Phase-Space Exploration in Nuclear Giant Resonance Decay

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The rate of phase-space exploration in the decay of isovector and isoscalar giant quadrupole resonances in ^{40}Ca is analyzed. The study is based on the time dependence of the survival probability and of the spectrum of generalized entropies evaluated in the space of one-particle–one-hole (1p-1h) and 2p-2h states. Three different cases for the level distribution of 2p-2h background states, corresponding to (a) high degeneracy, (b) classically regular motion, and (c) classically chaotic motion, are studied. In the latter case the isovector excitation evolves almost statistically while the isoscalar excitation remains largely localized, even though it penetrates the whole available phase space.

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Collective nuclear excitations occur on dynamical time scales which are short compared to those of the compound nucleus and, therefore, probe simple configurations. In response to an external perturbation a collective mode is initially formed as a nonstationary state which occupies a small fraction of the available phase space. The subsequent decay, on the other hand, involves much longer time scales and explores more complex nuclear configurations. The specific characteristics of this process may, of course, depend on the energy distribution of the initial state, its multipolarity, its isospin, character, or the degree of collectivity. The generic features of the time-dependent phase-space exploration through decay, however, are of a much more general nature [1] and relate to the quantum manifestation of classical chaos in the time-dependent picture [2–5]. The nucleus is especially well suited for addressing such questions because of the inherent quantum nature and a generic chaoticity of the dynamics. There is also a wealth of experimental data which could be useful in verifying some of the theoretical concepts.

The study of chaos in nuclear physics has been mostly based—so far—on level statistics [6]. In practical terms this is rather restrictive since a reliable statistical analysis requires very precise energy resolution. It also does not provide firm means for investigating the role of collectivity and mechanisms of its coexistence with chaos [7]. In this respect the study of temporal correlations between an initially prepared nonstationary state and a state to which it evolves seems to be much more appropriate. Nuclear giant resonances are of central interest in this connection because they are located in a region of high level density which is expected to be dominated by chaotic dynamics. To make the theoretical studies meaningful one needs a scheme which incorporates the relevant elements, such as the possibility of defining a physical collective state, a realistic modeling of the background states whose complexity is consistent with the Gaussian orthogonal ensemble (GOE) of random matrices [6], and, finally, the realistic coupling between the two.

The recently developed model [8], based on a diagonalization of the full nuclear Hamiltonian consisting of a mean field part and a residual interaction,

$$\hat{H} = \sum_i \epsilon_i a_i^\dagger a_i + \frac{1}{2} \sum_{ij,kl} v_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k \,, \qquad (1)$$

in the basis of one-particle–one-hole (1p-1h) and 2p-2h states

$$|1\rangle \equiv a_p^\dagger a_h |0\rangle \,, \quad |2\rangle \equiv a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1} |0\rangle \,, \qquad (2)$$

fulfills these requirements and proves numerically manageable [9]. A prediagonalization of the two-body interaction v in the 1p-1h and 2p-2h subspaces defines $|\tilde{1}\rangle = \sum_1 C_1^{\tilde{1}} |1\rangle$ and $|\tilde{2}\rangle = \sum_2 C_2^{\tilde{2}} |2\rangle$, and the coupling is mediated by the off-diagonal elements $\langle \tilde{1} | v | \tilde{2} \rangle$ and their complex conjugate. An initially excited state, in response to an external one-body field $\hat{F} = \sum_{ij} F_{ij} a_i^\dagger a_j$, can be represented as

$$|F\rangle = \hat{F}|0\rangle = \sum_{\tilde{1}} F_{\tilde{1}} |\tilde{1}\rangle \,. \qquad (3)$$

