Comment on "Method of Constrained Global Optimization"

In a recent Letter [1] Altschuler *et al.*, describe an optimization algorithm and illustrate its implementation with several examples, including the problem of finding electrostatic minimum energy configuration for N unit point charges constrained to lie on the surface of a sphere with unit radius. In the range $2 \leq N \leq 65$ their results agree with those found previously [2]. They also present new results "... (which) we believe to be the minimum energies for $66 \leq N \leq 100$...." However, for the particular values $N = 69$, 86, and 87 we have identified configurations with lower energies. Specifically, if $E_1(N)$ and $E_2(N)$ denote the lowest and next-to-lowest energy values of locally stable states found by computer searches, then

N	$E_1(N)$	$E_2(N)$
69	2064.533 483 23	2064.536 066 23
86	3258.211 605 71	3258.213 663 08
87	3337.000 750 02	3337.002 642.99

Comparing these results with the corresponding entries in Table I of Ref. [1], it is evident that their algorithm has converged to E_2 rather than E_1. In the absence of rigorous analytical bounds we cannot exclude the existence of states with even lower energies.

Computer trials indicate that in the range $70 \leq N \leq 112$, the number of distinct configurations associated with each value of N grows exponentially, i.e., $M(N) \approx 0.382 \times \exp\{0.0497N\}$. If this trend is sustained for larger values of N, identifying global minima among a large set of nearly degenerate states for complex systems of this type will pose formidable technical challenges.

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[1] E. L. Altschuler, T. J. Williams, E. R. Ratner, F. Dowla, and F. Wooten, Phys. Rev. Lett. **72**, 2671–2674 (1994).
[2] T. Erber and G. M. Hockney, J. Phys. A **24**, L1369–L1377 (1991).

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