Transition Radiation of the Neutrino Magnetic Moment

M. Sakuda* and Y. Kurihara

National Laboratory for High Energy Physics (KEK), Ibaraki-ken 305, Japan
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If the neutrino has a finite mass and a magnetic moment, it would produce transition radiation when crossing the interface between two media of which plasma frequencies are ω_1 and ω_2 ($\omega_1 \gg \omega_2$). We found that the probability of transition radiation is larger by an order of magnitude using the quantum theory than that recently reported by one of us using classical electrodynamics, and that the energy spectrum of the radiation is uniform up to $\sim \gamma \omega_1$, where γ is the Lorentz factor of the neutrino ($\gamma = E_\nu / m_\nu$).

In the standard model [1] with the right-handed neutrino singlet (ν_R) the magnetic moment of the neutrino is induced by radiative corrections, and is estimated to be negligibly small: $\mu_\nu = (3 \times 10^{-19} m_\nu) \mu_B$ [2], where m_ν is the neutrino mass in units of eV and μ_B is the electron Bohr magneton. Thus, the existence of a neutrino magnetic moment at an order of $10^{-10} \mu_B$ would require a modification of the standard model of the electroweak interaction [3]. It might also explain the solar-neutrino problem [4–6]; further, the plasmon decay into a neutrino-antineutrino pair ($\gamma^* \to \nu \bar{\nu}$) would play a more important role in the stellar cooling process [7]. The present experimental upper bounds on the neutrino magnetic moment are $\mu(\nu_e) \lesssim 10^{-10} \mu_B$ [8,9], $\mu(\nu_\mu) \lesssim 10^{-9} \mu_B$ [9,10], and $\mu(\nu_\tau) \lesssim 10^{-6} \mu_B$ [11] at the 90% C.L. These experimental searches have been performed using the process of neutrino-electron elastic scattering [12] and the $e^+ e^- \to \gamma \nu \bar{\nu}$ process. However, there are other important processes of the electromagnetic interaction of the neutrino with matter: Cherenkov radiation and transition radiation. The possibility of Cherenkov radiation of the neutrino magnetic moment in 1 km^3 of water has recently been studied by Grimus and Neufeld [13,14]. The transition radiation of the neutrinos having a magnetic moment and a mass was recently discussed by one of us using classical electrodynamics [15]. However, the previous calculation concerning the transition radiation is not appropriate for the case of neutrinos, since such quantum-mechanical effects as the change in the spin orientation and the recoil of the neutrino during the interaction were not taken into account. In this Letter we revise the calculation of the transition radiation of a neutrino magnetic moment using quantum theory.

Transition radiation (TR) is produced when a charged particle or a particle with a magnetic moment traverses the interface between two different media [16,17]. In quantum theory, the electromagnetic interaction of the neutrino is described in terms of the Lagrangian density,

$$\mathcal{L} = \frac{\mu_\nu}{2} \bar{\psi} \sigma_{\mu\nu} \psi F^{\mu\nu}, \tag{1}$$

where μ_ν is the magnetic moment defined at the rest frame of the neutrino, ψ is the neutrino wave func-

tion, $\sigma_{\mu\nu} = \frac{i}{2}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$, and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ is the electromagnetic tensor. The phenomenological quantum theory of the TR of a charged particle was first given by Garibyan [18]. It is quite different from the explanation given by classical electrodynamics. We will present a calculation of the TR of the neutrino magnetic moment following Refs. [18,19]. The process is illustrated in Fig. 1. The four-momentum vector of a photon in a medium having a refractive index of n and satisfying the Maxwell equations is given by

$$k^\mu = (\omega, \mathbf{k}), \text{ with } |\mathbf{k}| = n\omega, \tag{2}$$

where ω is the energy of the photon. The magnetic permeability is assumed to be unity. The effective mass-squared of the photon is thus given by

