Superconducting Vortex Avalanches

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We monitor the dynamics of superconducting vortices in the Bean state, as the system is driven to the threshold of instability by the slow ramping of an external field. Individual avalanches, containing as few as 50 vortices, are detected in real time. Thus our experiment is the superconducting analog of monitoring the granular avalanches produced by slowly dropping sand on a sandpile. The observed distribution of vortex avalanche sizes shows a power-law behavior over two decades, proving that the vortex dynamics in the Bean state is characterized by avalanches of many length scales.

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Some 30 years ago Bean [1] and de Gennes [2] noted the close analogy between the marginally stable state of vortices in a hard superconductor and the marginally stable slope of sand in a sandpile. We can picture building up a sandpile by slowly dropping grains on a flat surface. The slope of the pile soon reaches a certain "maximal angle of stability," determined by a balance between gravity and intergrain frictional forces. A similar situation is present in a hard superconductor (i.e., with strong pinning). Vortices nucleate at the surface as an external magnetic field is slowly ramped. In the simplest model, due to Bean [1], the vortex density decreases linearly with distance into the superconductor. This again is due to a balance between vortex density gradients which drive vortices into the bulk and pinning forces which hamper their entry. These early analogies were invoked mainly in order to understand the *static* distribution of flux in a hard superconductor. More recently, interest has focused on the *dynamics* of systems slowly driven to the threshold of instability. A large number of diverse physical systems are characterized by such dynamics, including charge-density waves, pinned Wigner crystals, earthquake faults, granular assemblies, and superconducting vortices. These systems have received renewed attention due to their relation to spatiotemporal dynamics, instabilities, and self-organized criticality. Sandpiles have received particularly intensive theoretical and experimental attention as a model system exhibiting such threshold dynamics. In light of the strong *static* analogies between sandpiles and the Bean state in hard superconductors, it is natural to ask whether there are quantitative similarities between the *dynamic* processes (e.g., similar avalanche size distributions) in the two systems.

Here we report results of an experiment on the dynamics of vortices, which is closely analogous with those done on sandpiles. The magnetic field outside a tubular superconducting sample is ramped slowly, driving flux into the tube's outer wall. Eventually, the flux front will reach the inner wall of the tube and spill out into the tube's interior. We can detect, in real time, this entrance of flux into the tube's interior (Fig. 1). In particular, we can measure whether flux leaving the superconductor does so in discrete bundles or avalanches; if so, we can measure the sizes, lifetimes, and arrival times of these avalanches. Our experiment is thus the superconducting analog [3] of sandpile experiments [4,5] where the sand is slowly added to the apex of the pile; any sand which falls off the edge in the form of avalanches is then measured. Sandpile experiments which use a slowly tilting table [6] or rotating drum [7] may be more closely analogous to current-driven depinning of vortices [8].

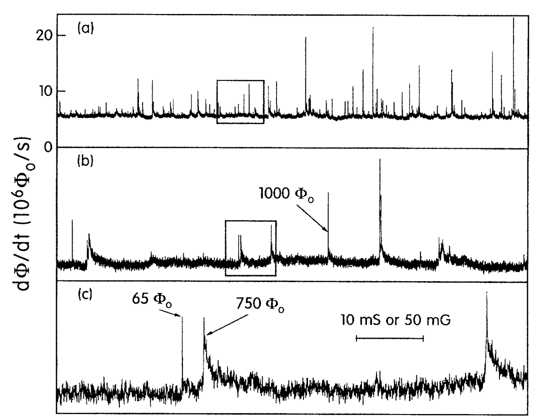

FIG. 1. The voltage measured on the pickup coil as the magnetic field is ramped at 5 G/s. Frame (a) shows a 30 G segment centered at $B = 7.55$ kG. There are 262 144 data points in this segment. The voltage trace consists of a series of many pulses, of widely varying sizes. Each pulse represents the sudden influx of a correlated vortex bundle or avalanche into the tube's interior. Frames (b) and (c) show successive magnifications of frame (a) by factors of 10 horizontally. The area under each pulse determines the number of vortices in the avalanche, as shown for several representative pulses.

