Vortex Flow in Helium-Filled Vycor Glass

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(Received 22 April 1994)

The system of helium-filled Vycor glass exhibits a regime of weak vortex pinning and dissipative superflow over an extended temperature region below the thermodynamic transition. This regime of dissipative superflow has an analog in the weak pinning and vortex liquid states found in high T_c materials. We have studied the ac flow of helium in this regime for the ^{4}He-Vycor system. The results are interpreted in terms of a complex response function with components reflecting the flow-induced polarization of the medium and the dissipation.

PACS numbers: 67.40.Hf, 67.40.Vs, 74.60.Ge

The phenomenon of dissipative superflow in high T_c superconductors associated with weak pinning and the proposed vortex liquid state is a subject of active current interest [1]. In our discussion, reported here, we explore the problem of dissipative superflow in an analogous system of superfluid ^{4}He in porous Vycor glass. As with the case of high temperature superconductors, the ^{4}He-Vycor system displays strong pinning of vortex lines at low temperatures and persistent currents can flow with essentially zero dissipation. However, at high temperatures, but well before the thermodynamic transition is reached, the strength of pinning becomes weak and even low velocity superflow leads to vortex motion and attendant dissipation. This regime of dissipative superflow in Vycor has an analog in the weak pinning and vortex fluid states of the high T_c materials. It is our belief that an understanding of the physics of dissipative superflow in the ^{4}He-Vycor system will be of general value and may provide useful insight into dissipative processes in the high T_c case.

For our discussion of dissipative superflow in the ^{4}He-Vycor system we have developed a phenomenological analysis which follows closely the ideas developed in the Kosterlitz-Thouless [2] model of the vortex unbinding transition in 2D He films and particularly its extension to finite frequencies by Ambegaokar *et al.* (AHNS) [3]. The essential ideas which emerge in the finite frequency model are that vortex pairs give rise to both dissipation and a reduction of the inertia of the superfluid. The combined effect is expressed in a complex dielectric function ϵ and is seen as a polarization phenomenon. Specifically, at finite frequencies dissipation results from vortex pairs separated by the diffusion length which are maximally out of phase with the oscillating flow, as well as from free vortices created by thermal activation. In addition, vortex pairs of separations up to the diffusion length can orientate themselves in the oscillating flow and produce a dissipationless coupling between the substrate and the superfluid thus reducing the measured superfluid density. Both of these results are succinctly expressed in Eq. (3.1) in the AHNS theory which relates the coupling of the space average of the superfluid velocity u_s to the substrate velocity v_n, through $u_s = (1 - \epsilon^{-1})v_n$. There is a close analogy here to a dielectric in an ac electric field, with u_s and v_n playing the roles of the polarization and external electric fields, respectively. In the AHNS treatment of vortex dynamics, the superfluid density is seen as a response function dominated by the factor ϵ^{-1} and has a rather broader interpretation than in the traditional two-fluid model. We will adopt a similar approach in this report of superflow in a porous medium and find that a generalized velocity-dependent dielectric function provides a consistent framework within which to analyze our results. In general terms, since the superfluid density in a porous medium is the stiffness coefficient [4], we find a "softening" of the medium resulting from vortices generated in large amplitude mass flows analogous to plastic deformation in a crystal containing a high concentration of stress-generated dislocations [5].

Nondissipative coupling of the superfluid to the motion of the substrate has long been known in porous media; a discussion of this has been given by Bergman *et al.* [6]. The effect results from the superfluid having to follow the complicated surface of the solid structure, which plays the role of a static impurity. In an acoustic experiment, this appears as a refractive index η which measures the longer path followed by a fourth sound wave. In an inertial experiment [7] it appears as a tortuosity factor ξ ($\xi = \eta^{-2}$), which is the fraction of the superfluid mass not coupled to the substrate. Tyler and Vavasour [8] have described the general case of a moving substrate together with a pressure gradient; their expression for the acceleration of the superfluid in a porous medium moving with velocity V_m is

$$\frac{\partial V_b}{\partial t} = -\frac{1}{\rho\epsilon}\frac{\partial P}{\partial x} + \left(1 - \frac{1}{\epsilon}\right)\frac{\partial V_m}{\partial t}. \qquad (1)$$

Here V_b (equal to u_s in the notation of AHNS) is the average flow velocity parallel to the axis of the superleak

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and we have replaced the η^2 factor in the Tyler-Vavasour result with ϵ in anticipation of our phenomenological description. Later, we will use V_b to calculate the pressure rise in a closed cavity into which the superleak empties. In the absence of a pressure gradient this equation reduces to that of AHNS; in our experiment we have only a pressure gradient. The essential assumption we now make, is that, as in the Kosterlitz-Thouless case, with vortices present the factor ϵ becomes a complex function and depends on the flow velocity relative to the substrate. The "bare" superfluid density is that measured at very low velocities where only the usual static impurity effect of the medium remains. With this generalization, Eq. (1) can be used to calculate mass flows in our experiment.

