Anderson-Mott Transition as a Random-Field Problem

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The Anderson-Mott transition of disordered interacting electrons is shown to share many physical and technical features with classical random-field systems. A renormalization group study of an order parameter field theory for the Anderson-Mott transition shows that random-field terms appear at oneloop order. They lead to an upper critical dimension $d_c^+ = 6$ for this model. For $d > 6$ the critical behavior is mean-field-like. For $d < 6$ an ϵ expansion yields exponents that coincide with those for the random-field Ising model. Implications of these results are discussed.

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It is a well-established result that electrons in a random potential at zero temperature undergo a metal-insulator transition as a function of the disorder, provided that the space dimension $d > 2$ [1]. This is true for both noninteracting [2] and interacting [3] electrons, but the respective transitions are quite different in nature. For the Anderson transition of noninteracting electrons no simple order parameter (OP) description, no upper critical dimension, and no Landau theory are known. The Anderson-Mott transition (AMT) of interacting electrons, on the other hand, has recently been shown to be conceptually simpler than the Anderson transition in that it allows for a simple OP description with the density of states (DOS) at the Fermi level as the OP. In this picture the metal (insulator), with a nonzero (vanishing) DOS at the Fermi level, represents the ordered (disordered) phase, respectively. This OP description leads to a finite upper critical dimension d_c^+ , and to a Landau theory of the AMT [4].

This existence of an OP description raises important questions about the nature of the fluctuations that drive the AMT, and about its relation to other phase transitions in random systems. Consider a static, spin-independent random potential $u(x)$ that couples to the electron density. In a fermionic field theory this gives rise to a term in the action [3]

$$
\int d\mathbf{x} \ u(\mathbf{x}) \sum_{n} \bar{\psi}_{n}(\mathbf{x}) \psi_{n}(\mathbf{x}), \qquad (1)
$$

where $\bar{\psi}$ and ψ are Grassmann fields and n is a Matsubara frequency index. Since the expectation value $\langle \bar{\psi}_n(\mathbf{x})\psi_n(\mathbf{x})\rangle$ determines the DOS, this means that the random potential couples to the OP for the AMT. Magnetic transitions where a random field (RF) couples to the OP are known to have peculiar properties: The RF fIuctuations are dominant over the thermal fluctuations, which leads to $d_c^+ = 6$ (rather than 4) [5], and to a violation of hyperscaling even for $d \leq 6$ [6]. An ϵ expansion of the critical exponents about $d = 6$ leads to the famous "dimensional reduction" problem [7], and the critical dynamics have been proposed to be anomalous [8]. It is then natural to ask whether similar phenomena are to be expected at the AMT. Physically it is plausible that interacting disordered electron systems should display RF effects, since they have the same type of frustration that occurs in RF magnets: The random potential favors a local electron arrangement that conflicts with the one favored by the electron-electron interaction.

In this Letter we show that there is indeed a close analogy between the AMT and RF problems. Within a renormalization group (RG) treatment of the AMT we find that RF-type terms appear which lead to $d_c^+ = 6$. A $6 - \epsilon$ expansion to first order in ϵ leads to critical exponents that are identical with those of the RF Ising model, and hyperscaling is violated due to a dangerous irrelevant variable.

ermionic fields. It carries two Matsubara frequency in-Our starting point is the nonlinear σ -model description of interacting disordered electrons [9]. This is a Gaussian field theory for a Hermitian matrix field $\tilde{Q}(\mathbf{x})$ with constraints $[\tilde{Q}(\mathbf{x})]^2 = 1$, with 1 as the unit matrix and tr $\tilde{Q}(\mathbf{x}) = 0$. \tilde{Q} is a classical field comprising two dices n, m and two replica indices α , β (quenched disorder has been incorporated by means of the replica trick). The matrix elements $\tilde{Q}_{nm}^{\alpha\beta}(\mathbf{x})$ are in general spin quaternions, with the quaternion degrees of freedom describing the particle-hole and particle-particle channel, respectively. For the sake of simplicity, in this paper we restrict ourselves to the particle-hole spin-singlet degrees of freedom, although the general model can be treated in the same way [10]. The matrix elements of \tilde{Q} can then be expanded as $\tilde{Q}_{nm}^{\alpha\beta} = \sum_{r=0,3} \tilde{Q}_{nm}^{\alpha\beta} \tau_r$, with $\tau_{0,1,2,3}$ as the quaternion basis. We write the action in the form

