Comment on "Quantum Chaos in the Born-Oppenheimer Approximation"

In their recent Letter [1] Blümel and Esser assume that a subsystem of a quantum system can be safely dealt with as a classical system. This, referred to by them as the mixed quantum-classical picture (MQCP), is the same approximation as that behind the discrete nonlinear Schrödinger equation (DNSE) [2]. It has been already remarked [3] that the rigorous microscopic foundation of the DNSE would also be equivalent to settling some fundamental problems which have been haunting quantum mechanics since its inception, insofar as the collapses of the wave function would be obtained from within quantum mechanics with no need of postulates [4], as well as to accounting for important biological processes [2,5]. One would also get, as Blümel and Esser [1] do, quantum chaos.

However, as pointed out in [6], we are compelled by quantum mechanics to average over the chaotic trajectories, thereby recovering the insensitivity of quantum mechanics to initial conditions. Furthermore, the error associated with the MQCP was recently carefully evaluated [7,8] with the following conclusions. These papers, and many others mentioned in [1] alike, concern the interaction between a two-level system and an oscillator. Within the Wigner formalism this interaction is described by the Liouvillian

$$\mathcal{L}_{\text{int}} = 2GQ\left(x_3 \frac{\partial}{\partial x_2} - x_2 \frac{\partial}{\partial x_3}\right) + Gx_1 \frac{\partial}{\partial P} + G \frac{\partial}{\partial P}\left[\frac{\partial}{\partial x_1}(1 - x_1^2) - \frac{\partial}{\partial x_2} x_1 x_2 - \frac{\partial}{\partial x_3} x_1 x_3\right], \quad (1)$$

where x_1, x_2, and x_3 describe the two-level system by means of the Bloch sphere while Q and P refer to the oscillator. Neglecting in $\mathcal{L}_{\text{int}}$ the term containing second-order derivatives, we obtain a classical-like Liouville operator that leads to a set of nonlinear equations of motion, equivalent to the nonlinear dimer of [5]. This is a factorization assumption equivalent to the MCQP used by Blümel and Esser [1] to derive their Eqs. (7) and (9), dealing with the oscillator as a classical variable in the Heisenberg equation of motion. The nonlinear and chaotic properties thus obtained are due to the presence of a reaction term, namely the second term on the right hand side (r.h.s.) of Eq. (1), which within the MCQP expresses the influence of the quantum subsystem on the classical oscillator. However, we stress that the MCQP is equivalent [4,7] to neglecting the third term on the r.h.s. of Eq. (1), which is as strong as the term responsible for all the interesting nonlinear properties of the quantum system [5]. Unfortunately, it has been remarked that some of the statistical properties stemming from this approximation are incorrect, as, for instance, the predictions on the onset of localization in the nonlinear dimer, conflicting with the prediction of exact equilibrium theories [8].

However, it must be stressed that, although the MCQP is untenable from a rigorous quantum mechanical point of view, some benefits can be derived from it. First of all, the oscillator in the work of Blümel and Esser is slow. In this specific condition it has been shown [8] that the equilibrium and dynamical statistical properties produced by the assumption that the third term on the r.h.s. of Eq. (1) can be neglected, are essentially correct. Furthermore, albeit the quantum chaos effects produced by the MCQP are not real, they serve the important purpose of explaining a quantum effect that would be incomprehensible without establishing a contact with the *wrong* prediction on the occurence of chaos. In Refs. [6] and [7] it was proved that the quantum counterpart of the chaotic properties produced by the MCQP is the anomalous increase of the quantum mechanical uncertainty of the oscillator. The results of [6] as well as those of Blümel and Esser [1] should contribute to making more popular the study of the quantum behavior of those systems which would be chaotic within the framework of the MCQP, while the attention of investigators is currently mainly devoted to quantum systems which are chaotic in the semiclassical limit [9].

Paolo Allegrini,[1] Luca Bonci,[1] Paolo Grigolini,[2,3] Riccardo Mannella,[1] Roberto Roncaglia,[2] and David Vitali[4]

[1]Dipartimento di Fisica dell'Università di Pisa
Piazza Torricelli 2, 56100 Pisa, Italy
[2]Istituto di Biofisica del C.N.R.
via San Lorenzo 28, 56127 Pisa, Italy
[3]Department of Physics, University of North Texas
P.O. Box 5368, Denton, Texas 76203
[4]Dipartimento di Fisica dell'Università di Camerino
via Madonna delle Carceri, 62032 Camerino, Italy

Received 28 June 1994
PACS numbers: 05.45.+b, 03.65.Bz

[1] R. Blümel and B. Esser, Phys. Rev. Lett. **72**, 3658 (1994).
[2] J. C. Eilbeck, P. S. Lomdahl, and A. C. Scott, Physica (Amsterdam) **16D**, 318 (1985).
[3] P. Grigolini, *Quantum Irreversibility and Measurement* (World Scientific, Singapore, 1993).
[4] L. Bonci, P. Grigolini, R. Roncaglia, and D. Vitali, Phys. Rev. A **47**, 3538 (1993).
[5] V. M. Kenkre and D. K. Campbell, Phys. Rev. B **34**, 4595 (1986).
[6] L. Bonci, R. Roncaglia, B. J. West, and P. Grigolini, Phys. Rev. Lett. **67**, 2593 (1991); Phys. Rev. A **45**, 12 (1992).
[7] L. Bonci, P. Grigolini, R. Roncaglia, D. Vitali, and B. J. West, Int. J. Mod. Phys. **B5**, 1175 (1993).
[8] D. Vitali, P. Allegrini, and P. Grigolini, Chem. Phys. **180**, 297 (1994).
[9] L. E. Reichl, *The Transition to Chaos in Conservative Classical System: Quantum Manifestation* (Springer, New York, 1992).

 0031-9007/95/74(8)/1484(1)$06.00