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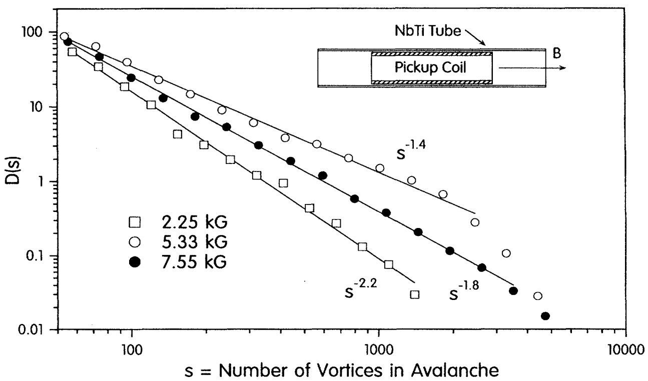

FIG. 2. The probability density $D(s)$ of measuring an avalanche of size s versus s. Distributions are shown for three magnetic fields, with the exponents shown determined from fits to the linear portion of each curve. The best-fit lines extend up to the points included in the fit. There are 5187 avalanches logarithmically binned for the 7.55 kG curve. The inset shows the arrangement of the tubular NbTi sample and the pickup coil.

The experimental arrangement is shown in the inset of Fig. 2 and is similar to those used in Refs. [9] and [10]. A thin-wall tube of conventional [11] superconductor $Nb_{47\%\,wt}Ti_{53\%}$, was prepared by cold swagging, with a reduction in diameter of $\approx 4:1$ to a final diameter of 6 mm and thickness 0.25 mm. A 3.4 cm length was cut from the original piece. Inside the tube is a 1.8 cm long coaxial pickup coil, consisting of $N = 1800$ turns of copper wire. All experiments were performed with the tube immersed in the helium bath. The Bean critical state forms in the tube's wall [12,13], as an external magnetic field, parallel to the tube's axis, is slowly ramped up from zero. Eventually, the external field reaches a value such that flux first enters the interior of the tube. This flux induces a voltage $N\,d\Phi/dt$ on the pickup coil, which we measured using an amplifier with a dc to 20 kHz bandwidth amplifier with 1 $\mathrm{nV}/\sqrt{\mathrm{Hz}}$ noise. The data were digitized at 40 kHz and recorded directly to computer disk.

If some flux should enter the tube as a vortex bundle or avalanche, this will induce a voltage *pulse* on the coil. The area under the pulse can be converted directly into the number of vortices s in the avalanche, as $s = \int V\,dt/N\Phi_0$, assuming that the vortex bundle cuts all N turns of the coil. There is evidence [14] that this may not be so. If not, then the actual number of vortices in the bundle is $s_{\mathrm{actual}} = s(L/l)$, where s is the measured value described above, and L and l are the length of the coil and of the vortex bundle, respectively. We may then write $s \propto l s_{\mathrm{actual}}$ which describes an effective vortex bundle *volume,* which is the natural extension of the meaning of the "size" of an avalanche to a three-dimensional system. With this caveat, however, we will still call s "the number of vortices in an avalanche."

Figure 1 shows the signal recorded by the pickup coil as the external field is ramped at a rate of 5 G/s, corresponding to about $6 \times 10^6 \Phi_0$ entering the tube per second. Figure 1(a) shows a 30 G segment centered at 7.55 kG, from a 0 to 8 kG field sweep at a temperature $T = 2.9 \pm 0.1$ K. The striking feature of this signal is that it is not at all smooth; instead, it consists of a succession of very pronounced pulses. Each pulse represents the sudden influx of many spatially correlated vortices, in what we term flux avalanches [15]. It is also quite apparent that there is a very wide distribution of pulse sizes and lifetimes. Figures 1(b) and 1(c) show successive enlargements of the boxed regions indicated. It becomes clear that individual pulses are reasonably well separated in time, and that their areas are well defined. Calibrations of the detection system bandwidth show that a step change in flux yields a pulse width of only some 50 μs, much shorter than the widths of pulses shown in Fig. 1. Thus the "tails" of the pulses in Fig. 1 represent an inherent feature of the vortex avalanches. In Figs. 1(b) and 1(c), several pulses (including the tails) have been integrated to yield the number of vortices shown. Several large events consisting of several thousand Φ_0 are visible, along with smaller events with only a hundred or so Φ_0. A careful analysis of the data indicates that we can reliably measure avalanches consisting of as few as 50 vortices; the largest avalanches found in this field range contained about 5000 vortices. The noise floor of the amplifier is responsible for the small-scale "fuzz" seen in these time traces.