The amplitudes $F_{\tilde{1}} = \langle \tilde{1} | \hat{F} | 0 \rangle$ contain the entire information about the strength of a given spectral line corresponding to the state $|\tilde{1}\rangle$ and about the phase coherence among these states. On the other hand, the level fluctuations of the states $|\tilde{2}\rangle$ can be used as a measure of the degree of complexity of the background states [8]. As soon as the coupling between the subspaces $|\tilde{1}\rangle$ and $|\tilde{2}\rangle$ is taken into account, the state originally localized in the 1p-1h subspace, as defined by Eq. (3), starts leaking into the 2p-2h space. The degree of mixing depends not only on the magnitude of v but also on the nature of the energy fluctuations in the 2p-2h space, which can significantly influence the distribution of the coupling matrix elements [9,10]. The most natural quantity for describing the leakage is the survival probability, defined as

$$P(t) = |\langle F(0) | F(t) \rangle|^2 . \qquad (4)$$

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The time evolution of the state vector $|F(t)\rangle$ can be expanded as

$$|F(t)\rangle = \sum_n a_n e^{-iE_n t/\hbar} |n\rangle, \quad (5)$$

where E_n and $|n\rangle$ are the eigenenergies and eigenstates of $\hat{H}$ in the space of 1p-1h and 2p-2h states. The expansion coefficients are determined by the initial state as $a_n = \langle n|F(0)\rangle$. The physical significance of $P(t)$ can be identified from its relation to the spectral autocorrelation function $G(E)$ [2,11,12],

$$P(t) = \int dE\, e^{-iEt/\hbar} G_F(E), \quad (6)$$

where

$$G_F(E) = \int dE'\, S_F(E') S_F(E' + E), \quad (7)$$

and $S_F(E)$ is the transition strength distribution $S_F(E) = \sum_n |F_n|^2 \delta(E - E_n)$. Thus $P(t)$ can be obtained from experiments which measure $S_F(E)$. Because of finite energy resolution the experiment determines only an envelope of $S_F(E)$ and consequently smooths out the fluctuations in $P(t)$. Average structures which are more interesting are preserved, however, provided that the resolution is not too coarse.

The calculations presented below for quadrupole excitations in ^{40}Ca are performed in the same basis as in Ref. [9], i.e., including all 1p-1h and 2p-2h states up to 50 MeV and using the same residual interaction. We then have 26 1p-1h and 3014 2p-2h states, which ensure a realistic description of the transition strength distribution. Since the present study concentrates on the phase-space exploration and the role played by chaotic dynamics, we distinguish three cases corresponding to different classes of the spectral fluctuations in the 2p-2h subspace. As established in Ref. [8] one finds (a) with no residual interaction in the 2p-2h subspace there are many degeneracies in $|\tilde{2}\rangle$ $(= |2\rangle)$ and the nearest-neighbor spacing is strongly peaked near zero; (b) inclusion of particle-particle and hole-hole matrix elements in $\langle 2|v|2'\rangle$ removes all degeneracies and leads to a Poissonian distribution of the nearest-neighbor spacings, characteristic of generic integrable systems; and (c) use of the full residual interaction yields GOE fluctuations [6], characteristic of chaotic dynamics.

The initial state $|F(0)\rangle = \hat{F}|0\rangle$ [Eq. (3)] is already nonstationary in the 1p-1h subspace and therefore oscillates within the limits set by this subspace. The time evolution of the resulting survival probabilities ($|F\rangle$ is normalized to unity) is shown in the upper panels of Figs. 1 and 2 for the isovector and isoscalar quadrupole excitations, respectively. It is interesting to note that the isoscalar excitation, being more collective, overlaps—on average—more frequently with its initial state, as a comparison of the horizontal lines in Figs. 1 and 2 indicates. Including the mixing with 2p-2h states [making use of Eq. (5)] the results (Figs. 1 and 2) show that the isovector excitation mixes much more efficiently with the background 2p-2h states, both concerning the oscillatory behavior and

