Positivity Constraints for Spin-Dependent Parton Distributions

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We derive new positivity constraints on the spin-dependent structure functions of the nucleon. These model-independent results reduce considerably their domain of allowed values, in particular, for the chiral-odd parton distribution $h_1(x)$.

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Nucleon parton distributions are important physical quantities which contain crucial information about the fundamental properties of the nucleon structure. Precise knowledge of parton distributions is also needed if one wants to explore hard scattering processes at future hadron colliders. For many years, spin-independent parton distributions have been accurately measured in a large number of experiments, in particular, deep inelastic scattering, and they are now known in a wide kinematic range. The experimental program going on at HERA (hadron electron ring accelerator, Hamburg, Germany) will further increase this kinematic domain with smaller x and larger Q^2. The situation is rather different for spin-dependent parton distributions whose experimental determination has been improved only recently with new measurements [1] of $g_1^p(x)$ and $g_1^n(x)$, at both CERN and SLAC by means of proton and neutron polarized deep inelastic scattering. These polarized structure functions provide us with some insight into the quark (or antiquark) helicity distributions usually called $\Delta q(x)$ [or $\Delta\overline{q}(x)$]. But in addition to the spin average quark distributions $q(x)$ and these helicity distributions $\Delta q(x)$, there is a third class of quark distributions called transversity distributions and denoted $h_1^q(x)$. These physical quantities which violate chirality [2–4] decouple from deep inelastic scattering but can be measured in Drell-Yan processes with both beam and target transversely polarized. So far there is no experimental data on these distributions $h_1^q(x)$ [or $h_1^{\overline{q}}(x)$], but there are some attempts to calculate them either in the framework of the MIT bag model [3] or by means of QCD sum rules [5].

The purpose of this Letter is to use positivity to derive model-independent constraints on $h_1^q(x)$, which will restrict substantially the domain of allowed values [6]. Similar constraints can be obtained for higher-twist parton distribuions, as we will see below.

Let us consider quark-nucleon elastic scattering $q(h) + N(H) \longrightarrow q(h') + N(H')$ (h, h' and H, H' are the helicities of the quark and nucleon, respectively) which is described in terms of five the s-channel helicity amplitude, denoted by $\langle h'H'|hH\rangle$ [7]. In the forward direction, as a consequence of helicity conservation, only three independent amplitudes are nonvanishing, namely, $\varphi_1^s = \langle ++|++\rangle$, $\varphi_3^s = \langle +-|+-\rangle$, and $\varphi_2^s = \langle +-|-+\rangle$, whose imaginary parts are simply related to total cross sections by the optical theorem.

The forward quark-nucleon amplitude is a 4×4 matrix M in the space where the basis states are $|++\rangle$, $|+-\rangle$, $|-+\rangle$, and $|--\rangle$. Positivity requires that $a^+Ma \geq 0$, where "a" is any 4-component vector in this space. This implies essentially three conditions [8],

$$\mathrm{Im}\varphi_1^s|_{t=0} \geq 0\,, \quad \mathrm{Im}\varphi_3^s|_{t=0} \geq 0\,, \tag{1}$$

and

$$\mathrm{Im}\varphi_3^s|_{t=0} \geq |\mathrm{Im}\varphi_2^s|_{t=0}\,. \tag{2}$$

Now the three quark distributions considered above, $q(x)$, $\Delta q(x)$ [denoted $f_1(x)$ and $g_1(x)$ in Ref. [3]], and $h_1^q(x)$, are defined by the light-cone Fourier transformation of bilinear quark operators between nucleon states [3]. In fact these quark distributions are related to the corresponding u-channel quark-nucleon helicity amplitudes φ_1^u's, which are simply obtained from the φ_i^s's by quark line reversal and we have

$$\begin{aligned} q(x) &= \tfrac{1}{2}\,\mathrm{Im}(\varphi_1^s + \varphi_3^s)|_{t=0}\,, \\ \Delta q(x) &= \tfrac{1}{2}\,\mathrm{Im}(\varphi_3^s - \varphi_1^s)|_{t=0}\,, \\ h_1^q(x) &= \tfrac{1}{2}\,\mathrm{Im}\varphi_2^s|_{t=0}\,. \end{aligned} \tag{3}$$

