Sudden Replacement of a Mirror by a Detector in Cavity QED: Are Photons Counted Immediately?

Heidi Fearn
Department of Physics, California State University, Fullerton, California 92634-9480

Richard J. Cook
Department of Physics, United States Air Force Academy, Colorado Springs, Colorado 80840

Peter W. Milonni
Theoretical Division (T-4), Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 19 September 1994)

We consider an excited atom in a cavity such that spontaneous emission is inhibited, and address the question of whether a sudden replacement of one of the cavity mirrors by a detector can result in a photon count immediately or only after some retardation time. The feasibility of an experiment of this type has led to considerable discussion as to its outcome. Following a brief summary of the conflicting arguments, we show that it is possible to count a photon immediately following the substitution of a photodetector for a mirror.

PACS numbers: 42.50.–p, 12.20.–m

Recent experiments on two-photon down-conversion [1] have extended the domain of cavity QED to include nonlinear optical processes and much larger emitter-mirror separations than have been possible in experiments with atoms in cavities [2]. Such experiments also allow the possibility, through the use of polarization-sensitive mirrors and fast Pockels cells, of investigating effects associated with the sudden replacement of a cavity mirror by a detector [3]. This possibility has stimulated considerable discussion about whether photons, under conditions of cavity-inhibited emission, can be counted immediately following the substitution of a photodetector for a mirror or only after some retardation time relating to the propagation of light between the emitter and the detector.

The same question can be raised in the context of ordinary cavity QED involving a single excited atom in a cavity. Suppose that spontaneous emission is completely inhibited by the cavity and that at time T one of the mirrors is suddenly replaced by a photodetector. Can a photon be counted immediately at time T, or is the photon count ideally zero until some time $T' = T + T_R$, where T_R is a retardation time determined by the distance between the atom and the detector that has replaced the mirror [4]?

Two plausible explanations, leading to different answers, have been proposed. According to one argument, the inhibited atom cannot "know" the mirror has been removed until the time $t = T + d/c$, where d is the atom-mirror distance, and the atom can begin to radiate only after this time. Since the propagation time to the detector is d/c, a photon can be detected only after a time $t + d/c = T + 2d/c$, i.e., after a time $2d/c$ following the mirror switchout.

The second viewpoint holds that, as in the case of a classical dipole radiator in a cavity, there are always fields (or, more precisely, probability amplitudes) propagating from the atom to the removable mirror and back to the atom, and that the inhibition of spontaneous emission implies a destructive interference of the two counterpropagating fields. The sudden removal of the mirror allows that part of the field propagating toward the mirror to escape from the cavity, so that a photon can be counted immediately following the switchout of the mirror.

In the absence of detailed calculations or an experiment, objections can be raised against either prediction. The first argument makes no reference to counterpropagating, destructively interfering waves or probability amplitudes. The second argument might appear to violate energy conservation, since it apparently predicts an immediately nonvanishing photon counting rate at time T while the atom is held in its excited state, spontaneous emission being inhibited until the atom can somehow receive the information that the mirror has been switched out.

We will show that a photon can be counted immediately following the replacement of a mirror by a detector. Because the analysis of any specific, real experiment will involve complications irrelevant to the question of interest, we consider an idealized model. This model consists of a two-level atom in the presence of a single plane mirror, and an electric-dipole atom-field interaction restricted to singly polarized field modes propagating only in the two directions normal to the mirror (Fig. 1). The Heisenberg-picture electric field operator is $E(z,t) = E_0(z,t) + E_s(z,t)$, where $E_0(z,t)$ is the free field in the absence of any sources and

$$E_s(z,t) = 2\pi i \mu \int_0^t dt' [\sigma(t') + \sigma^\dagger(t')] \times \sum_k \omega_k U_k(z) U_k(z_0) e^{i\omega_k(t'-t)} + \text{H.c.} \quad (1)$$

