Angular Distributions in the Drell-Yan Process: A Closer Look at Higher Twist Effects

A. Brandenburg, S. J. Brodsky, V. V. Khoze, and D. Müller
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 28 March 1994)

We calculate the angular distribution of the lepton produced in the Drell-Yan reaction taking into account pion bound state effects. This is particularly interesting since the standard perturbative QCD prediction is inconsistent with the data. We work in the kinematic region where one of the pion constituents goes far off shell, which allows us to treat the bound state problem perturbatively. We show that the angular distribution is very sensitive to the shape of the pion distribution amplitude. The model we discuss fits the data if we choose a two-humped pion distribution amplitude suggested by QCD sum rules.

PACS numbers: 13.85.Qk, 12.38.Cy

Lepton pair production in hadron-hadron collisions provides a basic testing ground for our understanding of quantum chromodynamics (QCD)—the theory of strong interactions. Extensive experimental and theoretical work has been done in the past two decades (for reviews, see Ref. [1]). In particular, the angular distribution of the lepton pair has been studied in detail, revealing a fatal disagreement of the QCD improved parton model prediction [2,3] with the data [4–6]. Recently it has been proposed that this problem may be resolved if the nontrivial structure of the QCD vacuum induces spin correlations between the initial state partons [7]. In this Letter we pursue another way to go beyond the standard parton model picture, namely, we consider contributions to the angular distribution induced by hadron bound state effects. Our approach is close in spirit to the higher twist model of [8,9].

The angular distribution of the μ^+ in

$$\pi^- + N \to \gamma^* + X \to \mu^+ + \mu^- + X \qquad (1)$$

may be parametrized in general as follows:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \sim 1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\,. \qquad (2)$$

Here θ and ϕ are angles defined in the muon pair rest frame and λ, μ, and ν are angle-independent coefficients. The naive parton model (Drell-Yan picture [10]) views the production of the virtual photon γ^* in (1) as originating from the annihilation of two uncorrelated constituent quarks, resulting in an angular distribution of the form $1 + \cos^2\theta$. This result follows simply from the fact that the virtual photon is produced transversely polarized in the annihilation of two on-shell fermions.

In order to describe the boson transverse momentum distribution $d^2\sigma/dQ_T^2$ one has to take into account radiative corrections to the Drell-Yan model. The Q_T distribution has been calculated in the QCD-improved parton model to the order of $O(\alpha_s)$ with resummation of the soft gluons at the leading double-logarithmic accuracy (see Ref. [11] and references therein). This approach was used in [3] to compute the angular distribution at fixed transverse momentum. The deviations from the $1 + \cos^2\theta$ behavior were found to be less than 5% in the range $0 < Q_T < 3$ GeV [3]. However, the NA10 measurements from CERN [4] and the Chicago-Iowa-Princeton collaboration [5,6] show a quite different behavior. In the limit where the momentum fraction x of one of the pion constituents is very close to 1 and for moderate transverse momenta of the muon pair, the value of λ turns strongly negative [6], consistent with a $\sin^2\theta$ distribution. This implies that in this kinematic limit the virtual photon is produced with longitudinal polarization, rather than transverse. Furthermore, the data [4–6] are observed to have a strong azimuthal modulation [nonzero μ and ν in (2)], an effect which is missing in standard QCD. The Lam-Tung sum rule [2], $1 - \lambda - 2\nu = 0$, which follows from the approach used in [3] is also badly violated by the experimental data.

One way to go beyond the standard treatment is to take into account the pion bound state effects [8,9]. We want to treat the bound state problem perturbatively; thus we will restrict ourselves to a specific kinematic region in which the momentum fraction x of one of the pion constituents is large, $x > 0.5$. In fact, in the large x region the off-shell nature of the annihilating quark from the projectile is crucial, and thus the operative subprocess must involve the correlated multiparton structure of the projectile. In effect the dominant subprocess in the off-shell domain is $\pi^- q \to \mu^+\mu^- q$. We resolve the pion by a single hard gluon exchange [12]. The main contribution reaction (1) then comes from the diagrams in Figs. 1(a) and 1(b) [8,9]. We see from Fig. 1(a) that the $\bar{u}$ quark propagator is far off shell, $p_{\bar{u}}^2 = -Q_T^2/(1 - x_{\bar{u}})$. The second diagram [Fig. 1(b)] is required by gauge invariance. [In a physical gauge the contribution of the second diagram is purely higher twist; that is, it contains extra powers of $\sqrt{Q_T^2/Q^2}$.] The leading contribution to the amplitude M for the reaction