$$k^2 = (1 - n^2)\omega^2. \tag{3}$$

In a uniform medium, the radiation process, $\nu(p_1) \to \nu(p_2) + \gamma(k)$, is kinematically allowed at the first order when n is greater than 1 and $n\beta > 1$ is satisfied, where β is the velocity of the neutrino [20]. This case leads to Cherenkov radiation of the neutrino magnetic moment. A detailed discussion for this case can be found elsewhere
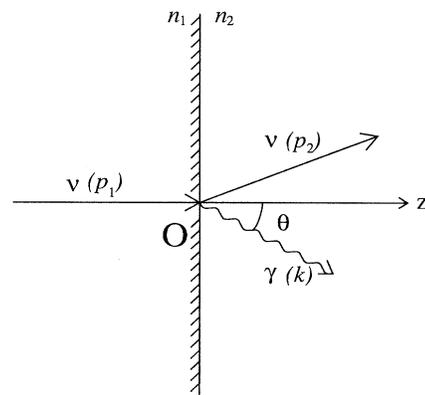
FIG. 1. Transition radiation at the interface of two media: $\nu(p_1) \to \nu(p_2) + \gamma(k)$. The refractive index changes from n_1 to n_2 at $z = 0$.

[14]. When the medium is uniform and n is less than 1, the effective mass-squared of the photon is positive and the radiation process $\nu \rightarrow \nu + \gamma$ is kinematically forbidden. However, as can be seen in the following, radiation becomes possible if there is a plane interface at $z = 0$, where the refractive index suddenly changes from n_1 ($z < 0$) to n_2 ($z > 0$). A transition probability for the radiation process $\nu \rightarrow \nu + \gamma$ at the lowest order is calculated by using the formula [21]

$$\Gamma = |S_{fi}|^2 \frac{V d^3 \mathbf{p}_2}{(2\pi)^3} \frac{V d^3 \mathbf{k}}{(2\pi)^3}, \tag{4}$$

with

$$S = i \int d^4 x \mathcal{L}, \tag{5}$$

where S is the S matrix, $V = L^3$ is the spacial volume of the interaction region, and $\mathcal{L}$ is the Lagrangian given in Eq. (1). We assume that the wave functions describing the initial-state and final-state neutrinos are given by

$$\psi_i(x) = \sqrt{\frac{m_\nu}{E_i V}} u(p_i, \lambda_i) \exp(-ip_i \cdot x) \qquad (i = 1, 2), \tag{6}$$

where m_ν is the neutrino mass, E_i is the neutrino energy, and $u(p_i, \lambda_i)$ denotes a positive-energy solution of the Dirac equation with four-momentum p_i^μ and helicity λ_i. Each of the wave functions $\psi_i(x)$ ($i = 1, 2$) is normalized to unit probability in a box of volume V. The S matrix is calculated from Eqs. (1) and (5) as

$$|S_{fi}|^2 = (2\pi)^3 L^2 T \delta(p_{1x} - p_{2x} - k_x)$$
$$\times \delta(p_{1y} - p_{2y} - k_y)\delta(E_1 - E_2 - \omega)$$

$$\times \frac{m_\nu}{E_1 V} \frac{m_\nu}{E_2 V} \frac{1}{2\omega n^2 V} \left| \int_{-L/2}^{L/2} dz \exp[i(p_{1z} - p_{2z} - k_z)z] M_{fi} \right|^2, \tag{7}$$

with

$$M_{fi} = \frac{\mu_\nu}{2} \bar{u}(p_2, \lambda_2) \sigma_{\mu\nu} u(p_1, \lambda_1) i(k^\mu \varepsilon^\nu - k^\nu \varepsilon^\mu), \tag{8}$$

where ε^μ is the unit polarization vector of the photon satisfying $k \cdot \varepsilon = 0$ and T is the time interval of the observation ($L = \beta T$). In connection with the phase in the integrand of Eq. (7), the formation-zone length of the medium is defined as

$$Z(n) \equiv (p_{1z} - p_{2z} - k_z)^{-1} = (p_{1z} - p_{2z} - n\omega \cos\theta)^{-1}, \tag{9}$$

where θ is the angle between the photon and the direction of the incident neutrino. The integral of Eq. (7) must be performed for the $(-L/2, 0)$ and $(0, L/2)$ regions separately. Since the integrand oscillates beyond the depth of the formation-zone length [$z \ll -Z(n_1)$ or $z \gg$