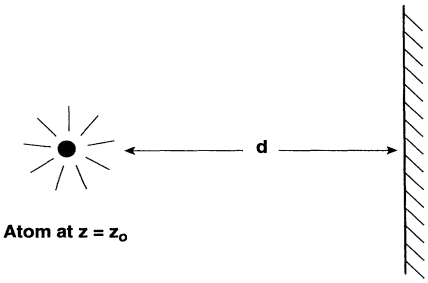

FIG. 1. Two-level atom with z coordinate z_0 near a plane infinite mirror at $z = L$. The field is restricted to modes with **k** vectors parallel to the z axis.

is the source field due to the atom. Here σ and $\sigma^\dagger$ are the usual two-level lowering and raising operators, respectively, μ is the electric-dipole matrix element for the two-level atom of transition frequency ω_0, and $U_k(z)$ is a mode function normalized in a volume of cross-sectional area A and length L. In our model $U_k(z) = (2/AL)^{1/2}\sin k(L-z)$, so that $U_k(L) = 0$ at the (perfectly conducting) mirror. In the limit $L \to \infty$, $\sum_k \to (L/\pi)\int dk = (L/\pi c)\int d\omega$ and

$$E_s(z,t) \to \frac{4\mu}{cA}\int_0^t dt'[\sigma(t') + \sigma^\dagger(t')] \times \frac{\partial}{\partial t'}\int_{-\infty}^{\infty} d\omega \sin\frac{\omega}{c}(L-z) \times \sin\frac{\omega}{c}(L-z_0)e^{i\omega(t'-t)}. \quad (2)$$

We let $z, z_0, L \to \infty$ in such a way that $z - z_0$, $L - z_0$, and $L - z$ remain finite and positive; these limits are those appropriate for an atom at a distance $d = L - z_0$ from a single mirror. In this limit [5]

$$E_s(z,t) = -\frac{2\pi\mu}{cA}\left[\dot{\sigma}_x\left(t - \frac{z-z_0}{c}\right) - \dot{\sigma}_x\left(t - \frac{2L-z-z_0}{c}\right)\right] \quad (z > z_0), \quad (3)$$

where $\sigma_x \equiv \sigma + \sigma^\dagger$. It is important to note that *no approximations have been made in the derivation of this result from the Hamiltonian for our model.*

We now make a rotating-wave approximation, exactly as in the case of an atom in free space, by taking $\dot{\sigma}(t) \cong -i\omega_0\sigma(t)$ and therefore

$$E_s(z,t) \cong \frac{2\pi i\mu\omega_0}{cA}\left[\sigma\left(t - \frac{z-z_0}{c}\right) - \sigma\left(t - \frac{2L-z-z_0}{c}\right)\right] - \frac{2\pi i\mu\omega_0}{cA}\left[\sigma^\dagger\left(t - \frac{z-z_0}{c}\right) - \sigma^\dagger\left(t - \frac{2L-z-z_0}{c}\right)\right] \equiv E_s^{(+)}(z,t) + E_s^{(-)}(z,t), \quad (4)$$

where the positive- and negative-frequency parts of the field are given approximately by [6]

$$E_s^{(+)}(z,t) \cong \frac{2\pi i\mu\omega_0}{cA}\left[\sigma\left(t - \frac{z-z_0}{c}\right) - \sigma\left(t - \frac{2L-z-z_0}{c}\right)\right] \quad (5)$$

and $E_s^{(-)}(z,t) = E_s^{(+)}(z,t)^\dagger$. In particular, at the position $z = z_0$ of the atom, we obtain from (5) the radiation reaction field

$$E_s^{(+)}(z_0,t) = \frac{2\pi i\mu\omega_0}{cA}\left[\sigma(t) - \sigma\left(t - \frac{2d}{c}\right)\right] \cong \frac{2\pi i\mu\omega_0}{cA}[1 - e^{2i\omega_0 d/c}]\sigma(t) = 0 \quad (6)$$

for $e^{2i\omega_0 d/c} = 1$, where $d = L - z_0$ is the distance of the atom from the mirror. In other words, if $e^{2i\omega_0 d/c} = 1$ the radiation reaction responsible for spontaneous emission [7] vanishes and spontaneous emission is inhibited.