$$u + \pi^- \to \gamma^* + X \to \mu^+ + \mu^- + X \qquad (3)$$

0031-9007/94/73(7)/939(4)$06.00

is obtained [12] by convoluting the partonic amplitude $T(u + \bar{u}d \to \gamma^* + d \to \mu^+ + \mu^- + d)$ with the pion distribution amplitude $\phi(z, \tilde{Q}^2)$,

$$M = \int_0^1 dz\, \phi(z, \tilde{Q}^2) T\,, \tag{4}$$

where $\tilde{Q}^2 \sim Q_T^2/(1-x)$ is the cutoff for the integration over soft momenta in the definition of ϕ. In the regime where $\tilde{Q}^2$ and Q^2 are compatible, one cannot use the usual probabilistic factorization of the structure functions and the hard annihilation subprocess [13].

For the hadronic differential cross section we have

$$\frac{Q^2\, d\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2\, dQ_T^2\, dx_L\, d\Omega} = \frac{1}{(2\pi)^4}\frac{1}{64}\int_0^1 dx_u\, G_{u/N}(x_u)\int_0^1 dx_{\bar{u}}\, \frac{x_{\bar{u}}}{1 - x_{\bar{u}} + Q_T^2/Q^2}\,|M|^2$$
$$\times\, \delta(x_L - x_{\bar{u}} + x_u - Q_T^2 s^{-1}(1 - x_{\bar{u}})^{-1})$$
$$\times\, \delta(Q^2 - s x_u x_{\bar{u}} + Q_T^2(1 - x_{\bar{u}})^{-1}) + \{u \to \bar{d}, \bar{u} \to d\}. \tag{5}$$

Here Q^μ is the four-momentum of γ^* in the hadronic center of mass system, $x_{u(\bar{u})}$ is the light-cone momentum fraction of the u ($\bar{u}$) quark, and $G_{u/N}$ is the parton distribution function of the nucleon. The longitudinal momentum fraction of the photon is defined as $x_L = 2Q_L/\sqrt{s}$, and it should be noted that its maximum value $x_L^{\max} = 1 - s^{-1}(Q^2 + 2Q_T^2)$ is slightly less than 1. The second term on the right hand side of (5) is the same as the first one with quark flavors interchanged. This term gives the contribution from the nucleon sea. In Fig. 1(c) we show a typical contribution to the hadronic cross section.

We note that no primoridal or intrinsic transverse momenta have been introduced. The single-gluon exchange is the only source of Q_T in the model discussed. We also neglected the quark masses and the mass of the projectile which are small compared to $\tilde{Q}$.

In analogy to Eq. (2) we parametrize the angular distribution as follows:

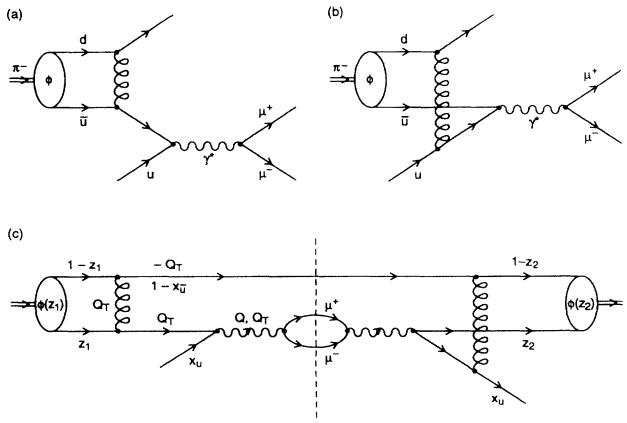

FIG. 1. (a) and (b) give the leading contribution to the amplitude of reaction (4). (c) gives a typical (1 out of 4) contribution to the cross section (6).

