Transverse Gauge Interactions and the Vanquished Fermi Liquid

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The interaction between a Fermi liquid and transverse gauge bosons is considered within the framework of the renormalization group. It is shown from an expansion in $\epsilon = 3 - d$, where d is the spatial dimension, that a nontrivial fixed point emerges for dimensions less than three, and that this fixed point signifies a critical Fermi system different from the conventional Landau-Fermi liquid. The dimension $d = 3$ is the upper critical dimension where the correlation functions contain logarithmic corrections; for $d > 3$, the system behaves like a Landau-Fermi liquid. Some of the consequences of this breakdown of Fermi liquid are discussed.

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It has been known for some time [1,2] that a system of fermions interacting with transverse gauge bosons does not behave like a Fermi liquid. For real electromagnetic interactions, this breakdown occurs at presumably unobservably low temperatures or energies. Thus, it would appear that this problem is of little practical consequence. However, in recent years, it has been repeatedly pointed out that a strongly correlated electron system can be equivalent to a problem of fermions interacting with a fictitious transverse U(1) gauge field, where the gauge coupling is not the fine structure constant ($\frac{1}{137}$) of the real electromagnetic interaction, but of order unity. We do not examine this equivalence, which can indeed be complex, but a vastly simpler well-defined model. It is hoped that a proper elucidation of this problem would lead to insights that may be useful in developing effective low energy theories of realistic physical problems, such as the normal state of high temperature superconductors [3], the state of half-filled quantum Hall systems [4], or quark-gluon plasma [5].

A normal Fermi liquid, at zero temperature, is a critical system; the excitations do not have a gap and are possible at all energy scales. However, despite its criticality, such a system does not normally pose any difficulty if these excitations remain decoupled [6]. The situation is reminiscent of the Gaussian model of statistical mechanics which is critical, but trivial, because all modes decouple. However, there are well-known examples, such as the Kondo problem, in which the nontrivial coupling between the excitations leads to interesting consequences [7]. Another interesting example of the critical nature of the Fermi liquid is Kohn and Luttinger's discovery [8] that, for a Fermi system, it is essential to pay attention as to how the zero temperature, low frequency, and infinite volume limits are approached. Clearly, a system with a gap in the excitation spectrum would behave more normally.

It is now well understood that the Landau-Fermi liquid interactions are marginal perturbations [9–11] that lead to no new conceptual modifications. Are there relevant perturbations that are physically interesting? Indeed, attractive coupling between electrons is relevant and leads to superconductivity with a broken symmetry ground state. Are there relevant perturbations that lead to a critical state which is described by fermion operators with scaling dimensions different from those of the Fermi liquid? It is this possibility in higher dimensions that is fascinating. In the present Letter we examine this possibility. The results are complementary to those obtained by Polchinski [12] and are also broadly consistent with those of Nayak and Wilczek [13] who examined a different model.

The Hamiltonian density for a system of nonrelativistic fermions in interaction with a vector potential $\mathbf{A}$ is

$$H = H_0^\gamma + \frac{1}{2m}(\boldsymbol{\nabla} + ig\mathbf{A})\psi^\dagger \cdot (\boldsymbol{\nabla} - ig\mathbf{A})\psi + U, \quad (1)$$

where we have set both $\hbar$ and the velocity of the gauge bosons to be unity. The effect of the potential interaction U will be assumed to be incorporated in the Hamiltonian of the fermions in the spirit of Landau theory and will not be discussed further, thereby simplifying the problem. H_0^γ is the Hamiltonian density of the free gauge field.

We shall treat the gauge group to be noncompact. This is not an entirely innocuous assumption. Polyakov [14] has shown that in a compact (2 + 1)-dimensional quantum electrodynamics, in the absence of the coupling to the matter field, the gauge quanta are massive due to instanton effects. Clearly, our perturbative renormalization group approach cannot capture these instanton effects, and if the instantons survive in the presence of the coupling to the fermions with a Fermi sea, then the results that we shall derive from an expansion in $\epsilon = 3 - d$ cannot be correct at $\epsilon = 1$. At this time, it is not known for sure if these instantons survive or not. To see why this is a subtle question, see Khlebnikov [15].