$$
S[\tilde{Q}, \Lambda] = \frac{-1}{2G} \int d\mathbf{x} \, \text{tr}\{\Lambda(\mathbf{x})[\tilde{Q}^2(\mathbf{x}) - 1] + [\partial_{\mathbf{x}} \tilde{Q}(\mathbf{x})]^2\} + 2H \int d\mathbf{x} \, \text{tr}[\Omega \tilde{Q}(\mathbf{x})] - \frac{\pi T}{4} K_s[\tilde{Q}(\mathbf{x}) \circ \tilde{Q}(\mathbf{x})].
$$
\n(2)

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Here G is a measure of the disorder, Ω is a diagonal matrix whose elements are the Matsubara frequencies ω_n , and H is proportional to the free electron DOS. $K_s < 0$ is a repulsive electron-electron interaction coupling constant in the particle-hole spin-singlet channel. We consider a short-ranged model interaction for simplicity. $[\tilde{Q} \circ \tilde{Q}]$ denotes a product in frequency space which is given explicitly in Refs. [3,9]. Notice that we have enforced the constraint $\tilde{Q}^2 = \mathbb{1}$ by means of an auxiliary, or ghost, matrix field $\Lambda(\mathbf{x})$.

The correlation functions of \tilde{Q} determine the physical quantities. Correlations of the \tilde{Q}_{nm} with $nm < 0$ determine the diffusive modes which describe charge and spin diffusion, while the DOS is determined by $\langle \tilde{Q}^{\alpha\alpha}_{nn} \rangle$ [3,9]. It is therefore convenient to separate \tilde{Q} into blocks:

$$
\tilde{Q}_{nm}^{\alpha\beta} = \Theta(nm)Q_{nm}^{\alpha\beta}(\mathbf{x}) + \Theta(n)\Theta(-m)q_{nm}^{\alpha\beta}(\mathbf{x})
$$

$$
+ \Theta(-n)\Theta(m)(q^{+})_{nm}^{\alpha\beta}(\mathbf{x}), \qquad (3)
$$

where Θ is the step function. We then integrate out the massless q field. Since the action, Eq. (2), is quadratic in q this can be done exactly. We note that this procedure leads to long-range interactions in the Q -field theory that result in an infinite set of relevant operators for $d < 4$. However, for the dimensionalities considered here, $d =$ $6 - \epsilon$ with $\epsilon \ll 1$, these interactions are easily shown to be RG irrelevant [10]. We further note that corrections to Eq. (2), which are, e.g., generated by the RG, can be taken into account and also shown to be RG irrelevant near $d = 6$. Physically, these results reflect the dominance of RF fluctuations over the q fluctuations, at least near $d=6$.

We next expand Q and Λ about their expectation values $\langle rQ_{12}(\mathbf{x})\rangle = \delta_{r0}\delta_{12}N_{n_1}$ and $\langle r\Lambda_{12}(\mathbf{x})\rangle = \delta_{r0}\delta_{12}l_{n_1}$, with $1 \equiv (n_1, \alpha_1)$, etc.,

$$
{}_{r}Q_{12}(\mathbf{x}) = \delta_{r0}\,\delta_{12}\,N_{n_1} + {}_{r}\phi_{12}(\mathbf{x}), \qquad (4a)
$$