$$\frac{Q^2\, d\sigma}{dQ^2\, dQ_T^2\, dx_L\, d\Omega}\left(\frac{Q^2\, d\sigma}{dQ^2\, dQ_T^2\, dx_L}\right)^{-1} = \frac{3}{4\pi}\frac{1}{\lambda + 3}(1 + \lambda\cos^2\theta + \mu \sin 2\theta\cos\phi + \tfrac{1}{2}\nu\sin^2\theta\cos 2\phi), \tag{6}$$

where the angular distribution coefficients λ, μ, and ν are now functions of the kinematic variables x_L, Q_T^2/Q^2, and Q^2/s.

We work in the Gottfried-Jackson frame where the $\hat{\mathbf{z}}$ axis is taken to be the pion direction in the muon pair rest frame and the $\hat{\mathbf{y}}$ axis is orthogonal to the $\pi^- N$ plane. With some algebra, using Eqs. (4)–(6), we arrive at the following expression for λ, μ, and ν:

$$\lambda(\tilde{x}, Q_T^2/Q^2) = 2N^{-1}\{(1 - \tilde{x})^2[(\mathrm{Im}I(\tilde{x}))^2 + (F + \mathrm{Re}I(\tilde{x}))^2] - 4(Q_T^2/Q^2)\tilde{x}^2F^2 + (Q_T^4/Q^4)\tilde{x}^2F^2\}, \tag{7}$$

$$\mu(\tilde{x}, Q_T^2/Q^2) = -4N^{-1}\sqrt{Q_T^2/Q^2}F\tilde{x}\{(1 - \tilde{x}) \times [F + \mathrm{Re}I(\tilde{x})] + (Q_T^2/Q^2)\tilde{x}F\}, \tag{8}$$

$$\nu(\tilde{x}, Q_T^2/Q^2) = -8N^{-1}(Q_T^2/Q^2)\tilde{x}(1 - \tilde{x})F[F + \mathrm{Re}I(\tilde{x})], \tag{9}$$

where

$$F = \int_0^1 dz\, \frac{\phi(z, \tilde{Q}^2)}{z}\,, \tag{10}$$

$$I(\tilde{x}) = \int_0^1 dz\, \frac{\phi(z, \tilde{Q}^2)}{z(z + \tilde{x} - 1 + i\epsilon)}\,, \tag{11}$$

and

$$N(\tilde{x}, Q_T^2/Q^2) = 2\{(1 - \tilde{x})^2[(\mathrm{Im}I(\tilde{x}))^2 + (F + \mathrm{Re}I(\tilde{x}))^2] + 4(Q_T^2/Q^2)\tilde{x}^2F^2 + (Q_T^4/Q^4)\tilde{x}^2F^2\}. \tag{12}$$

The variable $\tilde{x}$ acts to resolve the distribution amplitude much like the Bjorken variable resolves the structure functions,

$$\tilde{x} \equiv \frac{x_{\bar{u}}}{1 + Q_T^2/Q^2} = \frac{1}{2}\frac{x_L + \sqrt{x_L^2 + 4s^{-1}(Q^2 + Q_T^2)}}{1 + Q_T^2/Q^2}. \tag{13}$$

The factors $1/z$ in Eqs. (10) and (11) come from the gluon propagators and the factors $1/(z + \tilde{x} - 1 \pm i\epsilon)$ arise from the quark propagator in Fig. 1(b).

In contrast to Refs. [9] and [14] we did not omit terms $O((1 - x_{\bar{u}})\, Q_T^2/Q^2)$ and $O((1 - x_{\bar{u}})^{-1}\, Q_T^4/Q^4)$ and of higher orders. The nucleon distribution function $G_{q/N}$ does not appear in (7)–(9); thus only the pion distribution amplitude $\phi(z, \tilde{Q}^2)$ has to be specified. Before doing so we give some technical comments on our calculation.

