Impurities and Quasi-One-Dimensional Transport in a *d*-Wave Superconductor

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Impurity scattering in the unitary limit produces low-energy quasiparticles with an anisotropic spectrum in a two-dimensional d-wave superconductor. We describe a new *quasi-one-dimensional* limit of the quasiparticle scattering, which might occur in a superconductor with short coherence length and with *finite* impurity potential range. The dc conductivity in a d-wave superconductor is predicted to be proportional to the normal state scattering time and is impurity *dependent*. The *quasi-one-dimensional* regime will occur in high-T_c superconductors above critical impurity concentration. We argue that the impurities produce weak *orthogonal* localization of the quasiparticles.

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The symmetry of the pairing state in high-T_c superconductors was addressed in some recent experiments on the T^3 dependence of NMR [1], the phase shift by π in the flux dependence of the Josephson current [2], the linear temperature dependence of the penetration depth $\lambda(T)$ in single crystals [3], and the strong anisotropy of the energy gap in angular resolved photoemission [4]. All of them support $d_{x^2-y^2}$ symmetry of the gap. Theoretically the d-wave pairing state is predicted in spin-fluctuation exchange models [5,6] as well as in models with strong correlations [7].

It is well known that scalar impurities are pair breakers in d-wave and any other nontrivial pairing state superconductor [8–10]. They produce a finite lifetime of the quasiparticles in the nodes of the gap, a finite density of states at low energy, and a *finite* low frequency conductivity at low temperatures, ignoring localization effects. For the special case of a two-dimensional superconductor with a d-wave gap, a straightforward calculation yields the surprising result that the dc conductivity $\sigma(\omega \longrightarrow 0)$ is a "universal" number [10], *independent* of the lifetime of the quasiparticle (but dependent on the anisotropy ratio of the velocities of the quasiparticle in the node of the gap) [11]. However, experiments on microwave absorption in Y-Ba-Cu-O (YBCO) crystals with Zn impurities [12] show a *linear* temperature dependence of the conductivity and an *impurity-dependent* low-temperature conductivity. On the other hand, the measured conductivity [12] appears to be sample dependent and is about an order of magnitude higher than the predicted value [10]. One has to wait for settled experimental results to compare them with theoretical predictions [13].

The purpose of this Letter is to address *the role of strongly scattering disorder with finite potential range on the dc conductivity* at low temperatures in a short coherence length superconductor. (i) We will show that there is a new *quasi-one-dimensional regime* for the dc conductivity in superconductors with a short coherence length ξ, comparable to the range of the impurity potential λ. The quasiparticle contribution to the dc conductivity is governed by the self-energy $\Sigma(\omega \longrightarrow 0) = -i\gamma$ and by the phase space available for low-energy quasiparticles. The quasiparticle dispersion is strongly anisotropic in the vicinity of the nodes in a d-wave superconductor that has $E_k = \sqrt{v_1^2 k_1^2 + v_F^2 k_3^2}$ and $v_1/v_F \sim \Delta_0/\epsilon_F$ (see Fig. 1). We find that the overall contribution to the conductivity depends on the ratio of the energy of the quasiparticle to the scattering rate $v_1 \lambda^{-1}/\gamma = \Delta_0/\gamma(p_F\lambda)^{-1}$, and we get

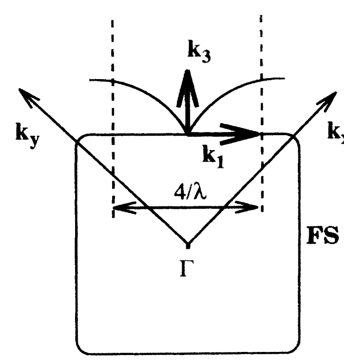

FIG. 1. Graphical presentation of the $d_{x^2-y^2}$ state, where $\Delta(\boldsymbol{k}) = \Delta_0(\cos k_x a - \cos k_y a)$. For clarity the gap function is only drawn in the neighborhood of one node $\boldsymbol{k}_0$. For calculational purposes a coordinate system $(k_1, k_3)$with the origin at $\boldsymbol{k}_0$ is used instead of (k_x, k_y) with the origin at the center of the Brillouin zone Γ. They are related by $k_1 = (k_x - k_y)/\sqrt{2}$, $k_3 = (k_x + k_y)/\sqrt{2} - |\boldsymbol{k}|0$. The FS denotes the Fermi surface of the tight-binding band $\xi(\boldsymbol{k}) = -t(\cos k_x a + \cos k_y a) - \mu$. After linearization around $\boldsymbol{k}_0$ we find $\Delta(\boldsymbol{k}) = v_1 k_1$, $\xi(\boldsymbol{k}) = v_F k_3$, where $v_1 = -\sqrt{2}\Delta_0 a \sin(k_{0x}a)$ and $v_F = -\sqrt{2} ta \sin(k_{0x}a)$. The momentum sums performed will be cut off at $|\boldsymbol{k}| = 2/\lambda$, where λ is the range of the impurity potential.