The first term in brackets in Eq. (5) is a retarded field propagating from z_0 to z. The second term involves propagation from z_0 to the mirror and then to z. These terms therefore correspond to fields propagating in the positive and negative z directions, respectively, as can also be seen from a plane-wave expansion of the field. The "forward"-propagating field

$$E_{s,F}^{(+)}(L,T) \cong \frac{2\pi i\mu\omega_0}{cA}\sigma\left(T - \frac{d}{c}\right) \quad (7)$$

for $z = L$ *can be measured instantaneously:* the photon count rate at an ideal broadband photodetector replacing the mirror at $z = L$ and $t = T$ is proportional to the normally ordered correlation function

$$\left\langle E_{s,F}^{(-)}(L,T)E_{s,F}^{(+)}(L,T)\right\rangle \cong \left(\frac{2\pi\mu\omega_0}{cA}\right)^2\left\langle\sigma^\dagger\left(T - \frac{d}{c}\right)\sigma\left(T - \frac{d}{c}\right)\right\rangle = \left(\frac{2\pi\mu\omega_0}{cA}\right)^2 P\left(T - \frac{d}{c}\right), \quad (8)$$

where $P(t)$ is the probability at time t that the atom is in the excited state. There is thus an immediately nonvanishing photon counting rate at $z = L$ when the detector replaces the mirror at time $t = T$. The rate at which field energy is lost from the cavity when the mirror is switched out at $t = T$ is

$$R(T) = \frac{cA}{4\pi}\left\langle E_{s,F}^{(-)}(L,T)E_{s,F}^{(+)}(L,T)\right\rangle = \frac{cA}{4\pi}\left(\frac{2\pi\mu\omega_0}{cA}\right)^2 P\left(T - \frac{d}{c}\right) = \beta\hbar\omega_0 P\left(T - \frac{d}{c}\right). \quad (9)$$

Here $\beta \equiv \pi\mu^2\omega_0/\hbar cA$ is half the spontaneous emission rate in the "free space" limit $d \to \infty$ in our model [8]. Both (7) and (9) are applicable at any time $t > d/c$ replacing T.

The probability that the atom at time t is in the excited state may be shown to be given by [8]

$$P(t) = \left| \sum_{n=0}^{\infty} \frac{\beta^n}{n!} \left(t - \frac{2nd}{c}\right)^2 e^{-\beta(t-2nd/c)} \theta\left(t - \frac{2nd}{c}\right) \right|^2 \tag{10}$$

for the case $e^{2ik_0 d} = 1$ of interest here. For times t sufficiently large compared with the "photon bounce time" $2d/c$, $P(t)$ reaches a steady-state value; for $2\beta d/c \ll 1$—the domain of standard cavity QED experiments—this steady-state value is $P_s \cong e^{-2\beta d/c}$. Assuming $T \gg 2d/c$, therefore, which must be the case if the mirror is switched out after a time when spontaneous emission is inhibited ($P > 0$, $\dot{P} = 0$), we have

$$R(T) = \beta\hbar\omega_0 P_s. \tag{11}$$

This has the following interpretation. The steady-state inhibition of spontaneous emission, i.e., the fact that the atom is not losing any net energy to the field by spontaneous emission, implies that any radiation emitted toward the mirror is exactly balanced by radiation reflected back from the mirror, a result consistent with (6). In particular, the rate at which radiant energy is transported toward the mirror is half the spontaneous emission rate, βP_s, times $\hbar\omega_0$, which is just (11). (The 1/2 is required because we are considering only the energy in the one-sided region $z_0 < z < L$.)