We note that the internal quark line of Fig. 1(b) can go on shell. The amplitude M of Eq. (4), however, is always regular due to the z integration [14] for realistic choices of $\phi(z, \tilde{Q}^2)$. This also can be read off from (11). The fact that the internal line goes on shell does not cause a Sudakov suppression since our diagrams are the lowest order contribution of an *inclusive* process. In other words, gluon emission to the final state will occur in the higher order corrections. Only when $x_{\bar{u}}$ approaches unity, where gluon emission is prohibited by kinematics, will the Sudakov suppression arise.

Our model and the parton model are not complementary, but rather different approximations to the Drell-Yan process. The diagrams in Figs. 1(a) and 1(b) give the *whole* leading order contribution in the specific kinematic region of large enough $x_{\bar{u}}$, $x_{\bar{u}} > 0.5$ [9]. This is so because the gluon exchange is the resolution of the pion bound state and not a radiative correction.

Now we can present our final results for λ, μ, and ν for different choices of the pion distribution amplitude $\phi(z, \tilde{Q}^2)$. We find in general that the values of μ and ν are very sensitive to the choice of $\phi(z, \tilde{Q}^2)$ which we always take to be positive, symmetric, i.e., $\phi(z, \tilde{Q}^2) = \phi(1 - z, \tilde{Q}^2)$, and normalized, $\int_0^1 dz\, \phi(z, \tilde{Q}^2) = 1$. Thus we will not restrict ourselves to the simplest case of $\phi(z) \sim \delta\left(z - \frac{1}{2}\right)$ considered in [9].

This sensitivity can be illustrated for the special case of $\tilde{x} = 0.5$ for which $\mathrm{Re}I = -2F$. From Eqs. (7)–(9) we get

$$\lambda(\tilde{x} = 0.5, Q_T^2/Q^2) = \frac{1 + 4\pi^2 a^2 - 4Q_T^2/Q^2 + Q_T^4/Q^4}{1 + 4\pi^2 a^2 + 4Q_T^2/Q^2 + Q_T^4/Q^4}, \tag{14}$$

$$\mu(\tilde{x} = 0.5, Q_T^2/Q^2) = \frac{2\sqrt{Q_T^2/Q^2}\,(1 - Q_T^2/Q^2)}{1 + 4\pi^2 a^2 + 4Q_T^2/Q^2 + Q_T^4/Q^4}, \tag{15}$$

$$\nu(\tilde{x} = 0.5, Q_T^2/Q^2) = \frac{4Q_T^2/Q^2}{1 + 4\pi^2 a^2 + 4Q_T^2/Q^2 + Q_T^4/Q^4}, \tag{16}$$

where $a \equiv \phi(z = 0.5)/F$.

From these formulas we see that μ and ν in this case are not suppressed only if a is a sufficiently small number. This is true for the so-called two-humped distribution amplitude [15] which has a dip around $z = 0.5$. On the other hand, the choice of a convex distribution amplitude, e.g., the asymptotic one, $\phi(z) = 6z(1 - z)$, will always produce suppressed μ and ν at $\tilde{x} = 0.5$.

We return now to our general results, Eqs. (7)–(9). In Fig. 2 we plot λ, μ, ν, and $2\nu - (1 - \lambda)$ versus $x_{\bar{u}}$ for $\sqrt{Q_T^2/Q^2} = 0.25$ for different choices of $\phi(z, \tilde{Q}^2)$ together with the data of Ref. [5]. For the two-humped distribution amplitude we have chosen the evolution parameter $\tilde{Q}^2$ to be effectively $\sim 4\ \mathrm{GeV}^2$. The solid line is the result for the two-humped $\phi(z)$ where powers of $(Q_T^2/Q^2)^{n/2}$ were dropped for $n \geq 3$ in Eqs. (7)–(9). We note that corrections to our model may induce such terms; thus the difference between the dashed and the solid lines should be viewed as the uncertainty of our predictions. We also show the data points of Ref. [5] averaged in the intervals $4.05 < \sqrt{Q^2} < 8.55$ GeV and $0 < \sqrt{Q_T^2} < 5$ GeV.