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at $T = 0$

$$\sigma(\omega \longrightarrow 0) = \frac{e^2}{2\pi\hbar}\frac{2}{\pi^2}\frac{v_F}{v_1}\left[1 + \left(\frac{\gamma}{2v_1\lambda^{-1}}\right)^2\right]^{-1/2}. \quad (1)$$

For $v_1\lambda^{-1}/\gamma \lesssim 1$ the quasiparticle dynamics is essentially *quasi-one-dimensional* and the conductivity *depends* on the impurity concentration. Our model predicts that the dc conductivity at low temperature should be *proportional to the scattering time in the normal state* and is *smaller* than the "universal" limit [10] by a factor $v_1\lambda^{-1}/\gamma \lesssim 1$. This limit might occur in high-T_c superconductors, for which we estimate $\lambda/a \sim 1-3$ and $\Delta_0/\epsilon_F \sim 10^{-1}$. In the limit $\lambda \longrightarrow 0$ Eq. (1) gives the universal dc conductivity, found in [10]. (ii) We argue that the origin of the strong potential scattering due to impurities in the high-T_c superconductors is the highly correlated antiferromagnetic nature of the normal state. The range of the impurity potential might be of the order of ξ_{AFM} and thus comparable to the superconducting coherence length ξ. Under these assumptions retaining a *finite* range of the impurity potential is required. (iii) We also discuss the localization of quasiparticles close to the nodes in a d-wave superconductor with scalar impurities. Scalar disorder leads to a weak *orthogonal* localization of quasiparticle states [10,11]. All of the results, presented here, are valid for *any* superconductor with nodes in 2D with a Dirac spectrum of quasiparticles.

Consider scalar impurities that give rise to the randomly distributed strong scatterers in 2D with a *finite* range λ: $\langle U(\boldsymbol{r})U(0)\rangle = (\eta/\lambda^2\pi)\exp(-r^2/\lambda^2) \xrightarrow{\lambda\to 0} \eta\delta(\boldsymbol{r})$ and dispersion η. The assumption of strong potential scattering off the impurity sites is well accepted for heavy fermion systems, where the Kondo effect plays an important role and thus any scalar impurity might produce the "Kondo hole" with s-wave scattering phase shift δ_0 close to $\pi/2$. The same assumption for the high-T_c superconductors seems to be justified by the experiments on Zn impurities in YBCO, which produce gapless superconductivity at 3% doping level and strongly change the NMR linewidth of ^{63}Cu [14]. A possible model describing the Zn impurities in high-T_c was proposed recently [15]. We therefore assume that scalar impurities are strong scatterers in these superconductors.

The second assumption of a *finite* range of the impurity potential is motivated by the observation that the high-T_c superconductors have a substantial antiferromagnetic coherence length $\xi_{\text{AFM}} \sim 3a$ at the transition temperature. Thus a scalar impurity *will* produce distortions in the magnetic correlations on the range of the ξ_{AFM}. On the other hand, the superconducting coherence length $\xi \sim 20$ Å is comparable to this scale, and thus the range of the potential is finite on the scale relevant for superconductivity. This point should be contrasted to the case of heavy-fermion superconductors, where the coherence length is $\sim 10^2$ Å, and, therefore, any potential impurity will have its range substantially shorter than the coherence length. In that case it is reasonable to use instead the assumption that the impurity has effectively *zero* range. We will retain λ finite below. As we will show, this leads to the new parameter $v_1\lambda^{-1}/\gamma$ and a dc conductivity that is dependent on this parameter.

The Bogoliubov Hamiltonian for quasiparticles in a d-wave 2D superconductor is

$$H = \int d\boldsymbol{r}\,\Psi^\dagger(\boldsymbol{r})[\xi(\boldsymbol{r})\tau_3 + \Delta(\boldsymbol{r})\tau_1 + U(\boldsymbol{r})\tau_3]\Psi(\boldsymbol{r}), \quad (2)$$