In the present Letter we calculate the three one-loop graphs shown in Fig. 1 to determine the renormalization group equations. The method is essentially the standard field theory technique [16]. It is important to note, however, that in a gauge theory single particle Green's function

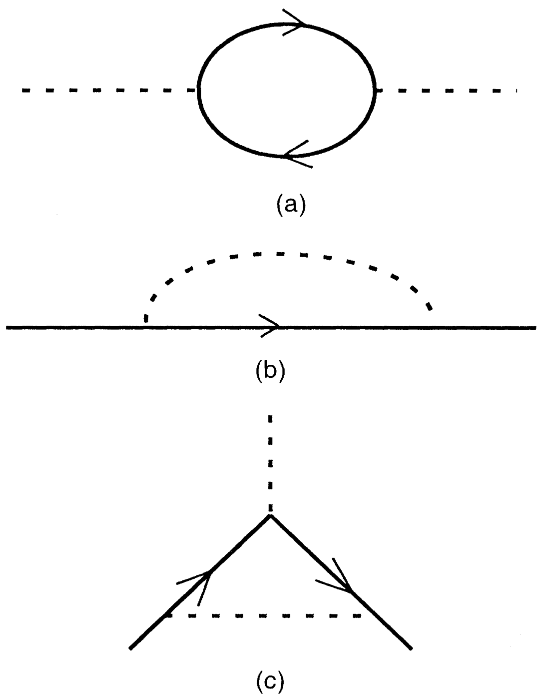

FIG. 1. One-loop diagrams: (a) the gauge boson self-energy, (b) the fermion self-energy, and (c) the one-loop vertex correction. The solid lines correspond to the fermions and the dashed lines correspond to the gauge bosons.

is not a gauge invariant quantity. The renormalization factors clearly depend on the choice of the gauge. One might ask then why we are interested in calculating a non-gauge-invariant quantity. The answer is that we use this calculation as a crutch to obtain the renormalization group β function which *is* gauge invariant, and therefore the critical exponents corresponding to the fixed points must also be independent of the choice of the gauge; below, we discuss further the question of gauge invariance.

First, consider the one-loop calculation of the photon self-energy [Fig. 1(a)] in arbitrary dimension d. To one-loop order, the photon propagator, analytically continued to real frequency ν at $T = 0$, is

$$\left\langle a_i(\mathbf{k}, \nu) a_j^\dagger(\mathbf{k}', \nu') \right\rangle = (2\pi)^{(d+1)} \delta(\nu - \nu') \times \delta^{(d)}(\mathbf{k} - \mathbf{k}') D_{ij}(\mathbf{k}, \nu), \quad (2)$$

where

$$D_{ij}(\mathbf{k}, \nu) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \frac{1}{k^2 - \nu^2 + A(k, \nu)}. \quad (3)$$

Here, $\mathrm{Re}A \sim (\nu/kv_F)^2$, when both ν and $\nu/kv_F \to 0$, and $\mathrm{Im}A \sim -(\nu/kv_F)$ as we approach the real axis from the upper half plane, and $\mathrm{Im}A \sim \nu/kv_F$ as we approach the real axis from the lower half plane; v_F is the Fermi velocity. Because of Landau damping of the gauge bosons, the low frequency behavior of the gauge propagator is significantly modified by the one-loop contribution. That the damping is proportional to ν/kv_F can be seen from the following simple argument. The lifetime of the particle-hole pairs is proportional to $(kv_F)^{-1}$, and we expect the $\mathrm{Im}A$ to contain the dimensionless combination ν/kv_F. We take the one-loop corrected propagator to be the effective gauge propagator for further calculation and consider the effective gauge propagator $D_{ij}^{\mathrm{eff}}(\mathbf{k}, \nu)$, where

$$D_{ij}^{\mathrm{eff}}(\mathbf{k}, \nu \pm i0) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) \frac{1}{k^2 \mp i\gamma(\nu/kv_F)}, \quad (4)$$

where $\gamma \propto (g^2 v_F p_F^{(d-3)}) p_F^2$. From this propagator, the engineering dimension of the gauge field is s^2, where s is the dilatation factor for the units of length and time. The engineering dimension of $g^2 v_F$ is $s^{(3-d)}$. We shall also show from the vertex graph that this is indeed the correct transformation law. Thus, the engineering dimension of the gauge field is given by $a_i(\mathbf{k}, \nu) \to a_i'(\mathbf{k}', \nu') = s^{-(d+3)/2} a_i(\mathbf{k}, \nu)$. To one-loop order, the photon wave function does not receive any anomalous dimension; in the language of quantum electrodynamics, $Z_3 = 1$.