After the mirror is switched out at $t = T$, the atom cannot begin to lose energy to spontaneous emission until a time d/c later. The fact that we can nevertheless count a photon *before* $t = T + d/c$ might therefore appear to violate energy conservation. Consider, however, the (cycle-averaged) energy $W_F(T)$ associated with forward-propagating radiation in the space $z_0 < z < L$ at time T:

$$\begin{aligned} W_F(T) &= \frac{A}{4\pi} \int_{z_0}^{L} dz \left\langle E_{s,F}^{(-)}(z,T) E_{s,F}^{(+)}(z,T) \right\rangle = \frac{A}{4\pi} \left(\frac{2\pi\mu\omega_0}{cA}\right)^2 \int_{z_0}^{L} dz\, P\left(T - \frac{z - z_0}{c}\right) = \beta\hbar\omega_0 \int_{T-d/c}^{T} dt'\, P(t') \\ &= \int_{T-d/c}^{T} dt'\, R(t') = \frac{d}{c} R(T) = \frac{\beta d}{c} \hbar\omega_0 P_s. \end{aligned} \tag{12}$$

There is similarly a nonvanishing energy associated with backward-propagating radiation, as well as an interference between forward- and backward-propagating radiation. It follows that the nonvanishing photon counting rate at $t = T$ occurs not at the expense of the atom, but rather as a depletion of *field* energy, i.e., a depletion of the energy associated with the backward-propagating field and the interference of the counterpropagating fields. We now take up this point in more detail.

Our analysis thus far has relied on the intuitive idea that the Poynting vector associated with the forward-propagating radiation alone gives the rate of energy depletion from the cavity when the mirror is suddenly removed. To better appreciate where the energy comes from to register a count at the detector, we consider now the time dependence of the cavity energy after the mirror is removed. The mirror switchout at $t = T$ will affect the backward-propagating field in such a way that the field (5) is replaced by

$$\begin{aligned} E_s^{(+)}(z,t) \cong \frac{2\pi i\mu\omega_0}{cA} \Bigg[&\sigma\left(t - \frac{z - z_0}{c}\right)\theta\left(t - \frac{z - z_0}{c}\right) \\ &- \sigma\left(t - \frac{2L - z - z_0}{c}\right)\theta\left(t - \frac{2L - z - z_0}{c}\right)\theta\left(T - t + \frac{L - z}{c}\right)\Bigg], \end{aligned} \tag{13}$$

where now we have explicitly included all appropriate step functions. The last step function accounts for the fact that backward-propagating waves persist at point z at times $t > T$ only if $L - z > c(t - T)$, i.e., if the information that the mirror is gone at $t = T$ has not yet propagated to z. Based on this expression we calculate, in a manner analogous to (12), the cavity energy associated with the backward-propagating waves plus the interference of the forward- and backward-propagating waves in the region $z_0 < z < L$:

$$W_{\mathrm{B,BFI}} \cong \frac{1}{c} \beta\hbar\omega_0 [d - c(t - T)] P_s \tag{14}$$

for $T < t < T + d/c$. Thus

$$\frac{d}{dt} W_{\mathrm{B,BFI}}(t) = -\beta\hbar\omega_0 P_s \quad (T < t < T + d/c), \tag{15}$$

which is just $-R(T)$ [Eq. (11)], i.e., the rate at which energy associated with the *forward-propagating* radiation will escape from the cavity when the mirror is switched out.

This confirms our assertion that the immediate detection of a photon, in spite of inhibited spontaneous emission, occurs at the expense of cavity field energy, or actually the change in field energy associated with the backward-propagating radiation and its interference with forward-propagating radiation when the mirror is switched out. It is precisely this change, according to (11) and

(15), that propagates *out* of the cavity and that can produce a photon count. We emphasize again that this occurs in spite of the fact that, back at the atom, there is still destructive interference and inhibited spontaneous emission, and a constant upper-state probability P_s, until time $T + d/c$. The *total* field energy has a constant expectation value up until this time. After $t = T + d/c$, of course, the atom radiates as it does ordinarily in free space. All these results are confirmed by a detailed microscopic model for a switchable mirror, analogous to the treatment in Ref. [8], which will be presented elsewhere.