$Z(n_2)$], the contribution of the lower and upper limits ($z = \pm L/2$) of the integral can be neglected [$L \gg Z(n_i)$ is assumed]. Only radiation from the volume near the interface [$-Z(n_1) \lesssim z \lesssim Z(n_2)$] is added coherently. This is the case with TR. A fraction of the momentum (z component) of the neutrino is lost in the volume near to the boundary between the two media. A detailed discussion of the energy-momentum (non)conservation in the process of TR can be found in Ref. [17]. We obtain the energy intensity S per interface from Eqs. (4) and (7) as

$$\frac{d^2 S}{d\theta \, d\omega} \equiv \omega \frac{d^2 \Gamma}{d\theta \, d\omega} = \frac{\mu_\nu^2 \omega^2 \sin\theta}{8\pi^2 \beta \beta_2 \gamma \gamma_2} |A_1 - A_2|^2, \tag{10}$$

with

$$A_\alpha \equiv \frac{1}{n_\alpha} \bar{u}(p_2, \lambda_2) \frac{\not{k}_\alpha \not{\varepsilon}}{p - p_{2z} - n_\alpha \omega \cos\theta} u(p_1, \lambda_1)$$
$$(\alpha = 1, 2). \tag{11}$$

If the momentum of the incident neutrino, $p_1^\mu = (E_\nu, 0, 0, p)$, is given, the other quantities in Eqs. (10) and (11) are calculated from the following equations:

$$E_2 = E_\nu - \omega, \qquad p_{2z} = \sqrt{E_2^2 - m_\nu^2 - n_\alpha^2 \omega^2 \sin^2\theta},$$
$$\beta_2 = p_{2z}/E_2, \quad \text{and} \quad \gamma_2 = E_2/m_\nu. \tag{12}$$

Since we are interested in the radiation in the x-ray region [$n_\alpha(\omega) \sim 1$], we assume that the refractive index can be expressed in terms of the plasma frequencies ω_α ($\alpha = 1, 2$) as $n_\alpha(\omega) = 1 - \omega_\alpha^2/2\omega^2$ for $\omega \gg \omega_\alpha$, and that the radiation from medium 1 ($z < 0$) propagates through the interface without any reflection or refraction. Thus, variables θ and ε^μ are independent of the medium α.