We now apply the ideas presented above to the analysis of dissipative flow data obtained with a double-channel Helmholtz resonator experiment [9]. The Helmholtz resonator was employed to provide an ac driving pressure across a thin slab of porous Vycor [10] glass superleak in parallel with an open channel. In a typical measurement the resonator was held at fixed temperature while the resonant angular frequency ω and total dissipation are determined as functions of the drive pressure ΔP. After correction for the dissipation and mass flow through the open channel, we obtain the current I passing through the Vycor superleak. Analysis of the two-channel resonator gives the following expression relating the Vycor supercurrent to the driving pressure ΔP,

$$\omega\left(\frac{I}{A_v}\right) = \left(\frac{\Delta P}{l_v}\right) G\sigma . \tag{2}$$

A and l are the open channel area and length, respectively, and A_v and l_v are the area and length of the superleak. The geometry constant $G = Al_v/A_v l$ and $\sigma = \sigma_D - i\sigma_I$ is the complex conductivity, where σ_I and σ_D are the inertial and dissipative components, respectively. These quantities are related to the measured frequency and dissipation by $\sigma_I = (\omega/\omega_1)^2 - 1$ and $\sigma_D = (\omega/\omega_1)^2 Q^{-1}$; here ω_1 is the resonant frequency measured with the superleak blocked and Q^{-1} is the dissipation attributed to the dissipative flow through the Vycor. In the temperature range of our measurements we find that both components of σ are nonlinear functions of the supercurrent. In Fig. 1 we show the variation of the inertial and dissipative components, at fixed temperature, with a velocity parameter, $v = I_0/A_v \rho_{s0}$, where I_0 is the magnitude of the current and ρ_{s0} is the zero velocity limit of the superfluid density and includes the tortuosity of the medium. The value of reduced temperature, $t = 1 - T/T_c$, for this data set is 2.60×10^{-3}. At low values of the driving pressure the velocity and dissipation are small and the conductance is dominated by the inertial term σ_I. As the velocity is increased the dissipation increases passing through a maximum while the inertial component rapidly declines. The data of Fig. 1 bear a resemblance to torsional oscillator measurements of ^{4}He films at the Kosterlitz-Thouless transition, with the log of the velocity parameter substituting for temperature.

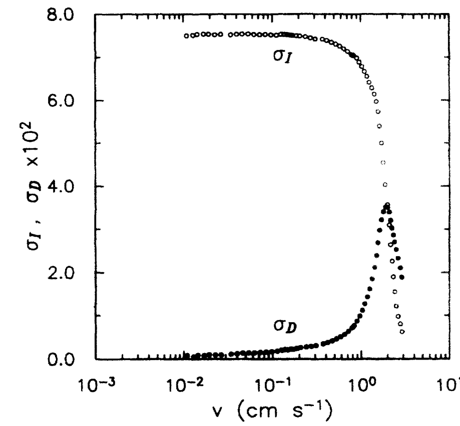

FIG. 1. The flow conductance coefficients σ_I and σ_D are plotted as functions of the velocity parameter v.

A more informative view of the data is given in Fig. 2 where we have plotted the dissipative component against the inertia term for data obtained at a number of reduced temperatures. Here the driving pressure or current can be viewed as the parametric variable. As the drive is reduced to zero the inertial component approaches a fixed value σ_{I0} proportional to ρ_{s0}. An important feature of these data is immediately evident. In the vicinity of the σ_{I0} for each data set, the dissipative component σ_D initially increases linearly with the reduction in the inertial component σ_I. A simple model offers an instructive contrast. If we assume that the superfluid mass is independent of the flow velocity while a dissipative friction or coupling between the superfluid and the Vycor increases with current then the values of σ_I and σ_D will trace a semicircular path of diameter $2\sigma_{I0}$. The dashed curve indicates the path that would be followed at constant superfluid density. The deviation of the actual

