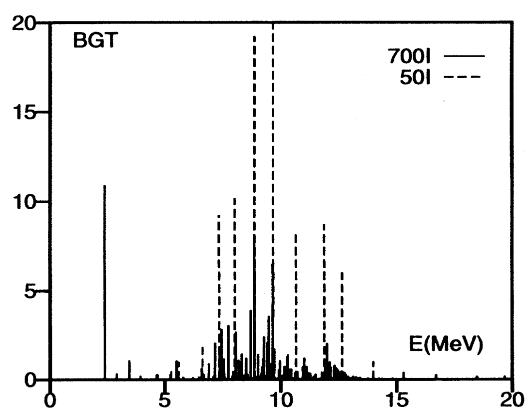

FIG. 1. ^{48}Ca$(p,n)^{48}$Sc GT % strength, $I=50$ and 700.

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their first 100 moments are identical. One would suspect that this enormous constraint would guarantee identical distributions for practical purposes, but this is not quite the case as can be gathered from Fig. 2 in which we have associated to each of the states at $I = 700$ a Gaussian width of 150 keV, to simulate the experimental situation. The corresponding strength function is what would be seen in an ideal experiment perfectly described by the calculation. To make the $I = 50$ peaks agree approximately with this profile, it is necessary to smooth them by Gaussians of 250 keV above 6 MeV. Two comments are called for.

(i) Dilution.—The 250 keV width depends on the density of levels, i.e., it corresponds to information brought in by the very high moments of the distribution. In situations of high level density the "dilution" may be so severe as to leave few traces of individual peaks. To give an example, in Fig. 1 only 32% of the strength is found in peaks with individual shares of less than 1%. For the $J = 3, 4, 5$, $T = 0, 1$ daughter states of the ^{48}Mn(GT)^{48}Cr process [19,20] the corresponding number is 82%.

(ii) Shifting.—The smoothed $I = 50$ peaks give a fair account of the exact situation but significant discrepancies show for the three peaks above 10 MeV that are out of phase.

The origin of quenching.—To understand the origin of the quenching effect we must first say a few words about the meaning of a shell model calculation. The basic idea (for abundant details and examples see [17,21]) is that the full space is divided into model ($|i\rangle$) and external ($|j\rangle$) states—also called intruders—that are then "dressed," i.e., correlated through the transformation

$$|\bar{i}\rangle = |i\rangle + \sum_j A_{ij}|j\rangle\,, \qquad |\bar{j}\rangle = |j\rangle - \sum_j A_{ij}|i\rangle\,, \tag{1}$$

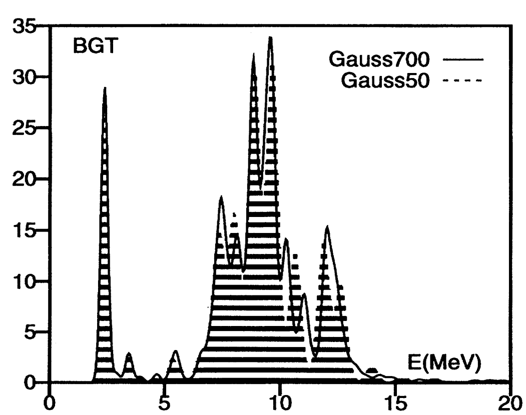

FIG. 2. Gauss700 (full line): $700I$ in Fig. 1, smoothed by Gaussians of 150 keV width. Gauss50 (shaded area): $50I$ in Fig. 1, smoothed by Gaussians of 150 keV width below 6 MeV and 250 keV width above 6 MeV.

which respects strict orthogonality, $\langle\bar{i}|\bar{j}\rangle = 0$. The amplitudes A_{ij} are obtained by demanding

$$\langle\bar{i}|H|\bar{j}\rangle = 0\,. \tag{2}$$

In practice this condition is treated in perturbation theory and, as a consequence, only the low lying states of the shell model diagonalization approximate well exact *eigenstates* of the system. The others, which are close in energy to the external ones, will have a status of *doorways*.

The calculation of the effective operator $(\sigma\tau)_{\rm eff}$ in the model space is more difficult than for the Hamiltonian, because we have to know the norms of the dressed states. Fortunately, there is a simple and rigorous argument that makes plausible the empirical result $(\sigma\tau)_{\rm eff} = \sqrt{0.6}\,(\sigma\tau)_m$.