We show the energy spectrum and the total energy per interface in Figs. 2 and 3 for the typical parameters: $E_\nu = 1$ MeV, $\omega_1 = \omega_p = 20$ eV (polypropylene), and $\omega_2 = 0$ eV (vacuum). In the calculation we average Eq. (10) over the helicity states of the incident neutrino, sum it over the helicity states of the outgoing neutrino, and sum it over two polarization states of the radiated photon. The probability is found to be the same as that in which the incident neutrino has a definite helicity of $\lambda_1 = -1$ or 1. The calculations of Eqs. (10) and (11) are performed numerically using the helicity amplitude subroutines [22,23]. The total energy S is obtained by integrating Eq. (10) over the ω and θ ranges $(0, E_\nu - m_\nu)$ and $(0, \pi/2)$, respectively [24]. A previous calculation using classical theory [15] is also shown for a comparison. The features of the TR of a neutrino magnetic moment are summarized as follows: (a) the majority of the radiation comes from the helicity-flip amplitude, and thus the effect is purely quantum mechanical; (b) the energy spectrum is flat up to $0.5\gamma\omega_p$, and then decreases rapidly; (c) the energy intensity is proportional to the Lorentz factor (γ)
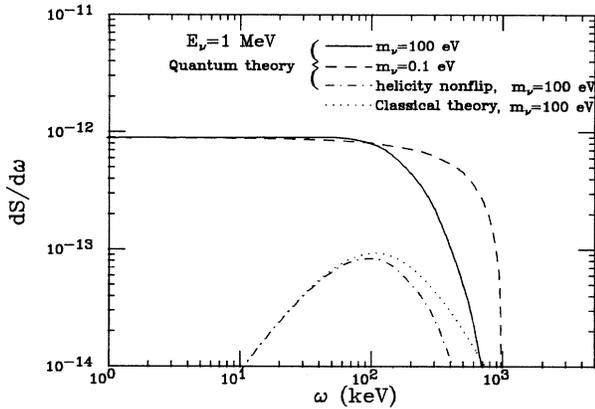
FIG. 2. Energy spectrum of the TR of the neutrino magnetic moment $(\mu_\nu = \mu_B)$ for $E_\nu = 1$ MeV, $m_\nu = 100$ eV (solid line), and $m_\nu = 0.1$ eV (dashed line). The plasma frequencies of media 1 and 2 are $\omega_1 = 20$ eV and $\omega_2 = 0$ eV, respectively. A calculation using classical electrodynamics [15] is also shown by the dotted line $(m_\nu = 100$ eV$)$, while the dash-dotted line indicates that of quantum theory when the helicity is not changed during the interaction, but the recoil effect is taken into account.

for $\gamma\omega_p \ll E_\nu$ (i.e., $m_\nu \gg \omega_p$) and begins to saturate for $\gamma\omega_p > E_\nu$ (i.e., $m_\nu < \omega_p$):

$$S = 1.7 \times 10^{-12}(\mu_\nu/\mu_B)^2\gamma\omega_p \quad \text{for} \quad \gamma\omega_p \ll E_\nu ,$$

(13a)

$$= 4.5 \times 10^{-13}(\mu_\nu/\mu_B)^2 E_\nu \quad \text{for} \quad \gamma\omega_p \gg E_\nu .$$

(13b)

The coefficients (equal to probability) originate from a dimensionless constant $\mu_B^2\omega_p^2 = 3.5 \times 10^{-11}$ for $\omega_p = 20$ eV [25]; and (d) the emitted angle has a peak at the forward direction, $\theta \sim 1/\gamma$.

First of all, the energy intensity turns out to be larger by an order of magnitude than that
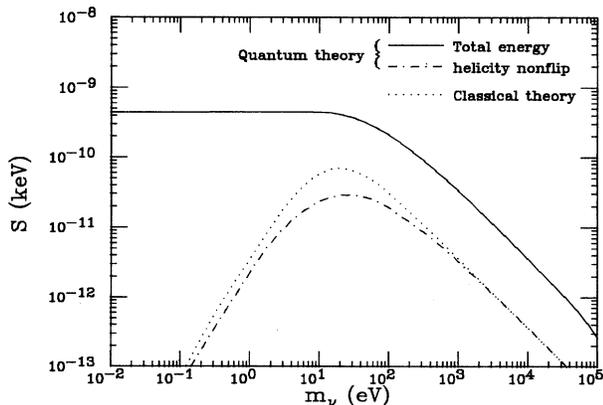
FIG. 3. Total TR energy of the neutrino magnetic moment $(\mu_\nu = \mu_B)$ as a function of the mass for $E_\nu = 1$ MeV (solid line). That using classical electrodynamics [15] is indicated by the dotted line, while the dash-dotted line is that of quantum theory when the helicity is not changed during the interaction, but the recoil effect is taken into account.

