Correlation Dimension and Largest Lyapunov Exponent for Broadband Edge Turbulence in the Compact Helical System

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The correlation dimension has been investigated in detail for broadband edge density fluctuations observed in the compact helical system. Fractal dimensions of 5–9 have been obtained, depending on plasmas produced, and hence, quantitatively characterize the attractor of the turbulence. The largest Lyapunov exponent has been also calculated, indicating that the observed chaos is deterministic and the attractor is strange.

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There has been considerable interest in edge plasmas of toroidal devices. It is especially of crucial importance, related to the anomalous radial transport, to investigate the properties of the broadband turbulence whose peak in amplitude is located in the plasma edge [1–3]. However, the turbulence properties are not clearly understood yet and are still open to studies of their underlying physics.

The broadband frequency spectrum is also well known to be characteristic of chaos, observed in a nonlinear physical system that usually has a small number of degrees of freedom [4–6]. Thus one important question is whether the broadband turbulence observed in the edge plasma is chaotic or not [7]; i.e., can the dynamical system be modeled by nonlinear equations with only a few variables [8]? To our knowledge, however, this problem is not clearly solved yet. There are no clear-cut publications to demonstrate that the attractor of the broadband edge turbulence is strange. The largest Lyapunov exponent, for example, has not been obtained at all, which is the quantity used to characterize the average divergence of neighboring trajectories on the attractor, and hence, the most important quantity to distinguish between chaos and white noise [4]. A correlation dimension is the only quantity that has been calculated for the edge turbulence, and besides, the results are not sufficient to solve the problem [7,9,10]; the dimension for the turbulence with specific values of wave number has been calculated in the TFR tokamak, but the dimension for the total turbulence has not been estimated [7], and it has been shown in the TEXTOR tokamak that the broader the frequency spectrum is, not surprisingly, the larger the dimension is [10]. In this paper, we study the correlation dimension and largest Lyapunov exponent of *broadband total* turbulence measured in the compact helical system (CHS) edge plasma, to understand better the properties of edge plasma turbulence from the chaotic point of view.

The most used algorithm of calculating the correlation dimension ν was proposed by Grassberger and Procaccia [11]. This algorithm requires the reconstruction of a trajectory in a p-dimensional space, which is done by taking as coordinates $I(t), I(+\tau), I(t+2\tau), \ldots, I[t_i + (p-1)\tau]$, where τ is an appropriate delay time [12,13]. In our experiments the time t is discretized, so that we obtain a series of p-dimensional vector $\mathbf{r}_i$ representing the phase portrait of the attractor:

$$\mathbf{r}_i = \{I(t_i), I(t_i+\tau), I(t_i+2\tau), \ldots, I[t_i+(p-1)\tau]\}, \quad i = 1,2,3,\ldots,m. \tag{1}$$

For proper reconstruction of the attractor, τ must be chosen not too small or not too large [11]. Denoting τ_{aut} as the autocorrelation time for $I(t)$, a value of τ should therefore be chosen such that $\tau = \tau_{\text{aut}}$ [9]. In our analysis, τ is in the range $\gtrsim 10\ \mu$sec. With the series of vectors $\mathbf{r}_i$, one can evaluate the correlation sum $C(r)$, defined by

$$C(r) = \lim_{m\to\infty} \frac{1}{m^2} \sum_{i,j=1}^{m} H(r - |r_i - r_j|), \tag{2}$$

where H is the Heaviside function defined by $H(r) = 1$ for positive r, and 0 otherwise. For an intermediate region of r, $C(r)$ will scale like $C(r) \propto r^{\nu}$. Thus, for each value of ν, we calculate $C(r)$ and determine the slope of the function g defined by $\log C(r) = g(\log r)$, arriving at an exponent ν. A more quantitative characterization of chaos comes from determination of a Lyapunov exponent; finding a positive Lyapunov exponent is an unambiguous signature of chaotic behavior. In this Letter, the Lyapunov exponent is calculated using the algorithm developed by Sato, Sano and Sawada [14], which is easy to implement and has less limitation even for high dimensional attractors.