Using Eq. (3), the constraints (1) and (2) read in terms of the parton distributions

$$q(x) \geq 0\,, \quad q(x) \geq |\Delta q(x)|\,, \tag{4}$$

and

$$q(x) + \Delta q(x) \geq 2|h_1^q(x)|\,. \tag{5}$$

This result can also be derived in a simple way, which we will indicate now. Since the quark distributions are related, via the optical theorem, to the forward quark-nucleon scattering amplitudes, one can write for the quark distribution q_+ (q_-), with helicity parallel (antiparallel) to that of the nucleon,

$$q_\pm(x) = \sum_X \langle N_+|O^+|q_\pm, X\rangle\langle X, q_\pm|O|N_+\rangle\,, \tag{6}$$

where one has to sum over all intermediate states X.

Then clearly one has

$$h_1^q(x) = \sum_X \langle N_-|O^+|q_-, X\rangle\langle X, q_+|O|N_+\rangle\,. \tag{7}$$

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Now if one denotes $a_X^\pm = \langle N_\pm | O | q_\pm, X\rangle$, by using parity conservation, one can write

$$q_+(x) = \frac{1}{2}\sum_X (|a_X^+|^2 + |a_X^-|^2)\,,$$
$$h_1(x) = \frac{1}{2}\sum_X (a_X^{+*}a_X^- + a_X^+ a_X^{-*})\,, \tag{8}$$

and, as a consequence of $|a_X^+ \pm a_X^-| \geq 0$, one gets immediately

$$q_+(x) > |h_1(x)|\,, \tag{9}$$

which is equivalent to Eq. (5), since $q(x) + \Delta q(x) = 2q_+(x)$.

Whereas the first two constraints (4) are familiar and quite obvious, the third constraint (5), which is much less trivial, was ignored so far. We show, in Fig. 1, the region allowed by Eq. (5) which is half the region obtained by considering instead

$$q(x) \geq |h_1^q(x)|\,, \tag{10}$$

as proposed in Ref. [3].

Clearly the same constraint (5) holds for all quark flavor $q = u, d, s$, etc. and for their corresponding antiquarks. Obviously any theoretical model should satisfy these constraints. In a toy model [9] where the proton is composed of a quark and a scalar diquark, one obtains the equality in Eq. (5) [10]. In the MIT bag model, let us recall that these distributions read [3]

$$q(x) = f^2(x) + g^2(x)\,, \quad \Delta q(x) = f^2(x) - 1/3g^2(x)\,,$$
$$h_1^q(x) = f^2(x) + 1/3g^2(x)\,, \tag{11}$$

and they also saturate (5). In this case, we observe that $h_1^q(x) \geq \Delta q(x)$, but this situation cannot be very general because of Eq. (5). As an example, let us assume $h_1^q(x) = 2\Delta q(x)$. Such a relation cannot hold for all x, and we see that Eq. (5), in particular, if $\Delta q(x) > 0$, implies $q(x) \geq 3\Delta q(x)$. This is certainly not satisfied for all x by the present determination of the u quark helicity distribution, in particular, for large x, where $A_1^p(x)$ is large [1]. The simplifying assumption $h_1^q(x) = \Delta q(x)$, based on the nonrelativistic quark model, which has been used in some recent calculations [11,12] is also not acceptable for all x values, if $\Delta q(x) < 0$ because of Eq. (5). To illustrate the practical use of Eq. (5), let us take, as an example, the simple relation

$$\Delta u(x) = u(x) - d(x) \tag{12}$$

proposed in Ref. [13] and which is well supported by the data [1]. It is then possible to obtain the allowed range of values for $h_1^u(x)$, namely,

$$u(x) - \frac{1}{2}d(x) \geq |h_1^u(x)|\,, \tag{13}$$

which is shown in Fig. 2, where Ref. [13] was used to evaluate $u(x)$ and $d(x)$. In this case, we see that for $x > 0.5$, the results of both the MIT bag model [3] and the QCD sum rule [5] violate our positivity bound, combined with low Q^2 data. A similar calculation can be done for the d quarks and the allowed region for $h_1^d(x)$ is shown in Fig. 3.