It is perhaps worth noting why, as a consequence of retardation, there will always be some field energy in the cavity before the mirror is removed. For a time $t = 2d/c$ after the atom is excited at $t = 0$, say, it will radiate uninhibitedly as if in free space. For times $0 < t < 2d/c$, therefore, the energy in the field is

$$W(t) = \hbar\omega_0[1 - P(t)] = \hbar\omega_0[1 - e^{-2\beta t}], \tag{16}$$

and the rate at which the field energy grows is

$$\dot{W}(t) = 2\beta\hbar\omega_0 e^{-2\beta t}. \tag{17}$$

At time $t = d/c$, when the radiated field reaches the mirror,

$$\dot{W}(d/c) = 2\beta\hbar\omega_0 e^{-2\beta d/c}, \tag{18}$$

which, for $2\beta d/c \ll 1$, is approximately $2\beta\hbar\omega_0 P_s$, as noted earlier. The rate at which the energy of the *forward-propagating* radiation grows is half this, i.e., $R(d/c) \cong \beta\hbar\omega_0 P_s = R(T)$, as given by Eq. (11). As the atom quickly attains a steady state for the special case $2\beta d/c \ll 1$ under consideration, the rate at which energy is put into the forward-propagating field will quickly equilibrate to the value $R(T)$. It is precisely this power that can be registered at the detector replacing the mirror. There is no contradiction with the fact of inhibited spontaneous emission because, although $\dot{P} = 0$, the steady-state probability P_s is always less than unity.

It also seems worth noting that, in the Schrödinger picture, the atom-field system at time $t < T$ is described to an excellent approximation by the state vector

$$|\psi(t)\rangle = a(t)|\text{atom excited}\rangle\,|\text{no photons}\rangle + \sum_k a_k(t)\,|\text{atom in lower state}\rangle \times |\text{one photon in mode } k\rangle, \tag{19}$$

where $P(t) = |a(t)|^2$ is the probability that the atom is excited at time t. In the steady state of inhibited spontaneous emission, $P = P_s < 1$, i.e., $a, a_k < 1$ and the atom-field system is in an "entangled" state reminiscent of the Schrödinger cat paradigm.

We thank R. Y. Chiao, F. De Martini, D. Greenberger, L. Hardy, T. J. Herzog, P. G. Kwiat, H. Weinfurter, and A. Zeilinger for particularly helpful and lively discussions. P. W. M. also thanks Professor A. Zeilinger and others for their hospitality at the University of Innsbruck, where the question addressed in this paper first came to his attention and where his unsure intuition led to detailed calculations culminating in the simplified model considered here.

[1] T. J. Herzog, J. G. Rarity, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **72**, 629 (1994).

[2] See, for instance, S. Haroche and D. Kleppner, Physics Today, 25 (January, 1989); E. A. Hinds, in *Advances in Atomic, Molecular, and Optical Physics,* edited by D. R. Bates and B. Bederson (Academic Press, Boston, 1990), Vol. 28; *Cavity Quantum Electrodynamics,* edited by P. R. Berman (Academic Press, San Diego, 1994).

[3] A. Zeilinger, private communication.

[4] We assume that the detector is at the same distance from the atom as the mirror was, thereby avoiding any additional, trivial retardation time.

[5] Unit step functions $\theta(t - (z - z_0)/c)$ and $\theta(t - (2L - z - z_0)/c)$ are implicit in the first and second terms, respectively. To simplify the equations we omit the step functions except where they are crucial to our discussion. A term involving a step function with argument $t - (2L - z + z_0)/c$, corresponding to propagation from the atom to the mirror at $z = 0$, reflection off this mirror and propagation to the mirror at $z = L$ followed by propagation to z, is absent in Eq. (3) as a consequence of our assumption $L \to \infty$, i.e., that the mirror at $z = 0$ is infinitely far from the atom in our model.

[6] Actually $E_s^{(+)}(z,t)$ has a small contribution from terms involving $\sigma^\dagger$, but such terms play no role in our discussion. See J. R. Ackerhalt, Phys. Rev. A **17**, 471 (1978).

[7] P. W. Milonni, J. R. Ackerhalt, and W. A. Smith, Phys. Rev. Lett. **31**, 958 (1973); P. W. Milonni and W. A. Smith, Phys. Rev. A **11**, 814 (1975).

[8] R. J. Cook and P. W. Milonni, Phys. Rev. A **35**, 5081 (1987).