We have tested our implementation of the algorithm by analyzing time series from mathematical systems with known attractors, and have obtained the already tabulated values for the correlation dimension and largest Lyapunov exponent. The algorithm also has been applied to mathematical and experimental data from an ion-sheath system [5,6,15]. Our use of 16 kilobytes data points has allowed us to measure the structure for correlation dimensions up to ~ 12.

Experiments have been performed on the CHS, which is a heliotron/torsatron device characterized by a low

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aspect ratio of 5 [16]. Various plasmas, e.g., neutral-beam injection (NBI), ion cyclotron range of frequency wave (hereafter referred to as RF), and NBI plus RF plasmas were produced for this investigation with magnetic fields of 0.95–1.4 T, and magnetic-axis positions of 91.1–94.9 cm. The port-through NBI power is 850 kW, and the RF power is 120–200 kW at 22 MHz. The line averaged electron density $\bar{n}_e$ was $(0.7-3) \times 10^{13}$ cm^{-3}, and the electron temperature at the plasma center $T_e(0)$ was 0.16–0.35 keV. The duration of discharge was 70–130 msec. All data shown here were taken in the flattop region of the plasma discharge.

A thermal neutral lithium beam of 1 cm diameter with an eight-channel optical detection system, which measures the rates of local photon emission $I(t)$ for the lithium resonance line (670.8 nm) due to electron impact excitation, was used to measure electron density fluctuations in edge plasmas, as shown in Fig. 1 [1,17]. Radial electron density profiles also can be obtained with this detection system [1,17]. The spatial resolution of each channel was ~5 mm and the distance between adjacent channels was 6–12 mm. The electron density fluctuations given below were measured between the outermost magnetic surface and the radial position 2 cm away from it towards the chamber wall. The signals were digitized at 2 MHz with a transient recorder, and 16 kilobytes of eight-bit data, which was the data length for analysis, were recorded for two channels and each shot. The high-frequency response of the signals was well above 2 MHz.

A typical wave form of electron density fluctuations observed in the RF plasma and a corresponding power spectrum S are shown in Figs. 2(a) and 2(b), respectively. The duration of record is 8.192 msec, about a quarter of which is shown. The power spectrum is plotted on logarithmic vertical and horizontal scales. In these figures, nonperiodicity and broadness can be observed. The power spectrum shows the occurrence of $(1/f)$-type noise in the frequency range of $20 < f < 250$ kHz, and the exponent of the power law is found to be ~ -2 in this frequency range from the log-log plots represented in Fig. 2(b). These features of the edge fluctuations are almost the same as observed in other toroidal devices [1–3].

Figure 3(a) shows log-log plots of $C(r)$ for the time series measured in the NBI plus RF plasma. The slopes of $C(r)$ on a log-log plot begin to saturate when p exceeds

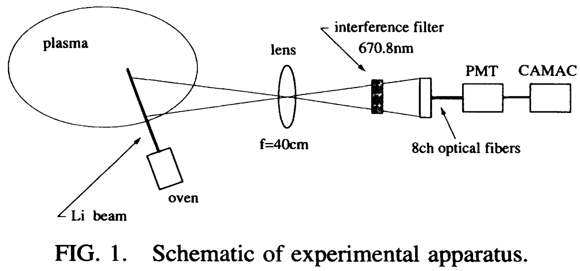

FIG. 1. Schematic of experimental apparatus.