The calculation of the single particle self-energy in arbitrary dimension d is similar to that described in Ref. [1]. From this calculation we obtain the wave-function renormalization factor Z of the fermion field and the renormalization factor Z_{v_F} for the Fermi velocity [13], i.e., $v_{F_0} = Z_{v_F} v_F$; actually, we obtain Z and the product ZZ_{v_F}. To obtain the renormalization group β function to one-loop order it is only necessary to compute the integrals at the fixed dimension $d = 3$. We impose a high frequency cutoff Λ and ask how the bare coupling constant $\alpha_0 = g_0^2 v_{F_0}/4\pi$ scales as we hold the renormalized coupling constant $\alpha = g^2 v_F/4\pi$ fixed at the renormalization point, at a low frequency μ, as we move the cutoff Λ.

The engineering dimension of the free fermion operator can be obtained from the definition of the Green's function and can be shown to be $\psi(\mathbf{p}, \omega) \to \psi'(\mathbf{p}', \omega') = s^{(d+2)/2} \psi(\mathbf{p}, \omega)$. The logarithmic part of the one-loop fermion self-energy Σ [Fig. 1(b)], when $p = p_F$, is

$$\frac{\partial}{\partial \omega} \Sigma(p_F, \omega) = -\frac{\alpha_0}{3\pi} \ln\left(\frac{\Lambda}{|\omega|}\right). \quad (5)$$

In calculating Eq. (5), the energy variable $\varepsilon(\mathbf{k})$ used in the free Green's function was *not* the linearized one [1]. The renormalization factor $Z = [1 - \partial\Sigma(p_F, \omega)/\partial\omega|_\mu]^{-1}$. Therefore,

$$Z = 1 - \frac{\alpha_0}{3\pi} \ln\left(\frac{\Lambda}{|\mu|}\right) + O(\alpha_0^2). \quad (6)$$

In fact, the above result holds as long as $\mu/E_F \gg (|p - p_F|/p_F)^3$. In the opposite limit, i.e., $\mu/E_F \ll (|p - p_F|/p_F)^3$, the self-energy does not contain any infrared anomaly, i.e., there are no logarithmic terms. The easiest way to see this is to calculate the imaginary part of the self-energy, which in this limit, is

$$\Sigma''(\mathbf{p}, \omega) \propto -f\left(\frac{|p - p_F|}{p_F}\right) \omega^2 \,\mathrm{sgn}(\omega), \quad (7)$$

identical to the behavior of a Fermi liquid. The function $f(x)$ is a smooth function of its argument and is given by $[K_{d-1}/(d-6)][1-(x/2)^{d-6}]$, where K_d is the surface area of the d-dimensional unit sphere divided by $(2\pi)^d$. Thus, the non-Fermi-liquid behavior is manifest only when $p \to p_F$ as fast as $\omega \to 0$. This can also be seen from an elegant argument by Polchinski [12]. We conclude, therefore, that, to one-loop order, $ZZ_{v_F} = 1$ because the factor ZZ_{v_F} multiplies the energy dispersion $\varepsilon(\mathbf{k})$ in the fermion propagator.

We now turn to the vertex correction. First, we would like to determine the engineering dimension of the coupling constant g_0. Each vertex corresponds to two fermion field operators (one creation and one destruction) and one gauge field operator. There are two momentum and frequency integrations, one set corresponding to the fermion operators and the other corresponding to the gauge field operator. In addition, the gauge field operator is dotted with the momentum of the incoming fermion and the overall vertex is multiplied by g_0/m. The entire combination should have zero engineering dimension. Thus, from the dimensions of the fermion and the gauge field operators that were determined previously, $g_0^2 v_{F_0}$ has the engineering dimension $s^{(3-d)}$; the engineering dimension of the Fermi velocity v_{F_0} is clearly zero.