where $\Psi = (c_\uparrow, c_\downarrow^\dagger)$ is the Nambu spinor, τ_i are the Pauli matrices, $\xi(\boldsymbol{k}) = -t(\cos k_x a + \cos k_y a) - \mu$ is the energy, counted from the Fermi surface, $\Delta(\boldsymbol{k}) = \Delta_0(\cos k_x a - \cos k_y a)$ is the $d_{x^2-y^2}$ energy gap, and $U(\boldsymbol{r})$ is the impurity potential. For the low-energy states, we linearize the Hamiltonian in the vicinity of the node close to the $(\pi/2, \pi/2)$ point. We find $\Delta(\boldsymbol{k}) = v_1 k_1, \xi(\boldsymbol{k}) = v_F k_3$ in the new coordinates, defined in Fig. 1 [10]. The resulting Dirac-like Hamiltonian takes the form

$$H = \int d\boldsymbol{r}\,\Psi^\dagger[v_F\hat{k}_3 + U(\boldsymbol{r})]\tau_3 + v_1\hat{k}_1\tau_1\Psi. \quad (3)$$

The self-consistent Green functions are given by

$$G = [i\tilde{\omega}_n + \xi(\boldsymbol{k})]/D, \quad F = -\Delta(\boldsymbol{k})/D, \quad (4)$$

with $D = \tilde{\omega}_n^2 + \xi^2(k) + \Delta^2(k)$, $i\tilde{\omega}_n = i\omega_n - \Sigma(i\omega_n)$, and we ignore the self-energy contribution to the anomalous Green function from impurity scattering, a contribution that vanishes upon angular integration [16]. For strong potential scattering impurities the normal self-energy is given by [17,18]

$$\Sigma(i\omega_n) = \Gamma g_0(i\omega_n)/[c^2 - g_0^2(i\omega_n)], \quad (5)$$

where $\Gamma = n_i/\pi N_0$ is the scattering rate in the normal phase, $c = \cot\delta_0$, with momentum-dependent phase shift $\delta_0(\mathbf{q}) = \pi/2$, $|\mathbf{q}| < 2\lambda^{-1}$, $g_0(i\omega_n) = 4(\pi N_0)^{-1} \times \sum_k' G(\boldsymbol{k}, i\omega_n)$. The factor of 4 in front of the last sum reflects the number of nodes in the gap, n_i is the impurity concentration, and N_0 is the density of states at the Fermi surface. The prime in $\sum'$ stands for the momentum sum up to the cutoff $|\boldsymbol{k}| < 2\lambda^{-1}$, which is implemented as a "hard" cutoff. This momentum cutoff follows immediately from the derivation of the self-energy with a finite range of the potential; see, for example, [18]. We find from the solution of the Dyson equation that $\Sigma(i\omega_n)$ is also momentum dependent with characteristic range λ^{-1}, which will be taken into account in the conductivity calculation.

The Born scattering limit is recovered from Eq. (5) for $c^2 \gg g_0(i\omega_n)$. We are interested in the case of *unitary* scattering, for which we take the s-wave phase shift

$\delta_0 = \pi/2$, $c = 0$. In this limit the solution of the Eq. (5) for $\Sigma(\omega \to 0) = -i\gamma$ is

$$\gamma^2 = \frac{\Gamma \pi N_0}{4} \left[\sum_k{}' (\gamma^2 + (v_1 k_1)^2 + (v_F k_3)^2)^{-1} \right]^{-1}. \quad (6)$$

The sum over momentum in this equation is logarithmically divergent at the upper limit, which is taken as $\min(\lambda^{-1}, \Delta_0/v_1)$ for the k_1 integral; the k_3 integral is always cut off by Δ_0/v_F, since $v_F \lambda^{-1} \sim \epsilon_F \gg \gamma, \Delta_0$. We find for $I = \sum_k{}' [\gamma^2 + (v_1 k_1)^2 + (v_F k_3)^2]^{-1}$

$$I = \ln\left(b + \sqrt{1 + b^2}\right)/2\pi v_1 v_F, \quad (7)$$

where $b = \min(2 v_1 \lambda^{-1}, \Delta_0)/\gamma$. I has two asymptotics:

(1) $I = (1/2\pi v_1 v_F)[\ln(\Delta_0/\gamma) + O(\gamma/\Delta_0)]$, for $v_1 \lambda^{-1}/\gamma \gg 1$. This limit corresponds to isotropic strong scattering with no momentum cutoff as $\lambda^{-1} \longrightarrow \infty$. It has been investigated previously [10,18,19]. The impurity self-energy is shown to be $\tilde{\gamma} = \Delta_0 \sqrt{\pi \Gamma / \Delta_0 \ln(\Delta_0/\Gamma)}$. We will not further discuss this case.