We also want to remark that Eq. (5) can be use to put a bound on the "tensor charge" δq, whose expression in terms of $h_1^q(x)$ and $h_1^{\bar{q}}(x)$ is

$$\int_0^1 \left[h_1^q(x) - h_1^{\bar{q}}(x)\right] dx = \delta q\,. \tag{14}$$

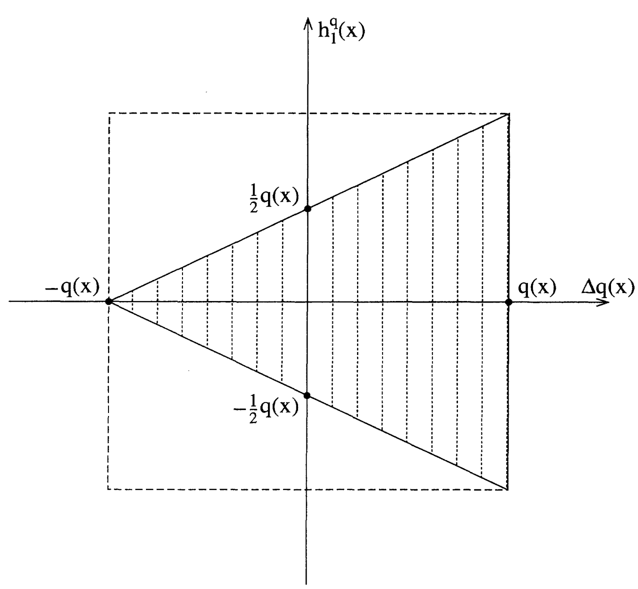

FIG. 1. The striped area represents the domain allowed by positivity.

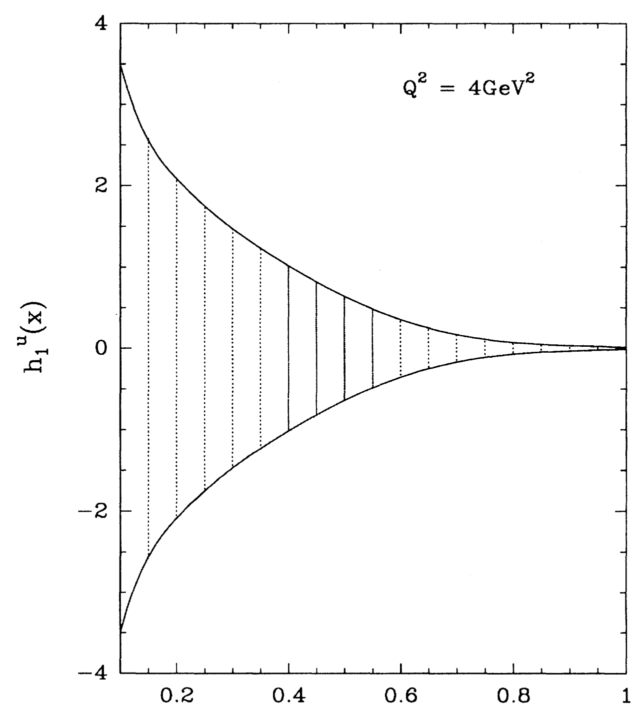

FIG. 2. The striped area represents the domain allowed for $h_1^u(x)$, using Eq. (13) and Ref. [13].