$$
{}_{r}\Lambda_{12}(\mathbf{x}) = \delta_{r0}\,\delta_{12}\,l_{n_1} + {}_{r}\psi_{12}(\mathbf{x}), \qquad (4b)
$$

with $\langle \phi_{12}(\mathbf{x}) \rangle = \langle \psi_{12}(\mathbf{x}) \rangle = 0$. Notice that N_n is proportional to the DOS at an energy ω_n measured from the Fermi surface [3]. The resulting action is quadratic in ϕ , but contains terms with arbitrary powers in ψ . If one formally integrates out ψ , one is left with an action in terms of the OP Q only. It can be shown that the terms of higher than second order in ψ do not change the structure of the resulting ϕ -field theory [10]. It is therefore sufficient to integrate out ψ in Gaussian approximation. This is easily accomplished with the result

$$
S[\phi] = -\int d\mathbf{x} \operatorname{tr} \left[\phi(\mathbf{x})(-\partial_{\mathbf{x}}^2 + \langle \Lambda \rangle) \phi(\mathbf{x}) + \frac{u}{G} \{ [\langle \mathcal{Q} \rangle \phi(\mathbf{x})]^2 + \langle \mathcal{Q} \rangle^2 \phi^2(\mathbf{x}) \} \right] + \frac{\Delta}{2} \int d\mathbf{x} \sum_{i = > , <} [\operatorname{htr}_i \phi(\mathbf{x})]^2
$$

\n
$$
-u \int d\mathbf{x} \operatorname{tr} \phi^4(\mathbf{x}) - \frac{2u}{\sqrt{2G}} \int d\mathbf{x} \operatorname{tr} [\langle \mathcal{Q} \rangle \phi^3(\mathbf{x}) + \phi(\mathbf{x}) \langle \mathcal{Q} \rangle \phi^2(\mathbf{x})]
$$

\n
$$
- \frac{u}{G} \int d\mathbf{x} \operatorname{tr} \left[A \left(\phi^2(\mathbf{x}) + \frac{2}{\sqrt{2G}} \langle \mathcal{Q} \rangle \phi(\mathbf{x}) \right) \right] - \frac{2}{\sqrt{2G}} \int d\mathbf{x} \operatorname{tr} [B \phi(\mathbf{x})],
$$
(5a)

where we have scaled ϕ by a factor of $\sqrt{2G}$. $tr = \sum_{r,\alpha,n}$ denotes a trace over all discrete degrees of freedom, and htr_{>,<} $\equiv \sum_{r,\alpha} \sum_{n \ge 0} n \ge 0$ denotes "half-traces" that extend only over positive and negative frequencies, respectively. A and B are functions of $\langle Q \rangle$ and $\langle \Lambda \rangle$, and are given by

$$
A(\langle Q \rangle, \langle \Lambda \rangle) = \langle Q \rangle^2 - 1 + f(\langle \Lambda \rangle), \qquad B(\langle Q \rangle, \langle \Lambda \rangle) = \langle \Lambda \rangle \langle Q \rangle - 2GH\Omega, \tag{5b}
$$

where $f(\langle \Lambda \rangle)$ is a matrix with elements $f_{12} = \delta_{r0} \delta_{12} f_{n_1}$, with

atrix with elements
$$
{}_{r}f_{12} = \delta_{r0} \delta_{12} f_{n_1}
$$
, with
\n
$$
f_{n} = -\frac{G}{4} \sum_{p} \sum_{m=-1}^{\infty} \frac{2\pi T G K_{s}}{[p^{2} + \frac{1}{2}(l_{n} + l_{m})]^{2}} \left[1 + \sum_{n_{1}=0}^{n_{2}=m-1} \frac{2\pi T G K_{s}}{p^{2} + \frac{1}{2}(l_{n_{1}} + l_{n_{1}-n+m})}\right]^{-1},
$$
\n(Sc)
\nmilar expression for $n < 0$. The bare value of the coupling constant u is $u = -G/(df/dl)$. The
\nzero. However, a term of this structure is generated by the RG at one-loop order, and it is crucial
\naction. A comparison with Refs [5,6] shows that the half trace terms have the characteristic replica

