QCD Analysis of the Mass Structure of the Nucleon

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From the deep-inelastic momentum sum rule and the trace anomaly of the QCD energy-momentum tensor, I derive a separation of the nucleon mass into contributions of the quark and gluon kinetic and potential energies, quark masses, and the trace anomaly. The separation is done in the rest frame of the nucleon and at the [modified minimal subtraction scheme ($\overline{\text{MS}}$)] renormalization scale 1 GeV^2.

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The nucleon derives its mass (939 MeV) from the quark-gluon dynamics of its underlying structure. However, due to complexity of low-energy quantum chromodynamics (QCD), a more detailed understanding of the nucleon mass seems difficult. Lattice QCD is successful in reproducing the measured mass from the fundamental Lagrangian [1], but the approach provides little insight on how the number is partitioned between the nucleon's quark and gluon content. Years after the advent of QCD, our knowledge of the nucleon's mass structure comes mostly from models: nonrelativistic quark models, bag models, the Skyrme model, string models, the Nambu–Jona-Lasinio model to name just a few. Though all the models are made to fit the nucleon mass, they differ considerably on its origin. Depending on different facets of QCD the models are created to emphasize, interpretations of the nucleon mass often go to opposite extremes.

In this Letter I show that an insight on the mass structure of the nucleon can be produced within QCD with the help of the deep-inelastic momentum sum rule and the trace anomaly. The result is a separation of the nucleon mass into the contributions from the quark, antiquark, gluon kinetic and potential energies, quark masses, and the trace anomaly. Numerically, the only large uncertainty is the size of $\langle P|m_s\bar{s}s|P\rangle$, the strange scalar charge of the nucleon. Some implications of this mass breakup are discussed following the result.

Let me begin with the energy-momentum tensor of QCD,

$$T^{\mu\nu} = \tfrac{1}{2}\overline{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi + \tfrac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}, \quad (1)$$

where ψ is the quark field with color, flavor, and Dirac indices; $F^{\mu\nu}$ is the gluon field strength with color indices and $F^2 = F^{\alpha\beta}F_{\alpha\beta}$, and all implicit indices are summed over. The covariant derivative $\overleftrightarrow{D}^{\mu} = \overrightarrow{D}^{\mu} - \overleftarrow{D}^{\mu}$, with $\overrightarrow{D}^{\mu} = \overrightarrow{\partial}^{\mu} + igA^{\mu}$ and $\overleftarrow{D}^{\mu} = \overleftarrow{\partial}^{\mu} - igA^{\mu}$, where $A^{\mu} = A^{\mu}_{a}t^{a}$ is the gluon potential. The symmetrization of the indices μ and ν in the first term is indicated by $(\mu\nu)$. Equation (1) is quite formal, for it contains neither the gauge fixing and ghost terms nor the trace anomaly. The first type of terms have exact Becchi-Rouet-Stora-Tyuton (BRST) symmetry [2] and have vanishing physical matrix elements according to the Joglekar-Lee theorems [3]. I will add the trace anomaly explicitly when the renormalization issue is dealt with.

A few results about the energy-momentum tensor are well known. First of all, it is a symmetric and conserved tensor,

$$T^{\mu\nu} = T^{\nu\mu}, \quad \partial T^{\mu\nu} = 0. \quad (2)$$

Because of the second property, the tensor is a finite operator and does not need an overall renormalization [4]. All the fields and couplings in Eq. (1) are bare and their divergences are canceled by the standard set of renormalization constants. The only complication is that the tensor cannot be renormalized with a vanishing trace (see below). Second, the tensor defines the Hamiltonian operator of QCD,

$$H_{\text{QCD}} = \int d^3x\, T^{00}(0,\mathbf{x}), \quad (3)$$

which is also finite and scale independent. Third, the matrix element of the tensor operator in the nucleon state is [5]

$$\langle P|T^{\mu\nu}|P\rangle = P^{\mu}P^{\nu}/M, \quad (4)$$

where $|P\rangle$ is the nucleon state with momentum P^{μ} and is normalized according to $\langle P|P\rangle = (E/M)(2\pi)^3\delta^3(\mathbf{0})$ and E and M is the energy and mass of the nucleon, respectively. Lastly, the trace of the tensor is [6]

$$\hat{T}^{\mu\nu} = \tfrac{1}{4}g^{\mu\nu}\left[(1+\gamma_m)\overline{\psi}m\psi + \frac{\beta(g)}{2g}F^2\right], \quad (5)$$

where m is a quark mass matrix, γ_m is the anomalous dimension of the mass operator, and $\beta(g)$ is the β function of QCD. At the leading order $\beta(g) = -\beta_0 g^3/(4\pi)^2$ and $\beta_0 = 11 - 2n_f/3$, where n_f is the number of flavors. The second term is called the trace anomaly and is generated in the process of renormalization.

