Dynamics of the Sawtooth Collapse in Tokamak Plasmas

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(Received 25 March 1994)

Fast reconnection such as driven by electron inertia qualitatively changes the Kadomtsev picture of the sawtooth collapse in tokamak discharges. The collapse occurs in two steps, a fast Kadomtsev-type reconnection followed by a rapid reformation of a $q_0 < 1$ configuration. The latter is driven by the strong flows generated during the Kadomtsev phase. The theory provides a natural explanation of the main experimental observations including the snake phenomena.

PACS numbers: 52.55.Fa, 52.30.−q, 52.35.Py

The nature of the sawtooth collapse in tokamak plasmas has been a major puzzle since the observations of Soltwisch [1] on the TEXTOR tokamak indicating that the central safety factor q_0 remains well below unity after the collapse. Initially $q_0 \sim 0.75$, and the change Δq during the collapse is small, $\Delta q \lesssim 0.1$. This behavior has since been corroborated by observations on several different devices [2,3] under various different discharge conditions. In the standard theoretical model by Kadomtsev [4] and its variants full reconnection of the helical flux inside the $q = 1$ surface takes place, bringing q_0 above unity. Efforts to reconcile this model with the observations have not been successful. The prevailing view is that full reconnection does not occur. Instead reconnection ceases after the $m = 1$ magnetic island reaches a finite amplitude. This partial reconnection model suffers from two distinct problems: (a) Why does reconnection halt and (b) if it does halt, how can the thermal energy in the nonreconnected region be released? While toroidally induced field line stochastization can provide a plausible energy escape process at least for the electron thermal energy [5], no mechanism has been found to stop the dynamics before full reconnection has occurred. Solutions of the full compressible resistive magnetohydrodynamics (MHD) equations in toroidal geometry [6,7] show $m = 1$ dynamics very similar to the original Kadomtsev reduced MHD model. There are in fact arguments [8,9] that the nonlinear reconnection process should become independent of the actual MHD free energy, for instance, in the presence of an ideal MHD instability. In addition, the recent tomographic analysis of the electron temperature distribution on the TFTR tokamak exhibits a sequence of states that are amazingly similar to a Kadomtsev-type full reconnection process. The collapse time, however, is much faster than predicted for resistivity dominated reconnection. Although the soft x-ray emission during a typical sawtooth collapse on the JET tokamak exhibits a "hot crescent" instead of a hot shifted circular core, there is no indication that the reconnection is not complete.

An alternative interpretation of the experimental observations has recently been suggested by Kolesnichenko *et al.* [10]. These authors point out that Kadomtsev's equations allow more general solutions than the one discussed by Kadomtsev, permitting also $q_0 < 1$ in the final state. The initial and the final q profiles can even be identical. Such solutions require, however, two distinct reconnection processes, where the second (partly) reverses the effect of the first. The authors, however, admit that they cannot provide a mechanism for the second reconnection. In fact, numerical simulations in the framework of resistive MHD at low values of resistivity do not reveal any indication of such a secondary process. Because of insufficient time resolution in the measurement of $q(r)$, the experimental observations cannot distinguish between partial reconnection and full reconnection followed by a rapid partial reversal. The statistical analysis by Soltwisch [1] seems to rule out a purely diffusive process as caused by neoclassical resistivity [7]. (However, such a process could add to the efficiency of the mechanism described in this Letter.) The emergence of the snake [11,12], a helical filament of high plasma density localized at the $q = 1$ surface, intact after the sawtooth crash, is, moreover, apparently evidence in favor of partial reconnection. Because of these observations and the lack of a plausible mechanism for the second reconnection, the two-stage model has not been widely accepted.