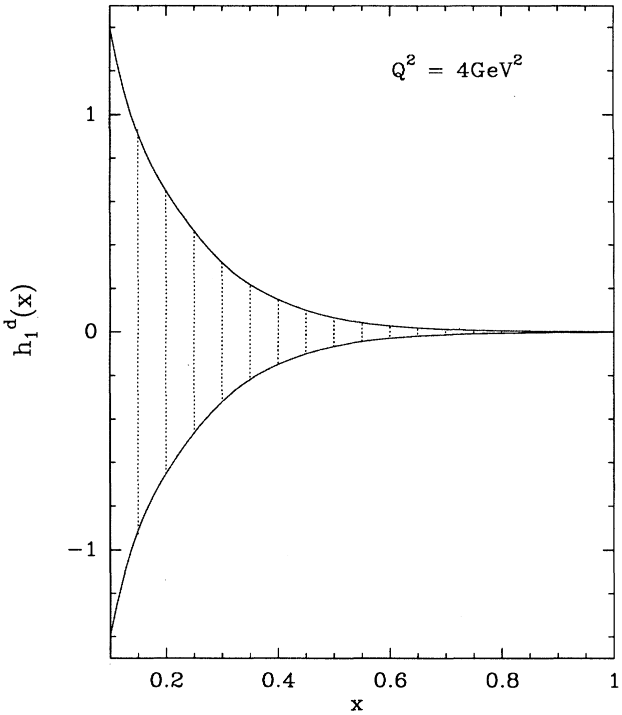

FIG. 3. The striped area represents the domain allowed for $h_1^d(x)$, using Eq. (5) and Ref. [13].

As noticed in Ref. [3], since it is a difference of quarks *minus* antiquarks, sea quarks do not contribute to δq. Now by making the reasonable assumption that Eq. (5) holds for valence quarks separately, one obtains

$$|\delta q| \leq \frac{1}{2} \int_0^1 [q_{\mathrm{val}}(x) + \Delta q_{\mathrm{val}}(x)]\, dx\,. \tag{15}$$

For u quarks we get

$$|\delta u| \leq 1 + \frac{1}{2} \int_0^1 \Delta u_{\mathrm{val}}(x)\, dx\,, \tag{16}$$

and for d quarks

$$|\delta d| \leq \frac{1}{2} + \frac{1}{2} \int_0^1 \Delta d_{\mathrm{val}}(x)\, dx\,. \tag{17}$$

By using the results of Ref. [13], one obtains

$$|\delta u| \leq \frac{3}{2} \quad \text{and} \quad |\delta d| \leq \frac{1}{3}\,. \tag{18}$$

These results are consistent with recent estimates of the tensor charges [14] and, in particular, in the MIT bag model one finds $\delta u = 1.17$ and $\delta d = -0.29$.

So far we have only considered the three twist-two quark (antiquark) distributions, but the above results, and in particular Eq. (5), are also valid for higher-twist distributions, which have been identified in Ref. [3]. For each twist, one has three independent amplitudes which have the same helicity properties as for twist two. Thus, up to twist four, one defines nine quark distributions, i.e., $q(x)$, $\Delta q(x)$, and $h_1^q(x)$ for twist two, and, following the notations of Ref. [3], $e(x)$, $g_T(x)$, and $h_L(x)$ for twist three, and $f_4(x)$, $g_3(x)$, and $h_3(x)$ for twist four.

So it is clear that we have the following constraints for the twist-three distributions:

$$e(x) + h_L(x) \geq 2|g_T(x)|\,, \tag{19}$$

and for the twist-four distributions

$$f_4(x) + g_3(x) \geq 2|h_3(x)|\,, \tag{20}$$

where we have used the notations of Ref. [3]. There are theoretical calculations based on the MIT bag model [3,15] for the twist-three distributions and we hope they satisfy the constraint (19).

None of the above generalized distributions, which are associated to quark-gluon dynamics, have been measured so far. As discussed in Ref. [3], the most natural place to learn about them is probably unpolarized and polarized Drell-Yan and semi-inclusive processes. We hope extensive studies both theoretical and experimental will be undertaken in the future, where full use will be made of our new significant constraints (5), (19), and (20).

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