bare value of Δ is zero. However, a term of this structure is generated by the RG at one-loop order, and it is crucial for $n \ge 0$, and a similar expression for $n < 0$. The bare value to include it in the action. A comparison with Refs. [5,6] shows that the half-trace terms have the characteristic replica structure of a RF term. Since it is quadratic in ϕ , this term contributes to the Gaussian (G) propagator. In the replica limit we find

$$
\langle \rho_{12}(\mathbf{k}) \rangle_{s} \phi_{34}(\mathbf{p}) \rangle^{(G)} = \delta(\mathbf{k} + \mathbf{p}) \delta_{rs} \frac{1}{16} \frac{1}{k^{2} + m_{12}} \times \left[\delta_{13} \delta_{24} + (-)^{r} \delta_{14} \delta_{23} + \frac{4\Delta \Theta(n_{1}n_{3})}{k^{2} + m_{12}} \delta_{r0} \delta_{12} \delta_{34} \right]. \tag{6}
$$

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Here $m_{12} \equiv (l_1 + l_2)/2 + u(N_1 + N_2)^2$. It is clear that the term proportional to Δ will increase the upper critical dimension by 2, as it does in the case of RF magnets. We therefore expect $d_c^+ = 6$ in the model given by Eqs. (5) rather than $d_c^+ = 4$ which one would conclude from a power-counting analysis of the action at zero-loop order, i.e., without the RF term [4,11].

Equation (5a) requires some explanatory comments. (1) $\langle Q \rangle$ and $\langle \Lambda \rangle$ are determined by the conditions $\langle \phi \rangle = 0$ and $\langle \psi \rangle = 0$. At zero-loop order, these two conditions yield $A(\langle Q \rangle, \langle \Lambda \rangle) = B(\langle Q \rangle, \langle \Lambda \rangle) = 0$. This is the zeroloop order equation of state that has been discussed in Ref. [4]. It yields mean-field exponents, which constitute the exact critical behavior for $d > d_c^+ = 6$. For $d <$ 6, renormalizations of the equation of state change the critical behavior. (2) In writing Eq. (5a) we have omitted some terms that are irrelevant by power counting for $d >$ 4. So are the terms of $O(\phi^4)$ which we kept. However, as we will see, the latter couple to the RF coupling constant Δ and therefore *must* be kept. We have verified that none of the terms omitted, and no other terms generated by the RG, couple to Δ [10]. These considerations are in direct analogy to the case of a magnet in a RF [5].

We now perform a one-loop RG analysis of the action, Eq. (5a), using a standard momentum-shell method [12]. It is convenient to first consider the theory at criticality. Then we can put $\langle Q \rangle = \langle \Lambda \rangle = 0$, and consider the renormalization of u for $d = 6 - \epsilon$. Since u is irrelevant for $d > 4$, we need to keep only contributions that are of or $d > 4$, we need to keep only contributions that are of order $g \equiv u\Delta$, Δ being relevant with a bare dimension of 2. Δ is not renormalized, and the ∂_x^2 term is not renormalized either to one-loop order, so the exponent $\eta = O(\epsilon^2)$. We obtain the following flow equation for g :

$$
\frac{dg}{d\ln b} = \epsilon g - \frac{9}{2}g^2 + O(g^3),\tag{7}
$$

with b as the RG length rescaling factor. Equation (7) possesses a fixed point $g^* = 2\epsilon/9 + O(\epsilon^2)$.