We start by noting that the $3(N - Z)$ sum rule is a consequence of the equality

$$\sigma\tau_+ \cdot \sigma\tau_- - \sigma\tau_- \cdot \sigma\tau_+ = 3(\hat{n} - \hat{z})\,, \tag{3}$$

where $\hat{n}$ and $\hat{z}$ are number operators for neutrons and protons, and the dots indicate that we keep the scalar term in spin space. Now let us separate the orbits into model, m (in our case the pf shell), and external ones, r. Please do not confuse orbits with states: $|j\rangle$ in Eq. (1) is an external state made of configurations that contain both m and r orbits.

The $\sigma\tau$ operator can be written as a sum of the model contribution plus the others

$$\sigma\tau_\pm = (\sigma\tau_\pm)_m + (\sigma\tau_\pm)_r\,, \tag{4}$$

then Eq. (3) splits in two, and we find immediately

$$\begin{aligned} S_- - S_+ &= (S_- - S_+)_m + (S_- - S_+)_r \\ &= 3\langle K|\hat{n}_m - \hat{z}_m|K\rangle + 3\langle K|\hat{n}_r - \hat{z}_r|K\rangle\,, \end{aligned} \tag{5}$$

where $|K\rangle$ is the *exact* target eigenstate, i.e., a normalized sum of $|\bar{i}\rangle$ states. The number operators are such that $\hat{n}_m + \hat{n}_r = \hat{n} = N$, $\hat{z}_m + \hat{z}_r = \hat{z} = Z$, but their expectation values are nontrivial because they are a measure of the correlations: Instead of being either filled or empty, orbits are partially full or partially empty. The hole occupancy, i.e., the filling factor immediately below the Fermi surface, can be extracted from transfer reactions or (e, e') scattering. The (d, p) results of Vold *et al.* [22] yield a value of 0.70(5) for ^{40}Ca, against 0.75(5) from the analysis of (e, e') experiments in ^{208}Pb [23–25], indicating a remarkable constancy for this fundamental quantity. Therefore, since we can equate terms separately in Eq. (5), we shall write

$$\begin{aligned} (S_- - S_+)_m &= 3\langle k|\hat{n}_m - \hat{z}_m|k\rangle 0.70(5) \\ &= 3(N - Z)0.70(5)\,, \end{aligned} \tag{6}$$

where $|k\rangle$ is the *model* (undressed) target eigenstate. Equation (6) can be read directly as meaning that some 70% of the strength is in the model space—assumed to be a full major shell—and some 30% outside. This interpretation is basically correct, but there is a subtle catch. When we act with $(\sigma\tau_\pm)_m$ on the exact ground state, there is no reason to suppose that the result is exactly a dressed model state: It may contain some external contribution. Therefore the strictly model strength, i.e., the one we have calculated, may not be 70% of the total, but somewhat less, and we propose to replace Eq. (6) by

$$(S_- - S_+)_{m'} \cong 3(N - Z)0.70\alpha\,, \tag{7}$$

where $\alpha \leq 1$, and m' stands for calculated model strength. Obviously if $\alpha \approx 6/7$, we would recover the standard quenching factor $\sqrt{0.6}$. Although a rigorous estimate would be hard, an experimental check is possible. What we would expect is an excess of 7/6 of the measured strength over the calculated one in the region where the latter is strong.

It should be stressed that we have by no means demonstrated that the effective operator is $(\sigma\tau)_{\rm eff} = \sqrt{0.6}\,(\sigma\tau)_m$, but we have certainly shown that the strength we have calculated accounts for at most 70% of the total.

Comparison with experiment.—In Fig. 3 the ^{48}Ca$(p,n)^{48}$Sc data have been scanned from Fig. 5 of Ref. [11]. The isobaric analog state at 6.7 MeV and the $J = 1$, $T = 4$ state at 16.8 MeV have been substracted: The former through a Gaussian of 200 keV width, the latter by continuing the background line. The $T = 3$ strength calculated with the effective operator $\sqrt{0.6}\,\sigma\tau$ is $22.7 \times 0.6 = 13.6$ GT units, and corresponds to an area of 100 in the figure. (The subtracted $T = 4$ contribution is $1.3 \times 0.6 = 0.78$ GT units.) The experimental area is 180, i.e., 24.5 GT units. Before we examine what the figure is telling us in the light of the analysis in the preceding discussion, we would like to explain the possible origin of the idea that much strength is missing.