In this paper, we show that the two-stage reconnection, with $q_0 < 1$ at the end, is a natural consequence of fast reconnection in high temperature tokamak plasmas. The critical ingredient is the kinetic energy of the flow generated during the first Kadomtsev-type reconnection. We first estimate the kinetic energy available at the end of a resistive Kadomtsev reconnection using the well-known properties of the Sweet-Parker layer. The bulk inflow velocity is $U \sim \eta^{1/2} V_A$ corresponding to an energy $\sim \eta$, which is negligible. Here V_A is the Alfvén velocity of the helical field B in front of the sheet, and η is the (normalized) resistivity. The kinetic energy resides primarily in the poloidal flow which is ejected at high velocity $V \sim V_A$ from the resistive layer of width $\delta \sim \eta^{1/2} r_s$ and length $L \sim r_s$ with r_s the radius of the initial $q = 1$ surface. At the end of the Kadomtsev reconnection the total kinetic energy remaining in this poloidal flow

0031-9007/94/73(7)/971(4)$06.00

is $\sim V_A^2 L\delta \sim \eta^{1/2}$, which is again small. In fact most of the magnetic energy liberated during the Kadomtsev process is dissipated Ohmically. Since $j \sim B/\delta \sim \eta^{-1/2}$, the rate of Ohmic dissipation is $\sim \eta j^2 L\delta \sim \eta^{1/2}$. The reconnection time is $\tau \sim U^{-1} \sim \eta^{-1/2}$. So the total Ohmic dissipation is ~ 1. These scaling laws explain the results of resistive simulations, that the dynamics essentially stops after the Kadomtsev reconnection phase, leaving the system with an extended central region with $q \gtrsim 1$.

Recently nondissipative effects in Ohm's law, electron inertia [13] possibly combined with electron pressure gradient [14], have been shown to give rise to different $m = 1$ mode reconnection dynamics characterized by a rapid nonlinear increase of the reconnection rate. Under conditions where dissipative processes (resistivity η and electron viscosity μ_e) are sufficiently weak such that the reconnection is dominated by the nondissipative effects in Ohm's law, most of the magnetic energy is transformed into kinetic energy. Typical tokamak plasmas fall in this regime [1–3]. Hence at the end of the first reconnection the system carries a strong convective plasma flow. It is this flow which leads to a partial inversion of the helical flux distribution and a final q value distinctly below unity. These arguments are substantiated by numerical simulations. To demonstrate the principle we restrict consideration to the simple reduced equations for the helical flux function ψ and the stream function ϕ, including electron inertia,

$$\partial_t \psi + \mathbf{v} \cdot \nabla\psi = d^2(\partial_t j + \mathbf{v} \cdot \nabla j) - \mu_e \nabla^2 j, \quad (1)$$

$$\partial_t \omega + \mathbf{v} \cdot \nabla\omega = \mathbf{B} \cdot \nabla j - \nu\omega, \quad (2)$$

$$\mathbf{B} = \hat{\mathbf{z}} \times \nabla\psi, \quad \mathbf{v} = \hat{\mathbf{z}} \times \nabla\phi,$$

$$j = \nabla^2\psi + 2, \quad \omega = \nabla^2\phi,$$

written in conventional dimensionless form with $d = c/\omega_{pe} r_s$, μ_e a phenomenological (perpendicular) electron viscosity representing the effect of weak field-line stochasticity [15] and/or current density gradient driven whistler turbulence [16], and ν a friction coefficient due to magnetic pumping or some flow instability. Figure 1 illustrates the time evolution of the helical flux $\psi(x,y)$ of a simulation with the initial current profile $j_0(r) = (2/q_0)(1 + r^2/r_0^2)^{-2}$, $q_0 = 0.67$, $d = 0.02$, and $\mu_e = 10^{-8}$. A friction coefficient $\nu = 0.05$ was switched on after the flow energy reached its maximum value simulating the onset of a shear flow instability. Figure 1(a) gives the initial flux distribution. The nonlinear evolution starts at $t \simeq 1100$ and the Kadomtsev phase, Fig. 1(b), terminates at $t \simeq 1230$. The dynamics resembles that of the resistive Kadomtsev process (except for the time scale). The current sheet, which forms at the X point, gradually decays due to dissipation by electron viscosity. Following the Kadomtsev phase the strong flow pulls the helical flux back into the central region [Figs. 1(c)–1(e)]. Finally, secondary reconnection leads to a partial reformation of the initial sheared state with $q_0 \simeq 0.75$ [Fig. 1(f)].