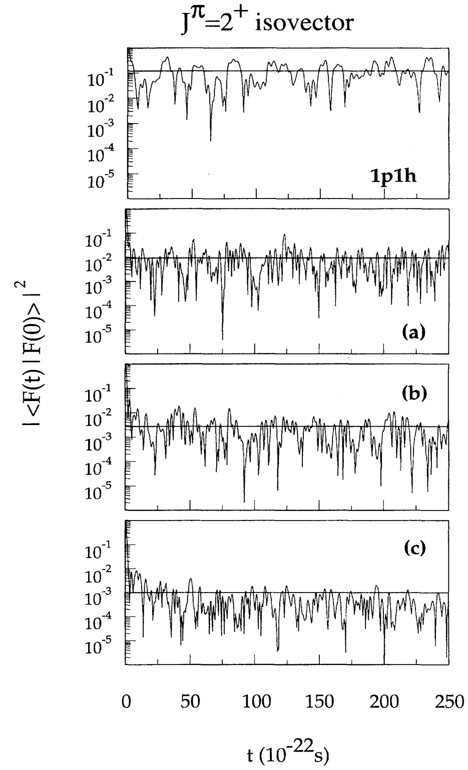

FIG. 1. The time dependence of the isovector quadrupole survival probability $P(t)$ in ^{40}Ca: (1p-1h) no coupling to the 2p-2h subspace; (a) no residual interaction in 2p-2h subspace; (b) including only particle-particle and hole-hole matrix elements in the diagonalization of the 2p-2h subspace; (c) diagonalization of the full residual interaction in the 2p-2h subspace. The solid horizontal lines indicate the time-averaged asymptotic values for the corresponding $P(t)$.

the asymptotic value of the survival probability $P(t)$. The latter systematically decreases with an increasing degree of complexity in the background states [going from (a) to (c)]. Most interestingly, for the isovector excitation in the chaotic case (c), $P(t)N$, where N denotes the total number (3040) of states in our space, reaches—on average—a value close to 3 (3.08). It is known [1] that a state evolving from generic initial conditions does not visit all the regions of the space with equal probability but overlaps more frequently with its initial value. This effect, characteristic of quantum ergodicity [1], is present even if the whole space is accessible. In this extreme case it is just the factor of 3 which prescribes the lowest limit on the average asymptotic behavior of $P(t)$. Such an "elastic enhancement" finds empirical evidence in nuclear physics [13], and the factor of 3 is considered as a quantum-mechanical signature of chaos [4]. While the factor of 3 is consistent with random-matrix-theory estimates [14], the same asymptotic value may, in principle, be associated with the regular dynamics [3] if the subspace is defined such that there are no other conserved quantum numbers than those defining it. An additional

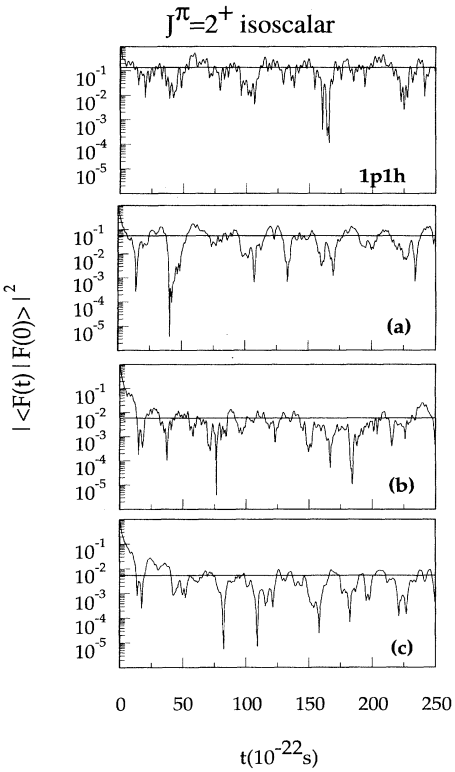

FIG. 2. Same as Fig. 1 but for the isoscalar quadrupole survival probability.