(2) $I = \lambda^{-1}/(\pi v_F)\gamma + (1/2\pi v_1 v_F) O((2 v_1 \lambda^{-1}/\gamma)^3)$, for $v_1 \lambda^{-1}/\gamma \lesssim 1$. In this case the lifetime is given by

$$\gamma = \frac{\Gamma \pi}{8} p_F \lambda. \quad (8)$$

To check the self-consistency of the assumption $v_1 \lambda^{-1}/\gamma \lesssim 1$ we use Eq. (8) to get $\Gamma (p_F \lambda)^2 \geq 8/\pi \Delta_0 \sim 8 T_c$. For an estimated $\lambda \sim 2a$ the condition for the *quasi-one-dimensional* regime of quasiparticle scattering is

$$\Gamma \geq 8 \times 10^{-2} \Delta_0 \sim 2.5 \times 10^{-1} T_c \simeq 20 \text{ K}. \quad (9)$$

This estimate is to be compared to the typical scattering rate in clean samples of YBCO $\Gamma/T_c \sim 5 \times 10^{-2}$. Furthermore, in the impure samples with *quadratic* temperature dependence of the penetration depth the estimates are $\Gamma/T_c \sim 0.1-1$ [20]. At low temperatures $T < \gamma$ with $\gamma(p_F \lambda) \geq \Delta_0 \geq \gamma$ the superconductor is in the *quasi-one-dimensional* regime and remains relatively clean for, say, $p_F \lambda \sim 6$.

The applicability of the *quasi-one-dimensional* vs the *isotropic* regime is governed by the ratio of the respective scattering rates $\gamma/\tilde{\gamma} \sim \sqrt{\Gamma/\Delta_0}\, p_F \lambda \simeq n_{\text{imp}}^{1/2}$. We conclude that the pure and the lightly doped d-wave superconductor with small n_{imp} is always in the *isotropic* scattering regime. The concentration of impurities required to produce *quasi-one-dimensional* scattering depends on Δ_0 and the range of the potential λ: for Zn doped YBCO $n_{\text{imp}} \sim 5\%$ and for impure $La_{1.86}Sr_{0.14}Cu_2O_4$ $n_{\text{imp}} \sim 0.16\%$ are required [21].

To explain this effective *change of dimensionality* we note that the transverse momentum in the sum in case (2) is limited by $k_1 < 2/\lambda$ and that the quasiparticle dispersion on such a small scale is irrelevant, compared to γ. The transverse scattering does not contribute effectively to the conductivity; that is why we call this case a *quasi-one-dimensional* limit. The existence of this limit is the result of the *finite* impurity range λ [22]. In this limit the scattering rate in the superconducting state is of the same order as the normal state scattering rate $\gamma \sim 2\Gamma \sim 40$ K for $p_F \lambda \sim 6$. The finite density of states $N(\omega \to 0)/N_0 = \Gamma/\Delta_0 \sim n_{\text{imp}}$, *linear* in the impurity concentration, is generated in case (2) as well.

We now turn to quasiparticle conductivity. We shall use the lowest order bubble diagram with self-consistent Green functions with *no vertex corrections* [10]. Taking into account the vertex corrections, which are small for dominant s-wave scattering, will change the numerical factor in the expression for conductivity. However, our main result, i.e., the functional dependence on the scattering rate and impurity concentration, will remain unchanged. For dc conductivity we get

$$\sigma(\omega \to 0) = \frac{e^2}{\hbar} \frac{4 v_F^2}{\pi^2} \sum_k{}' \int d\epsilon \, [-\partial_\epsilon n(\epsilon)] \times \left[|G''(\boldsymbol{k}, \epsilon)|^2 + |F''(\boldsymbol{k}, \epsilon)|^2 \right], \quad (10)$$

where, linearizing the quasiparticle spectrum in the vicinity of the nodes (see Fig. 1), $G''(\boldsymbol{k}, \omega = 0) = \gamma/[\gamma^2 + (v_1 k_1)^2 + (v_F k_3)^2]$, $F''(\boldsymbol{k}, \omega = 0) = 0$. The momentum integral in Eq. (10) yields the final formula Eq. (1) for $T = 0$ with $O(T^2)$ corrections. This formula holds for *any* model of disorder as long as $v_1 \lambda^{-1}/\gamma \lesssim 1$.

For the particular case of disorder considered in Eq. (8), the conductivity is

$$\sigma(\omega \to 0) = \frac{e^2}{\pi \hbar} \frac{16}{\pi^3} \frac{\hbar}{m \lambda^2 \Gamma}. \quad (11)$$

It is smaller than the normal state conductivity $\sigma(\omega \to 0)_{\text{normal}} = (e^2/\pi\hbar)\epsilon_F/\Gamma$ due to the small factor $\hbar/m\lambda^2\epsilon_F \lesssim 1$ with $\lambda > a$. The conductivity is also impurity *dependent*, $\sigma \sim \Gamma^{-1} \sim n_{\text{imp}}^{-1}$. This model predicts that the dc conductivity at low temperatures should be *inversely proportional to the scattering rate in the normal state and to the impurity concentration* [23].