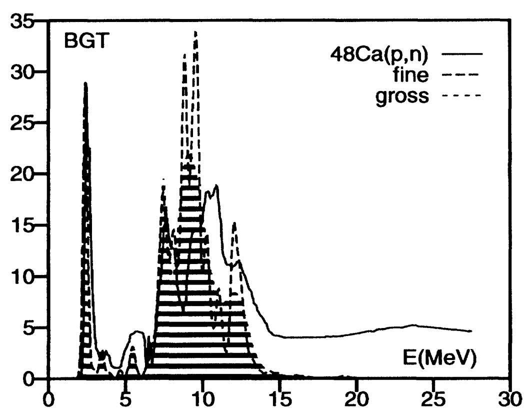

FIG. 3. ^{48}Ca(p,n) (full line): experimental GT strength from Ref. [11]; fine (dashed line): same as Gauss700 in Fig. 2; gross (shaded area): same as fine and Gauss700 up to 8 MeV, then 700*I* in Fig. 1 smoothed by Gaussians of 500 keV width. An area of 100 in the figure corresponds to 13.6 GT units (see text).

In the absence of any extra information, the experimental profile of Fig. 3 suggests that the tail above 15 MeV is background. Then in good logic it has to be extrapolated back under the resonance and subtracted, leading to a "missing" factor of 0.43 [11]. Osterfeld [26] argued that this "experimentalist's background" should be restored. Then, missing and quenching factors become about equal, and the standard picture emerges. However, it is clear from the figure that the tail cannot start abruptly at 15 MeV: It must originate somewhere between 5 and 10 MeV.

If now we turn to the calculations, we can understand quite clearly what is happening: Below 8 MeV, with the instrumental Gaussian smoothing of 150 keV (the "fine" curve in Fig. 3), they agree with the data perfectly. Above that energy, Gaussians of 500 keV are necessary (the "gross" shaded area), a clear indication that intruders are coming in, causing shifting and dilution of the model strength. The situation is similar to that of Fig. 2, but in the absence of constraints on the moments the effects are stronger. It is worth noting that we obtain as a fringe benefit a good estimate of the level density up to 8 MeV by direct counting in Fig. 1.

The interpretation is transparent: The experimental distribution is bimodal, the main group being mostly model strength and the bump above 15 MeV due to intruders, but, as we have just shown, the bump originates at around 8 MeV. Therefore the experimentalist's background is indeed present, although it is genuine strength rather than background.

The intruders also carry some strength of their own below 15 MeV, where the calculated area is 97 and the experimental one 111. Therefore we find $\alpha \approx 97/111 = 0.87 \approx 6/7$, the ratio needed to explain standard quenching, as discussed after Eq. (7).

From Eq. (6), we can estimate the area beyond 15 MeV: $(0.3/0.7)111 = 47.5$. Therefore out of a total of 180, some 158.5 should be interpreted as genuine strength corresponding to 21.6 GT units, i.e., 95% of the $T = 3$ contribution to the $3(N - Z)$ bound.

The uncertainties in the numbers we have chosen are large enough to allow for estimates that will exceed the (lower) bound, especially if we remember that strength at higher energies may exist. It is difficult to escape the conclusion that *most of the strength that was thought to be missing is actually observed and due to intruders.* This possibility is not ruled out in the analysis of the data in [11].

The calculation for ^{90}Zr by Bertsch and Hamamoto [27], which indicated that much strength is above the GT resonance, is basically consistent with our results, the main difference being in the location of intruder strength, which we expect at lower energies.

It seems quite evident that the missing factor should be much closer to 1 than hitherto suspected: In the absence of further experimental evidence the observed profile above 15 MeV in Fig. 3 must be assumed to contain mostly genuine GT strength. As S_+ has the same origin as intruder strength, results on the ^{48}Ca(n,p) reaction would be welcome.

To sum up, Eq. (6) is a model independent result assigning some 70% of the GT strength to the resonance region, and 30% outside of it and due to intruders. State of the art shell model calculations give perfect agreement with the observed profile up to 8 MeV, and then unmistakable signs of a rapid increase of the density of intruder states. The observed tail of the resonance is naturally interpreted as containing the bulk of the strength hitherto thought to be missing. Therefore the (p,n) reaction probably tells us little about isobars in nuclei—which was taken to be its main interest. As compensation it may become a tool to measure a fundamental quantity, the level density.

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