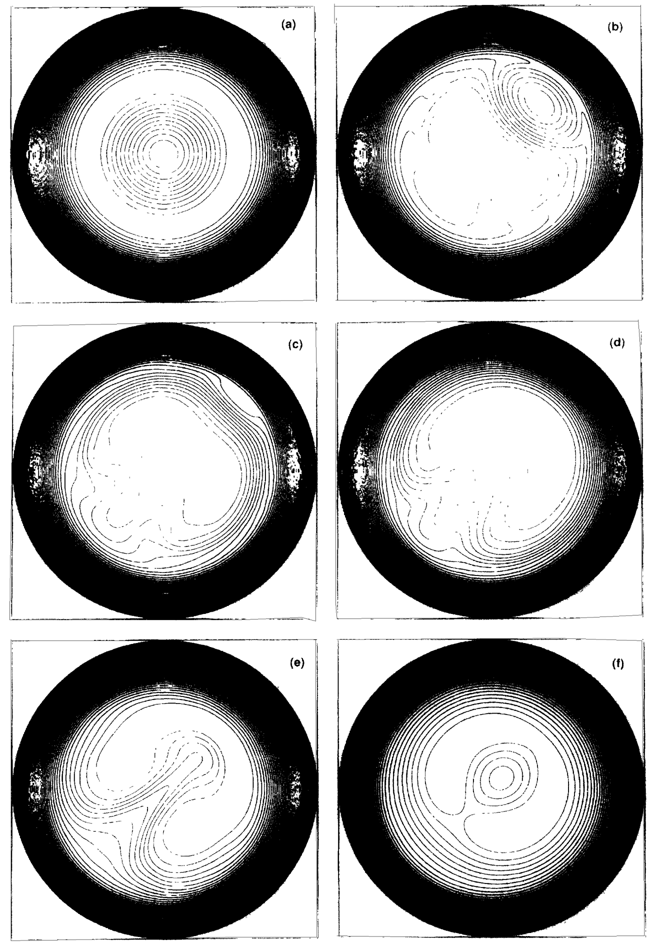

FIG. 1. Simulation of the time evolution of the helical flux function in the sawtooth collapse taken at $t = 0$, 1200, 1230, 1248, 1266, and 1350, showing the Kadomtsev reconnection, the reversal of the Kadomtsev process, and the secondary reconnection regenerating an almost symmetric configuration with $q_0 < 1$.

Figure 2 gives the current density distribution showing the current sheets corresponding (a) to the Kadomtsev reconnection and (b) to the secondary reconnection. Note that in the first case j is negative compared with the equilibrium current direction, while it is positive in the second case. Several points are noteworthy.

(a) The dynamics illustrated in Fig. 1 leads to the formation of an extended shear-free belt of $q \simeq 1$ around the sheared central region. Such a flat q belt has a strongly stabilizing effect on the $m = 1$ mode [17]. Thus, the final state with $q < 1$ remains and does not begin a second Kadomtsev-type reconnection leading to $q > 1$.

(b) Because of this broad $q \simeq 1$ belt in the final state only part of the magnetic energy set free during the Kadomtsev phase is reinvested in the formation of the final magnetic state, even if q_0 is not much different from

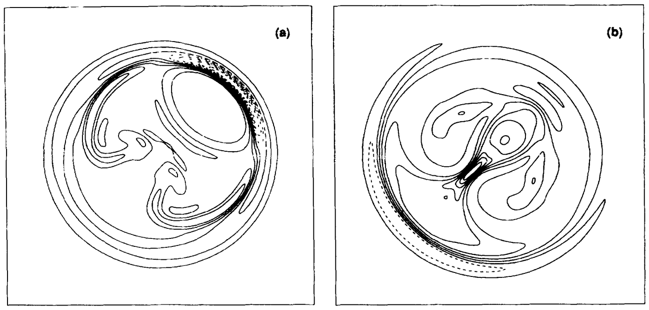

FIG. 2. Current density distribution corresponding to Kadomtsev reconnection $t = 1200$ (a) and secondary reconnection $t = 1302$ (b).

its original value. Hence there is an excess of kinetic energy.