$[S = 1.9 \times 10^{-13}(\mu_\nu/\mu_B)^2\gamma\omega_p$ for $\gamma\omega_p \ll E_\nu]$ estimated by classical theory. The TR yield is not reduced even for the case of a small mass under the condition that the magnitude of the magnetic moment is the same. The recoil effect becomes important for $\gamma\omega_p \gtrsim E_\nu$. A dominant helicity-flip amplitude is characteristic of the interaction of Eq. (1), which has already been pointed out concerning other processes [12,13]. A previous calculation using classical theory corresponds to the helicity-nonflip transition. To confirm this point, we also show the energy spectrum and the total intensity in Figs. 2 and 3 using quantum theory for the case when the helicity is not changed during the radiation process, i.e., $\lambda_1 = \lambda_2 = -1$ (dash-dotted line). The calculation using quantum theory takes into account the recoil effect, i.e., $p_2 \neq p_1$. The present calculation for the process disagrees with that of classical theory only for the region $(\omega \sim E_\nu)$ where the recoil effect is important. The classical calculation corresponds to the radiation of a particle with such a large magnetic moment (or spin) and large mass that the radiation has no effect on the spin state or the trajectory of the particle.

The sensitivity of a typical transition radiation detector has already been discussed [15]. The present work shows that the TR yield has increased by about 10, and that the sensitivity of the method to the neutrino magnetic moment for a small mass region $(m_\nu < \omega_p)$ is not as much decreased as that given previously. We now present a calculation of the TR yield for a practical detector containing many foils, where the interference effects between the individual interfaces (equal to the "formation-zone effect") must be taken into account [26]. For example, the TR yield per interface given in Eq. (10) must be corrected for a periodic radiator comprising N polypropylene foils $(N = 100-500, \omega_1 = 20$ eV, and thickness $\ell_1 = 0.1$ mm) stretched in air $(\omega_2 = 0.8$ eV and spacing $\ell_2 = 2$ mm) [27]. The average TR yield per interface at $E_\nu = 1$ MeV is estimated to be almost the same for $m_\nu \gg \omega_1$ and about a half for 0.01 eV $< m_\nu < \omega_1$ as compared to that given in Eq. (10). The reduction due to the formation-zone effect is not so large in this case [28].

In conclusion, we have revised the calculation of the transition radiation of a neutrino magnetic moment using quantum theory, where both the helicity-flip effect and the recoil effect are taken into account. We found that it is larger by an order of magnitude than that estimated using classical electrodynamics and that the energy spectrum of the radiation is uniform up to $0.5\gamma\omega_p$. The transition radiation of the neutrino magnetic moment is unique in that the energy intensity depends explicitly on the neutrino mass.

*Electronic address: sakuda@kekvax.kek.jp

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$$\frac{d^2 S}{d\theta\, d\omega} = \frac{\mu_\nu^2 \omega_p^4 \sin\theta \cos\theta (1 - \beta\beta_2 \cos^2\theta + \frac{1}{\gamma\gamma_2})}{16\pi^2 \beta\beta_2 (p - p_{2z} - n\omega \cos\theta)^2 (p - p_{2z} - \omega \cos\theta)}.$$

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$$\frac{d^2 S(N)}{d\theta\, d\omega} = \frac{d^2 S}{d\theta\, d\omega} 4 \sin^2\left(\frac{\ell_1}{2Z_1}\right) \frac{\sin^2[N(\frac{\ell_1}{2Z_1} + \frac{\ell_2}{2Z_2})]}{\sin^2(\frac{\ell_1}{2Z_1} + \frac{\ell_2}{2Z_2})},$$

where Z_1 and Z_2 are the formation-zone length of the two media and S is the TR yield per interface given in Eq. (10). The average TR yield per interface is given by $S(N)/(2N)$.

[28] We note that the gap length is usually smaller than the formation-zone length of the gaps for $\gamma \gg 10^5$ in most transition-radiation detectors and that the main formation-zone effect comes from the gaps and not from the foils.