According to the above, the mass of the nucleon is

$$M = \frac{\langle P|\int d^3x\, T^{00}(0,\mathbf{x})|P\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle, \quad (6)$$

in the nucleon's rest frame. Although I formally work with the matrix elements of the nucleon, it actually is the difference of the nucleon matrix elements and the vacuum matrix elements that enters all the formula (the vacuum has zero measurable energy density). According to (6), a mass separation can be found through a decomposition of $T^{\mu\nu}$ into various parts, which are then evaluated with the deep-inelastic momentum sum rule and the scalar charge of the nucleon. (Note that parts of the energy-momentum tensor are not separately conserved, so the breaking of the nucleon energy cannot be Lorentz covariant.)

First of all, let me decompose $T^{\mu\nu}$ into the traceless and trace parts,

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \hat{T}^{\mu\nu}, \tag{7}$$

where $\overline{T}^{\mu\nu}$ is traceless. According to Eq. (4), I have

$$\langle P|\overline{T}^{\mu\nu}|P\rangle = \left(P^\mu P^\mu - \tfrac{1}{4}M^2 g^{\mu\nu}\right)/M, \tag{8}$$

$$\langle P|\hat{T}^{\mu\nu}|P\rangle = \tfrac{1}{4}g^{\mu\nu}M. \tag{9}$$

Combining Eq. (6) with the above three equations, I get

$$\langle \overline{T}^{00}\rangle = \tfrac{3}{4}M, \tag{10}$$

$$\langle \hat{T}^{00}\rangle = \tfrac{1}{4}M. \tag{11}$$

Thus $\frac{3}{4}$ of the nucleon mass comes from the traceless part of the energy-momentum tensor and $\frac{1}{4}$ from the trace part. The magic number 4 is just the space-time dimension. This decomposition, a bit like the virial theorem, is valid for any bound states in field theory.

The traceless part of the energy-momentum tensor can be decomposed into contributions from the quark and gluon parts,

$$\overline{T}^{\mu\nu} = \overline{T}_q^{\mu\nu} + \overline{T}_g^{\mu\nu}, \tag{12}$$

where

$$\overline{T}_q^{\mu\nu} = \tfrac{1}{2}\overline{\psi}i\vec{D}^{(\mu}\gamma^{\nu)}\psi - \tfrac{1}{4}g^{\mu\nu}\overline{\psi}m\psi, \tag{13}$$

$$\overline{T}_g^{\mu\nu} = \tfrac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}. \tag{14}$$

Although the sum of $\overline{T}_q^{\mu\nu}$ and $\overline{T}_g^{\mu\nu}$ with bare fields and bare couplings is finite (now neglecting the trace anomaly), individual operators are divergent and must be renormalized. Under renormalization, they mix with each other, and with other BRST-exact operators and with the operators of the equations of motion which have vanishing physical matrix elements [2,3]. For my purpose, I regard both the operators renormalized and dependent on a renormalization scale μ^2. Define their matrix elements in the nucleon state,

$$\langle P|\overline{T}_q^{\mu\nu}|P\rangle = a(\mu^2)\left(P^\mu P^\nu - \tfrac{1}{4}g^{\mu\nu}M^2\right)/M, \tag{15}$$

$$\langle P|\overline{T}_g^{\mu\nu}|P\rangle = [1 - a(\mu^2)]\left(P^\mu P^\nu - \tfrac{1}{4}g^{\mu\nu}M^2\right)/M, \tag{16}$$

where I have used Eq. (8) to get the second equation. $a(\mu^2)$ is related to the deep-inelastic sum rule [7],

$$a(\mu^2) = \sum_f \int_0^1 x[q_f(x,\mu^2) + \overline{q}_f(x,\mu^2)]dx, \tag{17}$$

where the sum is over all quark flavors and $q_f(x,\mu^2)$ and $\overline{q}_f(x,\mu^2)$ are quark momentum distributions in the nucleon in the infinite momentum frame. Again, according the Eq. (6), I find the contributions to the nucleon mass,

$$\langle \overline{T}_q^{00}\rangle = \tfrac{3}{4}a(\mu^2)M, \tag{18}$$

$$\langle \overline{T}_g^{00}\rangle = \tfrac{3}{4}[1 - a(\mu^2)]M. \tag{19}$$

Finally, I turn to the trace part of the energy-momentum tensor, $\hat{T}^{\mu\nu}$. According to Eq. (5), I decompose it into $\hat{T}_m^{\mu\nu}$ and $\hat{T}_a^{\mu\nu}$, the mass and trace anomaly term, respectively. Both operators are finite and scale independent. If I define