It is important to note that most of the flux does not actually enter the tube as discernible avalanches. In Fig. 1(a) we see that the avalanche activity rides on top of a dc value of $d\Phi/dt$, which corresponds to the mean ramp rate of the external field. We find that the fraction of flux entering in measurable avalanches is about 3% of the total. We expect that some fraction of this dc signal is due to large numbers of avalanche events too small and too frequent to be measured with our apparatus. The bulk of this dc signal, however, is likely due to thermally activated vortices moving into the tube in a fluidlike manner. Thermally activated flux motion can be large near the critical state, where the activation energy for hopping goes to zero. In their noise measurements, Yeh and Kao [16] also find that only a fraction of the flux moves in the form of flux bundles.

The concept of self-organized criticality [17] was introduced to explain the dynamics of certain slowly driven dissipative systems with many degrees of freedom. A sandpile was used as a model system in which to explore these ideas. Such systems are critical in the sense of a second-order phase transition, and so exhibit long-range spatial and temporal correlations. They are self-organized in that the critical state is an attractor of the dynamics, and so the system evolves toward criticality without tuning an external parameter. For example, in sandpiles, the long-range correlations are presumably manifested as power-law distributions of avalanche sizes and lifetimes, which occur at the critical angle to which the sandpile naturally evolves [4–6].

The Bean state is certainly self-organized in the sense that the vortices are dynamically driven toward a marginally stable state. We may ask whether it is a critical state as well, characterized by power-law distributions of bundle or avalanche sizes. The distribution of flux avalanche sizes for our experiment is shown in Fig. 2, where we plot the normalized probability $D(s)$ that an avalanche will contain s vortices, versus s. The normalization is such that $D(s)\,ds$ is the average number of avalanches recorded with sizes between s and $s + ds$, and we have used logarithmic binning. Here we show distributions of events occurring in three 450 G wide windows centered at 2.25, 5.33, and 7.55 kG. At all fields, $D(s)$ has a power-law behavior, $s^{-\nu}$, up to some cutoff size. The low field data, at 2.25 kG, appear straight up to avalanche sizes of about 1500 vortices. The cutoff here is very sharp, with no events recorded above $s \approx 1500$. The middle field data at 5.33 kG show power-law behavior up to $s \approx 2000$ vortices. Above this size, there is a gradual decay away from the power-law curve. Still, the power-law dependence extends over 1.6 decades of s. Finally, at the highest field of 7.55 kG, the distribution has two full decades of power-law behavior for avalanches up to 5000 vortices.

It is interesting to compare these results with those obtained in sandpile experiments. Most closely analogous to our work are experiments [4,5] in which the sand is slowly added to the apex of the pile. Here, a broad distribution over several decades of avalanche sizes in *small* piles was found. For *large* piles, the systems reverted to the relaxation oscillator behavior of rotating drum experiments [7], although even here close inspection of the pile mass indicates the presence of many small avalanches [5]. It is important to note that the linear extent of "large" piles from apex to base is still only some 50 particles [5]. In our vortex experiment, the distribution is broad at all fields studied, with power-law behavior over as many as two decades. Here, finite-size effects are presumably minimal, as there are some 9000 vortices across the tube's wall at the highest field of 7.55 kG; overall, the tube contains about 3×10^9 vortices at this field. Thus our largest avalanches span only about 1% of the tube's width and contain only about 10^{-6} of the total number of vortices in the tube. No relaxation-type behavior was observed in our experiments. An important difference between vortices and sand is the crucial role that inertial effects play in granular systems. Since vortices have essentially no inertia, it is reasonable to expect their dynamics to be much closer to ideal sandpile models [17,18].

