ERRATA

Reversible Quantum Measurements on a Spin 1/2 and Measuring the State of a Single System [Phys. Rev. Lett. 73, 913 (1994)]

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In [1], a reversible measurement on a spin 1/2 was introduced, and it was concluded that this would allow us, with a very small but finite probability of success, to measure the state of a single system. This conclusion is not correct. Let me recapitulate the argument: The measurement yields one of two outcomes, T_1 or T_2, with respective probabilities P_1 or $P_2 = 1 - P_1$, which depend on the diagonal elements of the initial density matrix

$$\rho^i = \begin{pmatrix} a & c \\ c^* & 1-a \end{pmatrix}.$$

The number "a" can be deduced from P_1, hence from many measurements on *different* systems all in the same state. By also using suitably rotated measurements, one can determine c, hence the state ρ^i. Suppose that N measurements are required to achieve some prescribed accuracy. Now, the measurement is reversible in the sense that the initial state (even if unknown) can be *knowingly* recovered by means of other similar measurements, with a sizable probability of success, P_1^{rev} or P_2^{rev}, depending on whether the outcome was T_1 or T_2. Thus, a sequence of successive measurements on a *single* system may, with a probability which is very small, but nonzero, contain N reversals to the initial state, hence N "basic" measurements performed on the initial state. Because the outcome of each basic measurement is independent of any other measurements, one might think, as I did implicitly in [1], that the numbers N_1 and N_2 of outcomes T_1 and T_2 should occur in their "natural" proportions, $N_1/N_2 \sim P_1/P_2$, allowing us to deduce the initial state. But it was pointed out by Finkelstein, Huttner, and Gisin that this is not correct. In fact one has

$$\frac{N_1}{N_2} \sim \frac{\text{Prob}\{T_1 \text{ \& reversal}\}}{\text{Prob}\{T_2 \text{ \& reversal}\}} = \frac{P_1 P_1^{\text{rev}}}{P_2 P_2^{\text{rev}}}. \tag{1}$$

The additional factor $P_1^{\text{rev}}/P_2^{\text{rev}}$ reflects the fact that the outcome of each basic measurement influences the probability of subsequent reversal (which is not the same after T_1 or T_2), so that N_1/N_2 is skewed in the direction increasing the chances of reversal. Now, from Eqs. (4.10) and (5.5) of [1], one gets, assuming that reversal is achieved after just one measurement,

$$P_1 P_1^{\text{rev}} = k^2 T_{1+}^2 T_{1-}^2, \quad P_2 P_2^{\text{rev}} = k^2 T_{2+}^2 T_{2-}^2, \tag{2}$$

which are *independent of the initial state* (this is easily seen to remain true if reversal is achieved after *any* number of measurements). As stated by Huttner, the price to pay for a successful reversal is that you do not gain any information on the system. Thus, (1) yields no information on the initial state, and it is not possible to measure the state of a single system in this manner. Gisin points out that the possibility of doing so, even with an extremely low probability of success, would allow, by means of EPR-type experiments, to signal faster than light, and thereby "end the peaceful coexistence between quantum mechanics and relativity." Fortunately, this is not the case.

[1] A. Royer, Phys. Rev. Lett. **73**, 913 (1994).

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