It is possible to see that the vertex does not receive any anomalous dimension to one-loop order. From the Ward identity, the second order contribution to the vertex is

$$\hat{\mathbf{p}} \cdot \nabla_{\mathbf{p}} \Sigma(p,\mu)|_{p_F} = \Gamma^{(2)}(\mathbf{p}_F, \mathbf{p}_F, \mu), \tag{8}$$

where μ is the renormalization point, smaller than any relevant frequency in the problem. From the previous considerations of the self-energy, it is clear that there is no $\ln|\Lambda/\mu|$ contribution to $\nabla_{\mathbf{p}}\Sigma(p,\mu)$. Thus, to one-loop order, the vertex renormalization constant Z_g is 1 [17]. The same conclusion was also reached by Polchinski [12] using a different argument.

The bare fermion-gauge coupling is given by g_0, which is given by $g(Z_g/ZZ_{v_F})$ in terms of the renormalized coupling g. However, because both Z_g and ZZ_{v_F} are unity, we have $g_0 = g$. The actual coupling constant that characterizes the physical properties is, however, not g_0 but $\alpha_0 \equiv g_0^2 v_{F_0}/4\pi$. This leads us to the simple relation $\tilde{\alpha}_0 \Lambda^\epsilon Z = \mu^\epsilon \tilde{\alpha}$, where we have defined the dimensionless bare coupling constant $\tilde{\alpha}_0$, by $\alpha_0 = \tilde{\alpha}_0 \Lambda^\epsilon$, and similarly the dimensionless renormalized coupling constant $\tilde{\alpha}$, by $\alpha = \tilde{\alpha}\mu^\epsilon$. The renormalization group β function is obtained by taking the derivative with respect to Λ, holding μ, $\tilde{\alpha}$, and v_F fixed. We get

$$\beta(\alpha_0) \equiv \Lambda \frac{d\tilde{\alpha}_0}{d\Lambda} = -\epsilon\tilde{\alpha}_0 + \frac{\tilde{\alpha}_0^2}{3\pi}. \tag{9}$$

The β function implies that for $d < 3$ there is a nontrivial infrared stable fixed point, which for infinitesimal ϵ is $\tilde{\alpha}_0 = 3\pi\epsilon$. The critical exponent corresponding to this fixed point is universal and is ϵ. Therefore, the quasiparticle weight at $p = p_F$ vanishes as ω^ϵ, as $\omega \to 0$, unlike Fermi liquid theory. As $\epsilon \to 0$, there would be logarithmic corrections and the weight will vanish only as $[\ln(\Lambda/\omega)]^{-1}$, found previously in Ref. [1]. At first sight, one might be tempted to conclude that these results are gauge dependent. However, this is not so. The density of states $\rho(\omega)$ defined by $\rho(\omega) = \pi^{-1}\sum_{\mathbf{k}} \mathrm{Im} G_R(\mathbf{k},\omega)$, where $G_R(\mathbf{k},\omega)$ is the retarded fermion Green's function, is, in fact, invariant with respect to time independent gauge transformations that do not mix the Coulomb part of the interaction with the magnetic part; because of the sum over $\mathbf{k}$, the fermion operators refer to the same spatial point. Note that the infrared anomaly is, to a good approximation, independent of the momentum, and, therefore, to obtain the singular part of the density of states, it is adequate to replace the Green's function by its singular part, which is independent of momentum; the remaining sum over $\mathbf{k}$ simply gives the total number of degrees of freedom. The result is clearly gauge invariant, as it should be. We have also explicitly checked the gauge independence of the infrared anomaly by considering a family of time independent gauge transformations, where the projection operator is $\delta_{\alpha\beta} + (\lambda - 1)q_\alpha q_\beta/q^2$, λ being a gauge parameter.

To avoid misunderstanding, it is important to make a comment here. In our ϵ expansion, the coupling constant is marginally irrelevant in the infrared at $d = 3$. The relevancy is defined with respect to the trivial fixed point. As in φ^4 field theory, we find the nontrivial fixed point by expanding in ϵ and α about the trivial fixed point. We do not attempt an explicit construction of a nontrivial fixed point Hamiltonian. For even the relatively well-understood φ^4 field theory, the direct construction of the nontrivial fixed point Hamiltonian is not known, though all correlation functions can be obtained from the ϵ expansion.