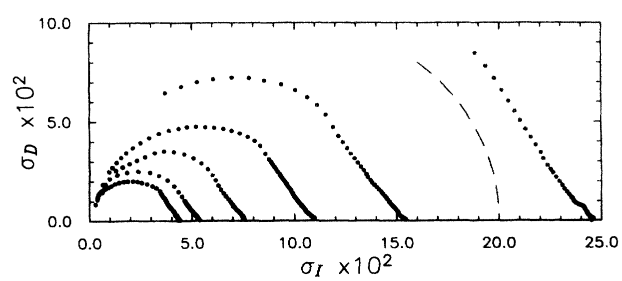

FIG. 2. The conductance coefficients σ_I and σ_D are plotted against each other for a series of runs taken at fixed reduced temperatures. The values of $t \times 10^3$ are 1.05, 1.53, 2.60, 4.56, 8.56, and 16.65 progressing from left to right. The dashed curve starting at 0.2 indicates the path followed by σ_I and σ_D at constant superfluid density.

data from such a path indicates that the magnitude of the superfluid density is being reduced below its zero velocity value. The reduction can be estimated in the context of the constant superfluid mass model and results from real part of ϵ, $\mathrm{Re}(\epsilon)$. Given the values of $\sigma_I(v)$ and $\sigma_D(v)$ for velocity v, the zero velocity limit for the inertial term $\sigma_{I0}(v)$ in this model is given by $\sigma_{I0}(v) = (\sigma_I^2 + \sigma_D^2)/\sigma_I = |\sigma(v)|^2/\sigma_I(v)$. Such a reduction in the superfluid density with increasing flow has long been known from studies of persistent currents in porous media. Kojima *et al.* [11] first reported a fractional reduction as large as 1.2% in the presence of a persistent current. This work has been extended to a greater level of sensitivity by Carey and Pobell [12].

The quantity of interest, in our case, will be the relative reduction of the superfluid density as a function of the flow velocity or driving pressure. The fractional shift is given by

$$\delta\rho_s(v)/\rho_{s0} = 1 - \sigma_{I0}(v)/\sigma_{I0}(0). \quad (3)$$

From Eq. (1), for a static superleak, multiplying by the superfluid density and superleak area we get the expression for the current

$$i\omega\epsilon(I/A_v) = (\rho_{s0}/\rho)\Delta P/l_v. \quad (4)$$

Here tortuosity has been absorbed into ρ_{s0} so that ϵ depends only on velocity and is unity at zero flow. From this we see by comparison with Eq. (2) that $iG\sigma = (\rho_{s0}/\rho)\epsilon^{-1}$. Expressed in terms of the dielectric function, the fractional shift in the superfluid density then becomes

$$\delta\rho_s(v)/\rho_{s0} = 1 - \mathrm{Re}[\epsilon(v)]^{-1}. \quad (5)$$

We will postpone a consideration of dependence of the ρ_s shift on driving pressure and superflow velocity until a later point.

In Eq. (4) the quantity $i\omega\epsilon$ may be viewed as a complex impedance (ω is almost constant across the data at 1600 rad s^{-1}). The dependence of the resistive component, $\omega\,\mathrm{Im}(\epsilon)$, on the flow velocity is of particular interest. In Fig. 3 we plot, on log-linear scales, this quantity as a function of the velocity parameter v defined earlier. The resistive component has units of inverse seconds and gives a measure of the reciprocal decay time for a persistent current. It is clear that for temperatures in the range shown in Fig. 3, a persistent current would be destroyed within a small fraction of a second. Recent experiments with a single channel resonator [13], with much higher sensitivity, show the dissipation to increase continuously from the lowest detectable velocities. The magnitude of the flow resistance increases almost exponentially with flow velocity indicating a rapid growth in the number and flow of unpinned vortices. This increase is smooth and monotonic without indication of special features such as the dissipation maximum seen in Fig. 1. Although we do not show it here, these data can be collapsed close to a universal plot through scaling of the velocity parameter v/t. This linear scaling of v and t can alternatively be