(c) In the simulation run presented in Fig. 1 a damping of the flow is needed to prevent the collimated flow across the center evident from Fig. 1(e) from driving reconnection at its head, thus splitting the flat q region into two parts. Subsequent sloshing and reconnection would further fragment the helical flux distribution into smaller pieces (islands) eventually leading to an average flat q profile. Sufficient damping could be generated by a Kelvin-Helmholtz instability of the sheared flow. The necessity of an efficient flow damping is, however, less stringent if we start from an initial q profile slightly flattened about $r = r_s$, as would be expected in a self-consistent simulation of the full sawtooth cycle. In this case a state corresponding to Fig. 1(f) is generated even without flow damping. The corresponding $q \simeq 1$ belt surrounding the central $q < 1$ region is also broader. Though the remaining flow energy must be ultimately dissipated, the evolution of ψ does not sensitively depend on this damping. Even magnetic pumping should suffice in this case.

(d) The system returns only slowly to a fully axisymmetric state, corresponding to a slowly decaying helical perturbation, which can be associated with the postcursor oscillations observed after the sawtooth collapse.

It should be noted that a morphologically very similar process occurs in the ideal $m = 1$ instability of a low-q nonreversed-field pinch configuration [18,19]. Starting with a nonresonant configuration $q_0 < 1/n$ [19], where n is the axial mode number, the system evolves into a reversed-field configuration with $q_0 > 1/n$, mainly by increasing B_z in the central region. In the case of the flux reversal phase on the sawtooth collapse in a tokamak, the plasma is also nonresonant, $q_0 \simeq 1$, at the beginning and evolves into a resonant state, $q_0 < 1$. The main difference is that while in the low-q pinch case the system is strongly unstable, in the tokamak case the system is stable before the reversal process, the latter being driven by the strong flows present. While in the pinch the final state is stable owing to the reversed B_z field in the outer region, it is stable in the tokamak case because of the extended shear-free $q \simeq 1$ belt around the central region. We should, however, note that because of the dense spacing of resonances in the reversed-field pinch, a single helicity behavior is not likely, while it dominates in the tokamak case.

Finally, we want to discuss qualitatively how the apparent survival of the snake during the sawtooth collapse can be reconciled with this picture. We claim that the snake, instead of remaining intact, breaks apart and then reforms. Since the density blob forms a substantial helical perturbation, it almost certainly controls the poloidal phase of the collapse dynamics. The symmetry of the equations (in the absence of diamagnetic drifts) implies that two situations are possible as indicated in Fig. 3. The current sheet forms either at the location of the snake [Fig. 3(a)] or just across from the snake [Fig. 3(b)]. In the latter case the snake could survive the collapse but its position would suffer a phase shift $\Delta\theta = \pi$. In the former case the snake splits, is transported along the sheet by the plasma flow, reforms in the center, and then shifts back to the original side. Hence there is no phase shift. Experimental observations [11] indicate that there is no phase shift, thus favoring the fragmentation and reformation picture. The long-time persistence of the snake requires a strong process of density condensation which should also be active after the fragmentation of the snake during reconnection. This picture of the snake behavior assumes that the sawtooth dynamics is not affected by the presence of the snake. It can, however, not be excluded that the dynamics is changed, since the density perturbation of the snake is much larger than the density change as a result of the sawtooth collapse.

