Blümel and Esser Reply: In our opinion, the ultimate aim of quantum chaos research is to confirm or to disprove the existence of fully quantized systems that exhibit exponential sensitivity and chaos. In the context of a classification proposed below we call such systems type III. Possible candidates for type III systems were recently discussed in the literature [1,2]. In the beginning of quantum chaos research it was hoped that the quantized versions of classically chaotic systems would be type III. This turned out not to be the case. Rather, these systems typically display quasiperiodic motion and no exponential sensitivity. We call these latter systems type I. They are investigated within the framework of quantum chaology [3]. Intermediate between type I and type III are type II systems. They are not fully quantized, but display exponential sensitivity and chaos in the quantum subsystem. Thus, the model discussed in [4] is an example of a type II system.

The Comment by Allegrini *et al.* [4] mainly addresses the question of the validity of the Born-Oppenheimer approximation used to obtain the type II system discussed in [5]. Questions about the quality of the approximations are undoubtedly important for practical applications in nuclear, solid state, and molecular physics. But they are not currently our primary concern. An example may clarify our intentions. Type I systems, e.g., are usually investigated by computing classical system trajectories irrespective of whether the classical approximation is justified or not. In a second step, the quantum version of the system is investigated, and the behavior of the classical system trajectories (chaotic or regular) is linked to the behavior of the operators and wave functions of the fully quantized system. Thus, classical mechanics is not considered as an approximation to the full quantum dynamics, but as an underlying skeleton in the sense of Feynman's path integrals, or Gutzwiller's periodic orbit expansions [6]. The properties of the classical skeleton influence the behavior of the quantum dynamics and give rise to the "fingerprints" of classical chaos on the quantum level.

We think that the same philosophy applies to type II systems. Consequently, a type II system can be taken as an indicator of the behavior of the fully quantized system in the same sense as the presence of classical chaos leads to observable consequences on the quantum level in type I systems. Moreover, the first step of the Born-Oppenheimer approximation, namely the mixed classical quantum description, can be considered as a preliminary step from which an ansatz for the wave function of the fully quantized system can be derived. Therefore, in the spirit of type I systems, the direction of our research program is to investigate the full quantum dynamics of the system presented in [5] and to look for the fingerprints of type II quantum chaos in the fully quantized system.

The nonlinear dimer model mention by Allegrini *et al.* (their reference [5]) is not a type II system. It is integrable because it reduces to two independent electronic variables on the Bloch sphere [7]. It can be extended to a type II system if, in analogy to the model discussed in [5], the vibronic energy of the polarization is taken fully into account in the classical approximation [8].

We agree completely with Allegrini *et al.* that the exponential sensitivity and the chaos observed in the quantum subsystem of our type II system will not survive the full quantization of the system. This is in complete analogy with type I systems where the classical chaos is eliminated by the quantization process. Nevertheless, it is a fact that type II systems exhibit genuine chaos in the quantum subsystem and may serve as valuable models for the investigation of concepts and methods needed for the discussion of type III systems.

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