In Fig. 3 the same quantities are shown versus $\sqrt{Q_T^2}$ for $x_{\bar{u}} = 0.6$ and $\sqrt{Q^2} = 6$ GeV. The data points in this case are averaged over intervals $4.05 < \sqrt{Q^2} < 8.55$ GeV and $0.2 < x_{\bar{u}} < 1$ and are taken from Ref. [5]. All the data points were averaged over the intervals defined above in Ref. [5]. We would prefer to use the unaveraged data which are not available. The use of the averaged over $x_{\bar{u}}$ data in Fig. 3 forced us to fix the value $x_{\bar{u}} = 0.6$ for our

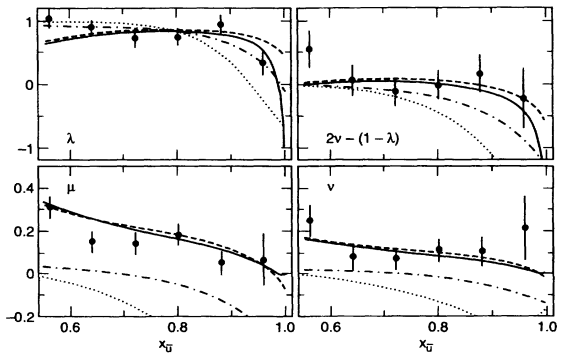

FIG. 2. The angular distribution coefficients λ, μ, and ν and the Lam-Tung combination, $2\nu - (1 - \lambda)$, in the Gottfried-Jackson frame, vs $x_{\bar{u}}$ for $\sqrt{Q_T^2/Q^2} = 0.25$. The dotted line corresponds to $\phi(z) = \delta(z - 1/2)$, the dash-dotted line corresponds to the asymptotic $\phi(z) = 6z(1 - z)$, and the dashed line shows the results for the two-humped distribution amplitude, $\phi(z) = 26z(1 - z)[1 - 50/13\, z(1 - z)]$. The solid line is the result for the two-humped $\phi(z)$ where powers of $(Q_T^2/Q^2)^{n/2}$ were dropped for $n \geq 3$ in Eqs. (13)–(15). The data points (averaged as explained in the text) are taken from Ref. [5].

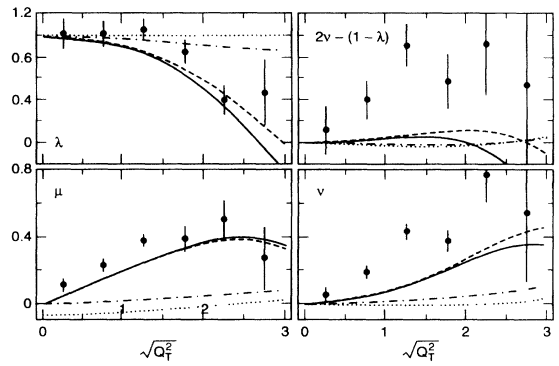

FIG. 3. The same quantities as in Fig. 2 are shown vs $\sqrt{Q_T^2}$ (in GeV) for $x_{\bar{u}} = 0.6$ and $\sqrt{Q^2} = 6$ GeV.

theoretical prediction which is rather low for our model and pushes it to the limits of its applicability.

Finally we would like to comment on the limitations of our model and corrections to it. The bound state effects considered here should have received a truly nonperturbative treatment. We have restricted ourselves to a perturbative approximation to the problem. This approximation makes sense only at large enough x which we have chosen to be > 0.5. The contribution of more than one hard gluon exchange will be suppressed by powers of α_s in this case. The contribution of soft gluons to the pion bound state is taken into account in the evolution of the distribution amplitude. The higher Fock states of the pion are expected to be suppressed when x is large enough [12]. The pion and the nucleon are not treated symmetrically in our model; namely, the nucleon bound state effects are not taken into account since in the kinematic region we consider, $x_{\bar{u}}$ is always large and x_u is always small. Gluon emission to the final state will first contribute to the evolution of the parton distribution functions. Hard gluon emission is suppressed by powers of α_s. No attempt of a systematic inclusion of higher order or mass effects was made.