We now turn to the disordered phase, i.e., the insulato where $N_{n=0}$ vanishes and $l_{n=0} \equiv l$ has a nonzero value. We thus put $\langle Q \rangle = 0$, and renormalize the mass term $\langle \Lambda \rangle$ or l in the action. We obtain

$$
\frac{dl}{d\ln b} = 2l - gl + O(g^2),\tag{8}
$$

and two equations that determine the renormalized equation of state,

$$
N^{2} = 1 - f(l) - \frac{Gg}{4u} \sum_{p} \frac{1}{(p^{2} + l)^{2}},
$$
 (9a)

$$
lN = 2GH\Omega - \frac{gN}{2} \sum_{p} \frac{1}{(p^2 + l)^2},
$$
 (9b)

where both N and l are to be considered as functions of Ω . If we replace g in Eq. (8) by its fixed point value g^* , we find that l scales like

$$
l(b) \sim b^{2[1-\epsilon/9 + O(\epsilon^2)]}.\tag{10}
$$

We next invoke the equation of state to find the relation between l and the distance from the critical point t . To

this end, we expand the right hand side (r.h.s.) of Eq. (9a) for small values of l . The l -independent contribution is t. At linear order in $d = 6$ one finds a term $\neg l$, and a term \sim l ln l. The prefactors of these two terms are related, since $df/dl = -G/u$. Replacing g by g^* , we can exponentiate and find

$$
l \sim t^{1+\epsilon/18+O(\epsilon^2)}.\tag{11}
$$

This holds for $\epsilon > 0$. For $d > 6$ one finds instead $l \sim t$ as one would expect within mean-field theory.

We can now combine Eqs. (10) and (11) to get the correlation length exponent ν to first order in ϵ . Since we also know η to this order, standard scaling arguments yield all other static exponents. We find

$$
\nu = \frac{1}{2} + \frac{\epsilon}{12} + O(\epsilon^2), \quad \eta = 0 + O(\epsilon^2),
$$

$$
\gamma = 1 + \frac{\epsilon}{6} + O(\epsilon^2), \quad \beta = \frac{1}{2} - \frac{\epsilon}{6} + O(\epsilon^2),
$$

$$
\delta = 3 + \epsilon + O(\epsilon^2).
$$
 (12)

In order to obtain β and δ we have used the fact that u is dangerously irrelevant, so hyperscaling is violated (see [6], and the discussion below), and have accordingly replaced d by $d - 2$ in all d-dependent scaling laws. Notice that Eqs. (12) are identical with the corresponding results for a RF Ising model [5,13].

We still need to determine the dynamical scaling exponent z . For this purpose we obtain a relation between l, N, and Ω from Eq. (9b). Expanding the r.h.s. for small *l*, going to criticality, and exponentiating, we find $Nl^{1+\epsilon/9+\overline{O}(\epsilon^2)} \sim \Omega$. If we combine this with Eq. (11), in which we substitute $t \sim N^{1/\beta}$, then we find

$$
z = 3 - \epsilon/2 + O(\epsilon^2). \tag{13}
$$

Notice that $z = \frac{\delta \beta}{v} = y_h$, with y_h the exponent of the field conjugate to the OP. This was to be expected, since our RG did not involve any frequency integrals.

Let us discuss our results. Grinstein [6] has shown that hyperscaling is violated in RF magnets because the quartic coupling constant is dangerously irrelevant. The same arguments apply here. In order to completely describe static scaling, we therefore need a third exponent, θ , in addition to the usual two independent exponents ν and η . θ describes the flow of u to zero, and enters all hyperscaling relations. Consider, for instance, the OP. Its scale dimension is $d/2 - 1 + \eta/2$, which in the absence of dangerous irrelevant variables leads to the scaling law $\beta = \nu(d - 2 + \eta)/2$. However, since u scales to zero like $u \sim b^{-\theta}$, and since $N(u \to 0) \sim 1/\sqrt{u}$, one has instead

$$
\beta = \nu(d-2-\theta+\eta)/2. \qquad (14)
$$

Our explicit one-loop calculation yields $\theta = 2 + O(\epsilon^2)$, but we expect $\theta = 2$ to all orders in ϵ as is the case in the RF Ising model [14]. Now consider the density

susceptibility $\partial n/\partial \mu$, or the specific heat coefficient $\gamma =$ $\lim_{T\to 0}C_V/T$, or the spin susceptibility χ_s . All scale like an inverse volume times a time, so their naive scale dimension is $d - z$. The violation of hyperscaling changes this to $d - \theta - z$. Above we have seen that z is equal to $y_h = \delta \beta / \nu$, and therefore,

$$
z = (d - \theta + 2 - \eta)/2.
$$
 (15a)