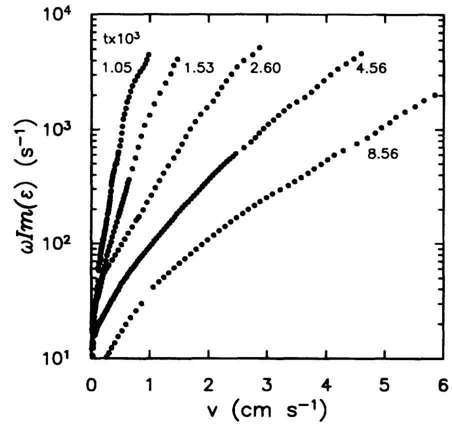

FIG. 3. The flow resistance, $\omega\,\mathrm{Im}(\epsilon)$, is plotted as a function of the velocity parameter v, for the five lowest reduced temperatures. The progression in temperature is from left to right with the lowest temperature data on the left.

expressed in a critical transfer velocity [9] corresponding to the saturation of the inertial component of the mass current, close to the peak of σ_D for which $\mathrm{Im}(\epsilon) \approx 1$ and ω is almost constant at 1600 rad s^{-1}.

In Fig. 4 we have plotted the fractional shift in the superfluid density $\delta\rho_s(v)/\rho_{s0}$ as a function of the flow resistance $\omega\,\mathrm{Im}(\epsilon)$. As the flow velocity and the resistance increase there is a rapid reduction in the value of the superfluid density, however, once the 10% to 12% level is reached, the shift is seen to saturate to a nearly constant value. While there is a tendency of these data to move to higher fractional shifts with increasing t (with exception of the data for $t = 16.65$) their close coincidence indicates a consistent picture across the temperature range.

A possible model to interpret our results at low velocities, up to the saturation in Fig. 4, might be of vortex loops of the effective medium generated by and respond-

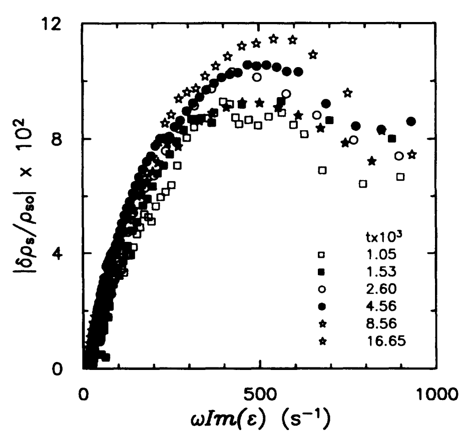

FIG. 4. The absolute value of the fraction shift in superfluid density is shown for the six reduced temperatures as a function of the resistance parameter $\omega\,\mathrm{Im}(\epsilon)$.

ing to the flow up to a size set by a diffusion scale as in AHNS. The importance of both dissipation and dissipationless coupling of the superfluid is indicated by the linear region at low velocities in Fig. 4. The model of Gillis *et al.* [14] for strong nonlinear response in the 2D case may be a useful guide here. Bowley and Giorgini [15] have also considered the nonlinear response through the whole velocity range. Gillis *et al.* [14] place the emphasis on the reduced activation energy of vortex pairs in the presence of a finite flow with the prediction that dissipation increases essentially exponentially with velocity following the increased density of polarizable pairs. In 3D this would translate to vortex ring density. The extension of William's model [16] has proposed a model of interacting vortex rings in 3D; the extension of this to finite velocities may also be useful in explaining our results. At high flows, beyond the peak of σ_D, where $\mathrm{Im}(\epsilon) = 1$, the flow is dominated by dissipation. At the highest dissipations shown in Fig. 3 the flow is approximately 75% resistive and can hardly be called superflow at all. In this regime we visualize almost viscous motion of a vortex tangle, where pinning plays only a small role and a better picture would be of a single component flow at high Reynold's number, essentially a state of fully developed turbulence with the normal fluid vortex line interaction providing a uniform drag.

We note that similar experiments using aerogel [17] have produced similar results, perhaps suggesting behavior generic to superleaks.

A. T. gratefully acknowledges extensive discussions with D. B. Hannon and is grateful to Professor R. M. Bowley for discussions and a preview of his analysis together with S. Giorgini of the resonator in terms of phase slippage. J. D. R. would like to thank Manchester University Physics Department for its hospitality during the period in which this work was initiated and the SERC(GR/F/45332) for its support to him as a Senior Research Fellow. The Cornell portion of this research has been supported by the NSF through the Cornell Material Science Center NSF/MSC-DMR-91-21654, Report No. 7741, and by the NSF through Grant No. NSF-DMR-8921733. We thank G. Zassenhaus for help in producing the figures.

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