The coefficient functions λ, μ, and ν are large $x > 0.5$ are very sensitive to the shape of the projectile's distribution amplitude $\phi(z, \tilde{Q}^2)$, the basic hadron wave function which describes the distribution of light-cone momentum fractions in the lowest-particle number valence Fock state. Measurements of meson form factors [12] and other exclusive and semiexclusive processes [16] at large momentum transfer can only provide global constraints on the shape of $\phi(z, \tilde{Q}^2)$; in contrast, the angular dependence of the lepton pair distributions can be used to provide local measurements of the shapes of these hadron wave functions. Detailed measurements of the angular distribution of leptons as a function of both x and Q_T for the reactions $Hp \rightarrow l + l^- X$ for the whole range of fixed target beams $H = \pi$, K, $\bar{p}$, p, and n will open up a new window on the structure of hadrons at the amplitude level.

Our analysis shows that the broad, two-humped distribution amplitude for the pion which was obtained within the context of QCD sum rules [15] can account for the main features of the data. In contrast, narrow momentum distributions, characteristic of weak hadronic binding, predict the wrong sign for the observed azimuthal angular coefficients μ and ν.

We wish to thank V. Telegdi for conversations regarding the experimental data, particularly the fact that the observed increase of the variable ν with the dimuon transverse momentum is at variance with perturbative QCD calculations. We are grateful to Yu. L. Dokshitzer, O. Nachtmann, and W. J. Stirling for useful suggestions and comments on the manuscript. We also benefited from discussions with H. Anlauf, V. M. Braun, T. Hyer, V. A. Khoze, and M. Peskin. This work was supported by the Department of Energy, Contract No. DE-AC03-76SF00515 and one of us (D. M.) is supported by Deutscher Akademischer Austauschdienst.

[1] K. Freudenreich, Int. J. Mod. Phys. A **5**, 3643 (1990); Yu. L. Dokshitzer, D. I. Dyakonov, and S. I. Troyan, Phys. Rep. **58**, 269 (1980); *Proceedings of the Workshop on Drell-Yan Processes, Fermilab, Batavia, 1982*, edited by E. Berger *et al.* (Fermi National Accelerator Laboratory, Batavia, 1983).
[2] C. S. Lam and W. K. Tung, Phys. Rev. D **21**, 2712 (1980).
[3] P. Chiappetta and M. Le Bellac, Z. Phys. C **32**, 521 (1986).
[4] NA10 Collaboration, S. Falciano *et al.*, Z. Phys. C **31**, 513 (1986); NA10 Collaboration, M. Guanziroli *et al.*, Z. Phys. C **37**, 545 (1988).
[5] J. S. Conway *et al.*, Phys. Rev. D **39**, 92 (1989).
[6] J. G. Heinrich *et al.*, Phys. Rev. D **44**, 44 (1991).
[7] A. Brandenburg, O. Nachtmann, and E. Mirkes, Z. Phys. C **60**, 697 (1993).
[8] E. L. Berger and S. J. Brodsky, Phys. Rev. Lett. **42**, 940 (1979); S. J. Brodsky, E. L. Berger, and G. P. Lepage, in *Proceedings of the Workshop on Drell-Yan Processes, Fermilab, Batavia, 1982* (Ref. [1]), p. 187.
[9] E. L. Berger, Z. Phys. C **4**, 289 (1980).
[10] S. D. Drell and T. M. Yan, Phys. Rev. Lett. **25**, 316 (1970).
[11] G. Altarelli, R. K. Ellis, M. Greco, and G. Martinelli, Nucl. Phys. **B246**, 12 (1984).
[12] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).
[13] S. J. Brodsky, P. Hoyer, A. H. Mueller, and W. K. Tang, Nucl. Phys. **B369**, 519 (1992); K. Eshola, P. Hoyer, M. Vanttinen, and R. Vogt (to be published).
[14] S. Matsuda, Phys. Lett. **119B**, 207 (1982).
[15] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
[16] A. V. Efremov and A. V. Radyushkin, Phys. Lett. **94B**, 245 (1980); M. K. Chase, Nucl. Phys. **B167**, 125 (1980); S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 1808 (1981). E. Braaten, Phys. Rev. D **28**, 524 (1983); E. Maina and G. Farrar, Phys. Lett. B **206**, 120 (1988); T. Hyer, Phys. Rev. D **48**, 147 (1993).