In conclusion, we have presented a mechanism, supported by numerical simulations, of rapid reformation of a $q_0 < 1$ configuration after full Kadomtsev-type reconnection in the sawtooth collapse. This solves a long-time puzzle in the understanding of the sawtooth collapse: How

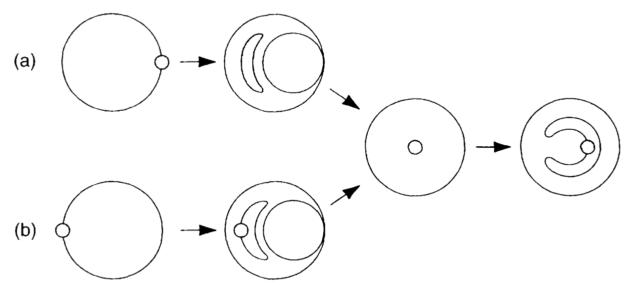

FIG. 3. "Survival" of the snake in the collapse. (a) Reconnection starts at the location of the snake. After the cycle is complete, the snake is shifted radially inward compared with its precollapse position. (b) If the snake is located on the opposite side, the snake remains intact during the collapse, but its poloidal location suffers a phase shift π.

can q remain below unity after the crash? The collapse occurs in two steps as has recently been suggested by Kolesnichenko *et al.* [10]. The important new ingredient is the high speed flow which results from fast weakly dissipative reconnection in high temperature tokamaks. The reformation of a region with $q < 1$ does not occur during the much slower resistive reconnection, where most of the free magnetic energy is Ohmically dissipated. The two-step process provides a natural explanation of both the T_e tomography and the observation of $q_0 < 1$, though it will be very difficult to observe the second step process directly in the experiments, since T_e has been essentially flattened by that time. The mechanism can also be reconciled with the apparent survival of the snake during the sawtooth collapse.

The authors gratefully acknowledge the assistance of Marianne Walter and Michael Tippett in carrying through the numerical computation. J. F. D. would like to acknowledge the support of the Alexander von Humboldt Foundation through a Senior Scientist Research Award at IPP, Garching.

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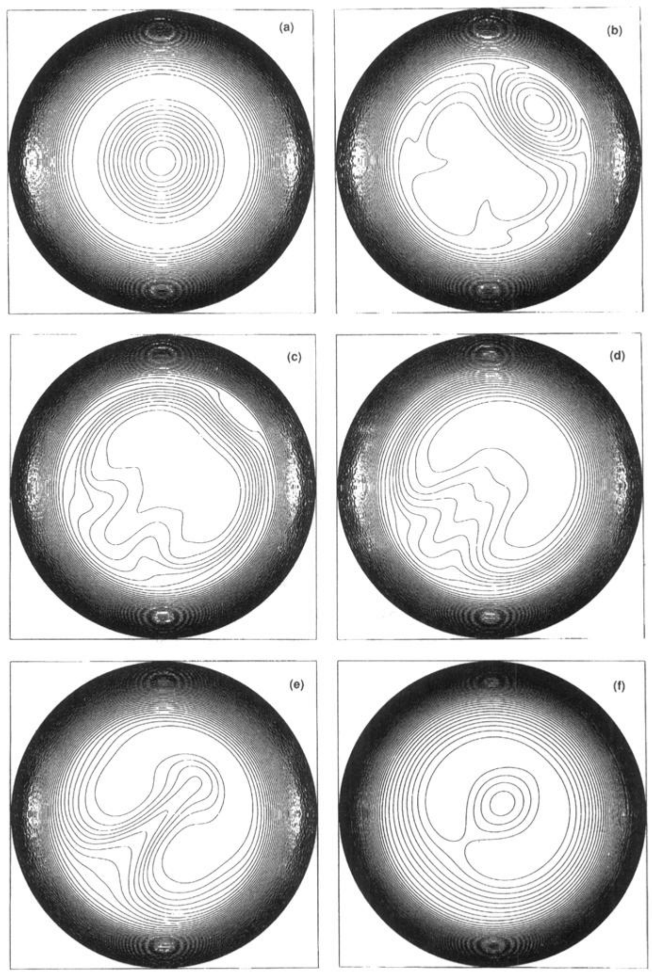

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