Returning to Eqs. (7) and (8), it is interesting to note that the parameter $|p - p_F|$ acts like a symmetry breaking parameter which cuts off the infrared anomaly, just as in statistical mechanics the magnetic field cuts off the critical fluctuations. In analogy with the magnetic transitions, one can define a critical exponent δ, defined there to be $M \sim H^{1/\delta}$. The crossover exponent δ in our case would be $\frac{1}{3}$; note that we are not implying the existence of an obvious symmetry breaking order parameter.

By integrating out the fermions and writing down an effective action containing only the gauge field, Gan and Wong [18] have argued that all terms beyond the quadratic term are irrelevant, i.e., the wave-function renormalization of the gauge boson Z_3 is unity. Thus, they have concluded, combined with the standard quantum electrodynamics argument $Z_1 = Z_2$, that the renormalization group β function is indentically zero in all dimensions. Z_1 is the renormalization factor for the electron-photon vertex and Z_2 is the same as the Z used here. To see the connection with quantum electrodynamics, we have to identify $Z_1 = Z_g/Z_{v_F}$. We agree that the charge g_0 does not acquire any anomalous dimension. However, the β function is not identically 0, as shown in Eq. (9).

That the wave-function renormalization of the gauge boson is indeed unity, to one-loop order, can also be explicitly checked by calculating the coefficient of the $-T\ln T$ term of the specific heat in $d = 3$. We have carried out this calculation without integrating out the fermions as in Ref. [1], ensuring that all $T\ln T$ terms are taken into account. The calculation is tedious, but we find that the specific heat is given by $C = (g_0^2 p_F^2/36\pi^2)T\ln T$. The result reported in Ref. [1] is incorrect by a factor of 4. According to Gan and Wong, however, the $-T\ln T$ term in the specific heat is entirely contained in the quadratic term in the effective gauge action. An explicit calculation of the specific heat using the quadratic effective gauge action shows that this is indeed true, confirming that, to one-loop order, the gauge field does not receive any anomalous dimension; whether or not this holds to higher-loop order is not clear.

For $d = 3$, an interesting consequence of the Ward identity $g_0 = g$ is the absence in the inverse fermion propagator G^{-1} of all leading logarithms proportional to $\alpha_0^n \ln^n \Lambda$, for all $n \geq 2$. Consider the renormalization group equation $[\Lambda\partial_\Lambda + \beta\partial_{\alpha_0} + \beta_F \partial_{v_{F_0}} - \eta]G^{-1} = 0$, where $\eta = d\ln Z^{-1}/d\ln\Lambda|_{\alpha,v_F,\mu}$, and $\beta_F = dv_{F_0}/d\ln\Lambda|_{\alpha,v_F,\mu}$. However, from the discussion above it follows that $\beta = \alpha_0\eta$ and $\beta_F = v_{F_0}\eta$. Therefore, $[\Lambda\partial_\Lambda + \eta(\alpha_0\partial_{\alpha_0} + v_{F_0}\partial_{v_{F_0}} - 1)]G^{-1} = 0$. If the leading logarithmic parts of G^{-1} are $\alpha_0^n G_n^{-1} \ln^n \Lambda$, then

$$\alpha_0^n G_n^{-1} \frac{d\ln^n \Lambda}{d\ln\Lambda} = -\xi_1\alpha_0\left(\alpha_0\partial_{\alpha_0} + v_{F_0}\partial_{v_{F_0}} - 1\right) \times \alpha_0^{n-1} G_{n-1}^{-1} \ln^{n-1}\Lambda, \tag{10}$$

where $\xi_1\alpha_0$ is the leading order ($\sim\alpha_0$) part of η. By explicit construction, G_1^{-1} does not depend on v_{F_0}. Then, because of the factor $\alpha_0\partial_{\alpha_0} - 1$, the right hand side is 0 for $n = 2$, implying $G_2^{-1} = 0$. Thus, Eq. (10), applied to successively larger values of n, implies that $G_n^{-1} = 0$ for all $n \geq 2$. The same should hold within the ϵ expansion.

In summary, we have shown from a renormalization group analysis that the coupling to a transverse gauge field can destroy the Fermi liquid behavior leading to a universality class with fermion operators having nontrivial scaling dimensions for $d \leq 3$. We have also shown how a systematic expansion in powers of $\epsilon = 3 - d$ can be carried out in a problem that superficially does not contain a small expansion parameter due to infrared anomalies in perturbation theory. In the future we plan to extend our present analysis.

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