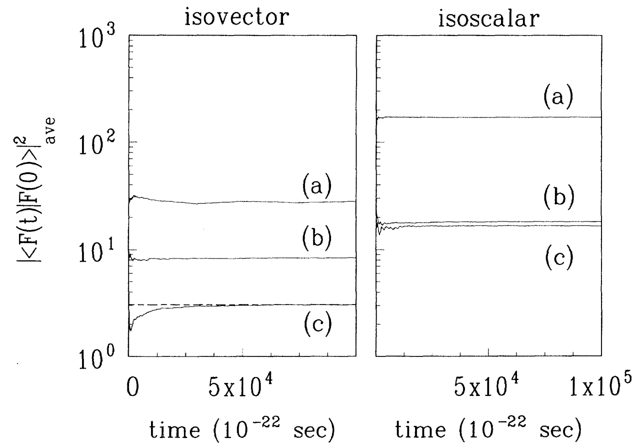

FIG. 3. The time-averaged survival probability $\overline{P}(t)$ in units of $1/N$ (see text) for the isovector (left-hand side) and isoscalar (right-hand side) quadrupole resonances: (a) no residual interaction in 2p-2h subspace; (b) including only particle-particle and hole-hole matrix elements in the diagonalization of the 2p-2h subspace; (c) diagonalization of the full residual interaction in the 2p-2h subspace.

requirement for chaoticity is the initial dephasing of $P(t)$ below its asymptotic value [2,3,5,12]. Such a dephasing, indeed, takes place for the isovector case, as can be seen from Fig. 3 where $\overline{P}(t) = \int_0^t dt' P(t')/t$ for all three types of the background spectra is displayed. As a consequence of the high density of states the corresponding "correlation hole" [2] extends over a time interval, 4 orders of magnitude longer than the characteristic "excitation time" of $\sim 10^{-22}$ sec. It should be emphasized that, contrary to all previous studies, no ensemble average over Hamiltonians or initial conditions has been employed in our investigations. The emergence of the correlation hole should not depend on the restrictions of the model space (1p-1h and 2p-2h). Its temporal extent may increase, however, in a more complete space due to an effective increase in the 2p-2h level density in the vicinity of the giant resonance. The isoscalar excitation (right panel of Fig. 3) only shows a trace of such a behavior, and the asymptotic values of $P(t)$ are systematically larger even though the initial state is coupled to the same background.

The question "What makes the isoscalar state so strongly localized?" then arises. Is it a manifestation of collectivity or, perhaps, is it that the specific properties of the coupling matrix elements block certain regions of the phase space and ergodization only occurs in the unblocked regions? A quantity which appears helpful in resolving this question originates from the concept of entropy. The information entropy of the state $|F\rangle$ can be defined, in a given basis $|k\rangle$, as

$$K = -\sum_k p_k \ln p_k \,, \tag{8}$$

where $p_k = |\langle k|F\rangle|^2$. It provides a quantitative measure of the complexity of the state $|F\rangle$ and its localization length in the basis $|k\rangle$ [15]. The so-defined K is, in principle, basis dependent, but the physically preferred basis is determined by the mean field [16]. As is well known, the mean field provides the dynamical parameters, such as the inertial mass or the restoring force for the collective motion. As the most smooth component of the nuclear Hamiltonian [17] it thus provides a natural reference for quantifying local GOE-type fluctuations. Further arguments for the appropriateness of the mean field basis relate to the degree of noncommutativity [18] between the operator whose fluctuations are to be studied and the one spanning the basis. Differences in eigenvalue spectral fluctuations [8] indicate that the degree of noncommutativity between our full Hamiltonian and the mean field is sufficiently high so that the mean field basis can be considered "random" with respect to the full wave function.