$$b = 4\langle \hat{T}_m^{00}\rangle/M, \tag{20}$$

then, according to Eq. (11), the anomaly part contributes

$$\langle \hat{T}_a^{00}\rangle = \tfrac{1}{4}(1 - b)M. \tag{21}$$

Thus, the energy-momentum tensor $T^{\mu\nu}$ can be separated into four gauge-invariant parts, $\overline{T}_q^{\mu\nu}$, $\overline{T}_g^{\mu\nu}$, $\hat{T}_m^{\mu\nu}$, and $\hat{T}_a^{\mu\nu}$. They contribute, respectively, $3a/4$, $3(1-a)/4$, $b/4$, and $(1-b)/4$ fractions of the nucleon mass. The corresponding breakdown for the Hamiltonian is $H_{\rm QCD} = H_q' + H_g + H_m' + H_a$, with

$$H_q' = \int d^3x\left[\psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi + \tfrac{3}{4}\overline{\psi}m\psi\right], \tag{22}$$

$$H_g = \int d^3x\,\tfrac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \tag{23}$$

$$H_m' = \int d^3x\,\tfrac{1}{4}\overline{\psi}m\psi, \tag{24}$$

$$H_a = \int d^3x\,\frac{9\alpha_s}{16\pi}(\mathbf{E}^2 + \mathbf{B}^2), \tag{25}$$

where I have consistently neglected γ_m and the higher-order terms in $\beta(g)$. One can put them back if a higher precision analysis becomes necessary. I also have taken $n_f = 3$. (Note that the heavy quarks do contribute to the mass term, the kinetic and potential energy term, and the trace anomaly term. However, the contributions cancel each other in the limit of $m_f \to \infty$, and for simplicity I neglected them.) If I rearrange the mass term by defining

$$H_q = \int d^3x\,\psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi, \tag{26}$$

$$H_m = \int d^3x\,\overline{\psi}m\psi, \tag{27}$$

then the QCD Hamiltonian becomes

$$H_{\rm QCD} = H_q + H_m + H_g + H_a. \tag{28}$$

Here H_q [Eq. (26)] represents the quark and antiquark kinetic and potential energies and contributes $3(a-b)/4$

fraction of the nucleon mass. H_m [Eq. (27)] is the quark mass term and contributes b fraction of the mass. H_g [Eq. (23)] represents the gluon energy and contributes $3(1-a)/4$ fraction of the mass. Finally, H_a [Eq. (25)] is the trace anomaly term and contributes $(1-b)/4$ fraction of the mass.

To determine the separation numerically, I need the matrix elements a and b. The deep-inelastic scattering experiments have determined $a(\mu^2)$ with an accuracy of a few percent. Using a recent fit to the quark distributions [8],

$$a_{\overline{\rm MS}}(1\ {\rm GeV}^2) = 0.55\,, \tag{29}$$

where $\overline{\rm MS}$ refers to the modified minimal subtraction scheme.

Without the heavy quarks, the matrix element b is

$$bM = \langle P|m_u\bar{u}u + m_d\bar{d}d|P\rangle + \langle P|m_s\bar{s}s|P\rangle\,. \tag{30}$$

The first term is the πN σ term apart from a small isospin-violating contribution of order 2 MeV. A most recent analysis gave a magnitude of 45 ± 5 MeV for this term [9]. So the only unknown in our analysis is the strange scalar charge $\langle P|m_s\bar{s}s|P\rangle$ in the nucleon. There are model calculations for this quantity in the literature [10]. Here I choose to estimate it using two standard approaches, though both of them are not completely satisfactory.

In the first approach [11], the strange quark mass is considered small in the QCD scale, and so the chiral perturbation theory can be used to calculate the SU(3) symmetry breaking effects. A recent second-order analysis on the spectra of the baryon octet combined with the measured σ term yields [9]

$$\langle P|\bar{s}s|P\rangle \simeq 0.11\langle P|\bar{u}u + \bar{d}d|P\rangle \tag{31}$$

$$\simeq 0.77\,, \tag{32}$$

where in the second line I have used $(m_u + m_d)/2 \simeq 7$ MeV at the scale of 1 GeV^2 [12]. Taking the strange quark mass to be 150 MeV at the same scale, I get

$$bM \simeq 160\ {\rm MeV}\,. \tag{33}$$

Using Eq. (11), I have

$$\langle P|(\alpha_s/\pi)F^2|P\rangle = -693\ {\rm MeV}\,. \tag{34}$$

In the second approach, the strange quark is considered heavy in the QCD scale. Using heavy-quark expansion, it was found [13] that

$$\langle P|m_Q\bar{Q}Q|P\rangle = -\tfrac{1}{12}\langle P|(\alpha_s/\pi)F^2|P\rangle\,. \tag{35}$$