Within this model the temperature dependence of $\sigma \sim T$ at low temperatures remains unexplained, since we obtain T^2 corrections at low enough temperature (see also [19]). If it is an intrinsic effect, it indicates an important physical effect, missing in this simple model.

We shall comment on quasiparticle localization in the d-wave superconductor (see also [10]). Originally the problem of weak localization of quasiparticles was considered for s-wave superconductivity [24].

Here we will show that the linearized Dirac-Bogoliubov equation in the *presence* of the impurity potential preserves time reversal ($\hat{T}$) symmetry, and thus the weak localization in this state belongs to the *orthogonal* universality class [10,11]. The linearized Bogoliubov equation (3) takes the form of a Dirac equation with the scalar impurity potential $U(\boldsymbol{r})$ playing the role of the gauge potential. Although it appears that this Hamiltonian violates $\hat{T}$, this is not the case. Time reversal for electron operators is defined as $\hat{T} c_\alpha = \epsilon_{\alpha\beta} c_\beta^\dagger$, α, $\beta = \uparrow\downarrow$, where $\epsilon_{\alpha\beta}$ is the antisymmetric tensor. Straightforward calculation yields for time inversion of the Nambu spinor $\Psi = (c_\uparrow, c_\downarrow^\dagger)$

$$\hat{T}\Psi = i\tau_2\Psi, \quad \hat{T}\Psi^\dagger = \Psi^\dagger(-i)\tau_2. \tag{12}$$

The Dirac-Bogoliubov Hamiltonian transforms under $\hat{T}$ as follows: $H = \Psi^\dagger\{[v_F\hat{k}_3 + U(\boldsymbol{r})]\tau_3 + v_1\hat{k}_1\tau_1\} \times \Psi \xrightarrow{\hat{T}} \Psi^\dagger(-i\tau_2)\{[v_F\hat{k}_3 + U(\boldsymbol{r})]\tau_3 + v_1\hat{k}_1\tau_1\}(i\tau_2)\Psi$, and

$$H \xrightarrow{\hat{T}} (-1)H. \tag{13}$$

Thus the Dirac-Bogoliubov Hamiltonian transforms under time reversal exactly as the energy, which changes sign under $\hat{T}$, and thus time reversal symmetry is preserved. This fact also follows from the observation that the original Bogoliubov Hamiltonian equation (2) for a d-wave superconductor preserves time reversal in the presence of scalar impurities. After linearization this symmetry should be preserved as well. Thus quasiparticle localization in the impure d-wave superconductor belongs to the *orthogonal* class of universality. Detailed analysis of this problem will be given elsewhere.

In conclusion, we find a new, *quasi-one-dimensional*, regime in the superconducting scattering rate γ and in the conductivity σ for strong impurity scattering. The highly anisotropic dispersion of the quasiparticle spectrum $v_1/v_F \sim \Delta_0/\epsilon_F$ and finite range of the impurity potential are *crucial* for this effect to take place. We argue that the finite range λ of the impurity potential is the result of the strong antiferromagnetic correlations $\xi_{\mathrm{AFM}} \sim 3a$ in the normal phase. This effect might occur in *any* superconductors with a linear quasiparticle spectrum in the nodes of the gap. We considered $d_{x^2-y^2}$ symmetry of the gap and find that for a reasonable scattering rate in the normal phase $\Gamma/T_c > 0.2$, the *quasi-one-dimensional* regime should occur at some critical impurity concentration where conductivity at low temperatures $\sigma \sim \Gamma^{-1} \sim n_{\mathrm{imp}}^{-1}$ is impurity *dependent* and is inversely proportional to the normal state scattering rate. We also show that even in the presence of scalar impurities, time reversal symmetry in the d-wave superconductor is preserved and argue that it leads to weak quasiparticle localization in the *orthogonal* universality class.

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[22] The fact that impurity scattering is *unitary* is crucial for this *quasi-one-dimensional* limit to occur. Indeed a simple estimate shows that for the *Born* limit with weak scattering $v_1\lambda^{-1}/\gamma \gg 1$.

[23] We consider here only elastic impurity scattering at low temperatures $T \ll T_c$. Inelastic spin fluctuations scattering, dominant in the normal state, is suppressed due to the superconducting gap. As a result of this suppression, the conductivity increases below the transition.

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