In our case the mean field basis corresponds to the unperturbed basis of states $|1\rangle$ and $|2\rangle$. Calculating $K(t)$ along the "trajectory" $|F(t)\rangle$ for the isovector and isoscalar states, we obtain asymptotically values of 7.10 and 6.53, while the corresponding initial values are 2.56 and 2.40, respectively. This is to be compared to $K^{\rm GOE} = 7.29$ [$K^{\rm GOE} = \psi(N/2 + 1) - \psi(3/2)$ [15], where ψ is the digamma function and N is the number of basis vectors]. A comparison of these numbers indicates nonuniformities in the p_k distribution, especially for the isoscalar excitations. Actually, even the GOE-type fluctuations result in a Gaussian distribution which is nonuniform (a uniform distribution maximizes the entropy and corresponds to $p_k = 1/N$ which, for $N = 3040$, gives $K \approx 8.02$).

In view of the above mentioned nonuniform phase-space exploration, we find it instructive to calculate the spectrum of q moments for $\{p_k\}$ and to introduce a generalized entropy [19]

$$K_q = \frac{1}{1-q} \ln \sum_{k=1}^{N} p_k^q . \tag{9}$$

From this definition it follows that $K_{q_1} \leq K_{q_2}$ if $q_2 < q_1$ (provided $\sum_k p_k = 1$). Equality holds for the uniform distribution. For $q \rightarrow 1$ Eq. (9) yields the information entropy [Eq. (8)]. The most important property of K_q is that with increasing q a higher weight is given to the largest components in the set $\{p_k\}$. For $q \rightarrow 0$, on the other hand, K_q just counts the number of sites (here the basis vectors $|k\rangle$) visited, irrespective of how frequently they are sampled. For this reason Eq. (9) also constitutes a basis for defining the multifractal dimensions of nonuniform fractal sets [20].

For selected q values Fig. 4 compares the time evolution of $K_q(t)$ for the isovector and isoscalar excitations when the background states have GOE fluctuations [case (c)]. As one can see from the large-q behavior of $K_q(t)$, which are systematically smaller for the isoscalar excitation, the large components of these remain much more localized (larger) than those of the isovector excitation. Since, by probability conservation, the number of significant components is smaller in the former case, the amplitude of oscillations is larger in the corresponding $K_q(t)$. On the other hand, the dynamics start to look similar in both cases as q decreases and, for $q \rightarrow 0$, K_q approaches a value of 8. This signals that, on the level of small probabilities, the whole space spanned by 3040 states is visited. This aspect of the dynamics is consistent with the scaling properties of the transition strength distribution for the isovector and isoscalar states discussed in Ref. [9]. On the level of small components they both scale. The similar calculation of $K_q(t)$ for the cases (a) and (b) shows that the states evolve to configurations characterized by significantly smaller entropies.

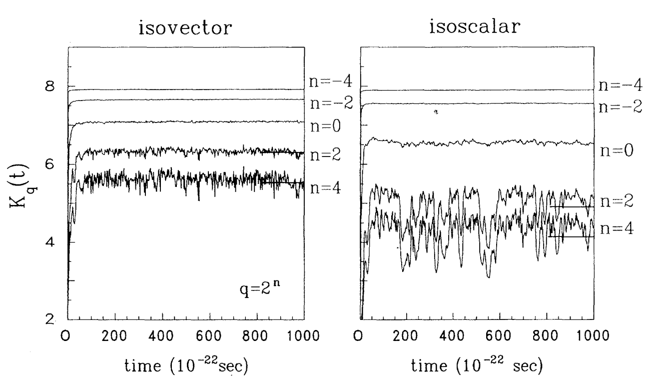

FIG. 4. The time evolution of the generalized entropies defined by Eq. (9) for the isovector (left-hand side) and isoscalar (right-hand side) giant resonances corresponding to case (c) in Fig. 2. The horizontal marks on the left-hand side of each panel denote the asymptotic values of K_q for $q = 4$ and $q = 16$.

It would be very interesting to verify some of the above predictions experimentally—especially the appearance of the correlation hole in the average survival probability $\overline{P}(t)$. This quantity is accessible through the convolution formula [Eq. (7)] in conjunction with Eq. (6). For the purpose of addressing specific questions concerning the quantum-mechanical phase-space exploration of collective modes, the full set of "generalized entropies" appears to be a useful theoretical tool.

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