Our experiment shows that the exponent ν measured in the dynamical critical state is not constant, since it ranges from -2.2 for the low field data to -1.4 for the middle field data (Fig. 2). Interestingly, the slopes are not monotonic in field, since ν for the highest field data increases again to -1.8. Since the dominant pinning centers in our sample are grain boundaries with spacings randomly distributed between about 30 and 50 nm [19], this variation in ν with magnetic field may be related in some way to the interplay between the interpin and intervortex spacing. The variation of ν is also reminiscent of the different exponents extracted in various sandpile experiments, as their "microscopic" parameters such as grain roughness are varied. Theoretically, ν has been found to vary in different sandpile models as the cellular automata "rules" are changed [18], as well as in a two-dimensional model of coupled blocks [20] as the coupling is varied. This later result is relevant to our experiment, since increasing the field changes the average vortex density and hence the vortex-vortex interaction. The vortex system is thus unique in that the interactions between the particles can be *continuously* tuned. The data shown in Fig. 1 were taken at a ramp rate of 5 G/s. At this rate, individual vortex avalanches can be readily identified, and distributions as in Fig. 2 formed. It is important to verify that the system is, in fact, in the slowly driven regime. We have thus studied the ramp-rate dependence of the avalanche activity. Field sweeps were taken at ramp rates R of 1, 5, 10, and 20 G/s. At low ramp rates, $R \leq 10$ G/s, the data look similar to that shown in Fig. 1. At the highest ramp rate of 20 G/s, however, the data look qualitatively different. Here it becomes difficult to identify individual avalanche events, since many avalanches are overlapping at this high rate. Thus we cannot form distributions of avalanche sizes at the highest ramp rate. Instead, for all rates we compute the power spectrum $S(f)$, defined here as the square of the Fourier transform of a field sweep similar to that of Fig. 1. Such power spectra are shown in Fig. 3. We see that, for low ramp rates, the spectra all show a power-law behavior over some three decades, with the same exponent of about -1.5. This power-law behavior is expected from a field sweep whose distribution of event sizes itself has a power-law behavior [21]. Thus we believe that the ramp rate of 5 G/s used to obtain $D(s)$ is well within the slowly driven regime. At the highest ramp rate of 20 G/s, however, there is a distinct Lorentzian-like knee [with $S(f) \approx f^{-2}$ for large f] emerging from the overall power-law behavior. This Lorentzian shape indicates an emerging field and time scale, so that the strongly driven system is no longer at the critical point and hence no longer spatially and temporally scaleless. Experiments in water-droplet avalanches [22] and noise measurements in type-II superconductors [16] have also shown such an emergent length scale as the system is strongly driven. It appears, then, that critical behavior is achieved only very near the marginally stable state, when the system is slowly driven.

We have presented evidence that the marginally stable (or Bean) state in hard superconductors exhibits flux

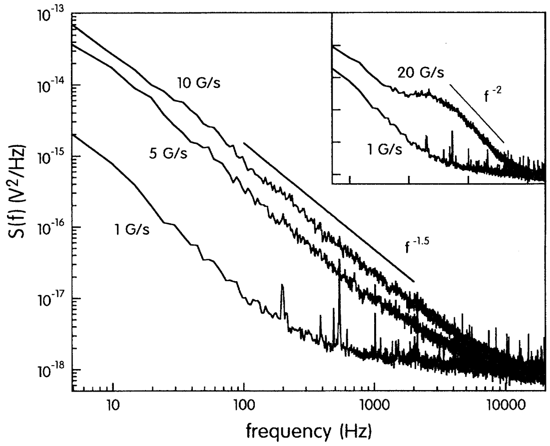

FIG. 3. Power spectra of magnetic field sweeps, at four different ramp rates. For low rates, the spectra show a power-law dependence, with the same exponent. At the highest rate of 20 G/s, however, a distinct knee is apparent (inset), indicating the emergence of a characteristic field or time scale. This data was taken over a 0.25 kG wide window centered at 7.55 kG for the highest rate; proportionally smaller windows were used at the lower rates.

avalanches with a power-law distribution of sizes when slowly driven *toward* the threshold of instability. This is consistent with theoretical predictions of flux-gradient-driven vortices [3]. We contrast our work with relaxation studies [23] in which the system instead moves *away* from the marginally stable state, where it is unclear that critical behavior should be observed. Further, in our experiment we were able to directly measure the sizes of individual avalanches, the power-law distribution of which is the hallmark of the critical state. Experiments which measure $1/f$ noise alone [24] cannot be taken as proof of criticality [25]. Our results thus indicate that the well-established static analogy between superconductors and sandpiles can be extended quantitatively to dynamical effects as well, with the two systems exhibiting similar threshold dynamical behavior.

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[13] The existence of the Bean state in this tube geometry may be directly confirmed by measuring the magnetic field difference ΔB between the inside and outside of the tube using a dc magnetometer. We find that this difference ranges from 0.36 kG at the lowest field (2.25 kG) studied down to 0.27 kG at the highest field of 7.55 kG. Thus, in the range of fields we investigate here, $\Delta B/B$ never exceeds 16%.
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