Thus the strange quark contribution in

$$\langle P|\bar{\psi}m\psi + [\beta(g)/2g]F^2|P\rangle = M\,, \tag{36}$$

which is an explicit form of Eq. (11), cancels. From the above equation and the σ term, I find

$$\langle P|(\alpha_s/\pi)F^2|P\rangle = -740\ {\rm MeV}\,. \tag{37}$$

This yields a strange matrix element $\langle P|m_s\bar{s}s|P\rangle = 62$ MeV. Together with the σ term, I determine

$$bM = 107\ {\rm MeV}\,. \tag{38}$$

The complete result of the mass separation at the scale of $\mu^2 = 1$ GeV^2, together with the two numerical estimates, is shown in Table I. I have not shown errors due to omission of higher-order perturbative effects and errors on the σ term and current quark masses. The total effect on individual numbers is about 5 to 10 MeV. Thus I have rounded up the numbers to nearest 10 MeV. The largest uncertainty is from the matrix element $\langle P|m_s\bar{s}s|P\rangle$, which could be larger than the difference of the two estimates shown. Nevertheless, I will argue below that the total strange contribution to the nucleon mass is quite small and with a smaller uncertainty.

The following comments can be made with regard to the numerical result.

(a) The quark kinetic and potential energies contribute about $\frac{1}{3}$ of the nucleon mass. Because the quark kinetic energy must be very large when confined within a radius of 1 fm, there must exists a large cancellation between the kinetic and potential energies. This may not be entirely surprising in the presence of strong interactions between quarks and gluons. Such strong interactions are clearly at the origin of the chiral symmetry breaking, embodied, for instance, in the Nambu–Jona-Lasinio model [10].

(b) The separation of the quark energy into different flavors is possible. Taking the number 270 MeV (the $m_s \rightarrow 0$ limit) as an example, I find the up-quark energy in the proton is 250 MeV using the momentum fraction carried by up quark 0.38 [8], the down quark energy, 105 MeV, and strange quark energy, -85 MeV. Further decomposition into valence and sea contributions cannot be made without knowledge of the separate valence and sea contributions to the scalar charge.

TABLE I. A separation of the nucleon mass into different contributions. The matrix elements a and b are defined in Eqs. (15) and (20).

Mass type	H_i	M_i	$m_s \rightarrow 0$ (MeV)	$m_s \rightarrow \infty$ (MeV)
Quark energy	$\psi^\dagger(-i\mathbf{D}\cdot\boldsymbol{\alpha})\psi$	$3(a-b)/4$	270	300
Quark mass	$\bar{\psi}m\psi$	b	160	110
Gluon energy	$\frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$	$3(1-a)/4$	320	320
Trace anomaly	$\frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2)$	$(1-b)/4$	190	210

(c) The normal gluon energy is about $\frac{1}{3}$ of the nucleon mass and the trace anomaly part contributes about $\frac{1}{4}$. From these two, I deduce the color-electric an color-magnetic fields in the nucleon separately [take $\alpha_s(1\ \text{GeV}) \simeq 0.4$],

$$\langle P|\mathbf{E}^2|P\rangle = 1700\ \text{MeV}, \tag{39}$$

$$\langle P|\mathbf{B}^2|P\rangle = -1050\ \text{MeV}. \tag{40}$$

So the magnetic-field energy is negative. This of course is due to a cancellation between the quark's magnetic field and that of the vacuum. On the other hand, the electric field in the nucleon is large and positive. This behavior of the color fields in presence of quarks is interesting, it may help to unravel the structure of the QCD vacuum.

(d) In the chiral limit, the gluon energy from the trace anomaly ($M/4$) corresponds exactly to the vacuum energy in the MIT bag model [14]. The role of such energy in the model is to confine quarks. Thus one sees a way here through which the scale symmetry breaking connects with quark confinement. To have confinement, a model should include H_a, the anomaly contribution to the effective Hamiltonian.

(e) The strange quark contributes about -60 MeV to the mass through the trace anomaly. When adding to the kinetic and potential energy contribution -85 MeV and the mass term 115 MeV (the $m_s \rightarrow 0$ limit), the total strange contribution to the nucleon mass is a mere -30 MeV. (The other limit gives a total of -45 MeV.) The smallness of the contribution is, to a large extent, insensitive to the matrix element $\langle P|m_s\bar{s}s|P\rangle$.

To summarize, I have found a separation of the nucleon mass into contributions from the quark kinetic and potential energy, gluon energy, and the trace anomaly. The largest uncertainty is from the strange matrix element $\langle P|m_s\bar{s}s|P\rangle$. The result has interesting implications on the quark-gluon structure of the nucleon and on the response of the QCD vacuum to color charges. Similar separation for the spin of the nucleon has been long sought in connection with polarized deep-inelastic scattering, where the axial anomaly seems to play an important role [15]. The present result encourages a vigorous study on the spin structure of the nucleon.

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