Somewhat surprisingly for a quantum phase transition, the dynamics are not independent from the statics. This is due to the RF fluctuations being stronger than the quantum fluctuations. We thus find that all thermodynamic susceptibilities scale like the OP, viz.,

$$
\chi(t,\Omega) = b^{(2+\theta-d-\eta)/2} \chi(tb^{1/\nu},\Omega b^z), \qquad (15b)
$$

where χ can stand for N, $\partial n/\partial \mu$, γ , or χ_s . Ω can stand for either external frequency or temperature, or, in the case of N, for the distance from the Fermi level.

RF problems contain an anomalously divergent correlation function [6]. In the present case this function describes "mesoscopic" fluctuations of the local DOS, $C(\mathbf{x}, \mathbf{y}) = [N(\mathbf{x})N(\mathbf{y})]_{\text{av}} - N^2$, where $N(\mathbf{x})$ is the (unaver- $\alpha(x, y) = [N(x)N(y)]_{av}$ N, where $N(x)$ is the en-
aged) local DOS at the Fermi level, and $[\cdots]_{av}$ is the ensemble average. At criticality C behaves like [6] $C(k \rightarrow$ $(0) \sim k^{-2+\eta-\theta}$. An experimental observation of such a strong divergence would indicate that RF features are indeed present at the AMT.

We now consider transport properties. The charge or spin diffusivity D has a scale dimension of $z - 2$. Since no *d* is involved, θ does not enter, and we have

$$
D(t, \Omega) = b^{2-z} D(tb^{1/\nu}, \Omega b^z).
$$
 (16)

One is also interested in the scaling behavior of the conductivity $\sigma = D\partial n/\partial \mu$. The behavior of σ depends on whether or not $\partial n/\partial \mu$ has an analytic background contribution in addition to the critical contribution given by Eq. (15b). Reference [4] has argued that for the present model it does not. This yields

$$
\sigma(t,\Omega) = b^{2+\theta-d} \sigma(tb^{1/\nu},\Omega b^z). \tag{17a}
$$

The conductivity exponent s is then

$$
s = \nu(d - 2 - \theta). \tag{17b}
$$

In more general models $\partial n/\partial \mu$ might have a noncritical background contribution. In that case σ will scale like the diffusivity, leading to

$$
s = \nu(z - 2) = \frac{\nu}{2}(d - 2 - \theta - \eta). \tag{17c}
$$

In either case, Wegner scaling [i.e., $s = v(d - 2)$ [15]], which in previous work in $d = 2 + \epsilon$ had been found to hold for the AMT as well as for the Anderson transition [3], is violated. This removes the requirement $s \ge 2(d 2/d$, which follows from Wegner scaling combined with the result of Ref. [16], and has led to severe problems with the interpretation of certain experiments [3].

Finally, we note that our results imply that all of the complications (most of them not quite understood) that are known to occur in the RF magnet problem should be expected for the AMT as well. For instance, although $\theta = 2$ to all orders in perturbation theory, this almost certainly changes due to nonperturbative effects [7]. $d =$ 4 is most likely some sort of a critical dimension. Finally, the non-power-law dynamical critical behavior that has been proposed for RF magnets [8] should be expected to occur at the AMT as well, giving the AMT some aspects of a glass transition, despite the conventional power law scaling encountered in perturbation theory.

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