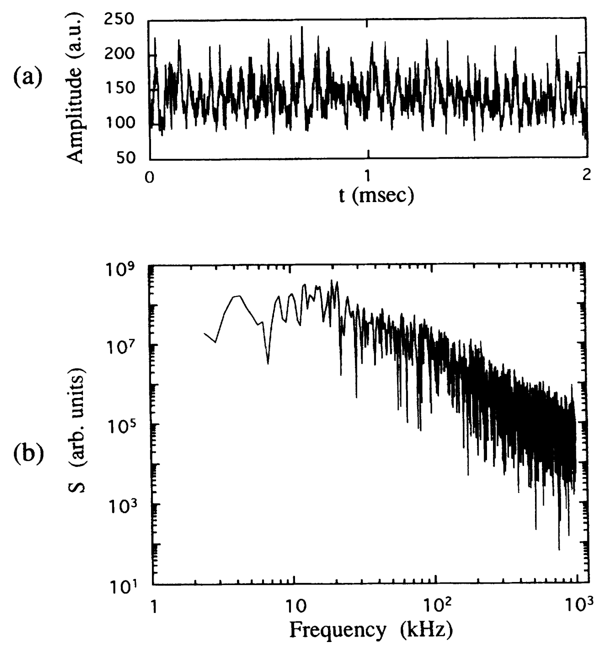

FIG. 2. (a) Real-time signal of electron density fluctuation I. (b) Power spectrum S of I.

8. The saturation of ν with an increase in p is illustrated by Fig. 3(b), representing that the correlation dimension is ~8.35. In this way, ν's are obtained to be 6.21 ± 0.10 for the NBI plasma, 6.05 ± 0.30 for the RF plasma, and 8.36 ± 0.15 for the NBI plus RF plasma, respectively. These fractal dimensions suggest that the attractor of the observed fluctuations is strange, and that the system has a relatively small number of degrees of freedom. Little difference in ν, within ± 0.3, is observed in the RF plasma when the RF power varies from 120 to 200 kW. It should be also noted that the plasma heated by two kinds of heating sources yields the larger ν than that heated by one kind, and this is acceptable since the turbulence results from a more complicated system might have a larger number of degrees of freedom. In a Navier-Stokes fluid at a high Reynolds number, the dimension of the attractor is reported to grow faster than a power (e.g., 4/3 [18]) of the Reynolds number. In plasmas, however, we have not identified parameters which control the correlation dimension of the edge turbulence, and hence, need a further investigation.

A temporal autocorrelation function C_{aut} of $I(t)$ is shown in Fig. 4(a). In this figure, C_{aut} tends to zero as the delay time increases. This suggests that the resemblance of the signal with itself in time diminishes, and even disappears for times that are sufficiently far apart. Therefore, there is a possibility that the behavior of the edge fluctuations is chaotic, since a chaotic regime is intrinsically unpredictable, by progressive loss of self-similarity, and it follows that no finite interval of observation of the signal suffices to predict its future behavior [4]. The data

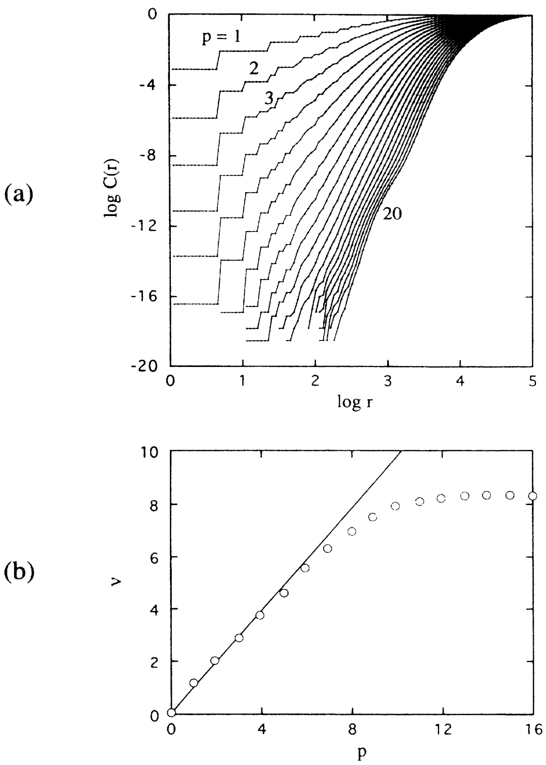

FIG. 3. (a) Log-log plot of $C(r)$ for the strange attractor. (b) Variation of ν as a function of the dimension p. The solid straight line shown in (b) is for random noise.

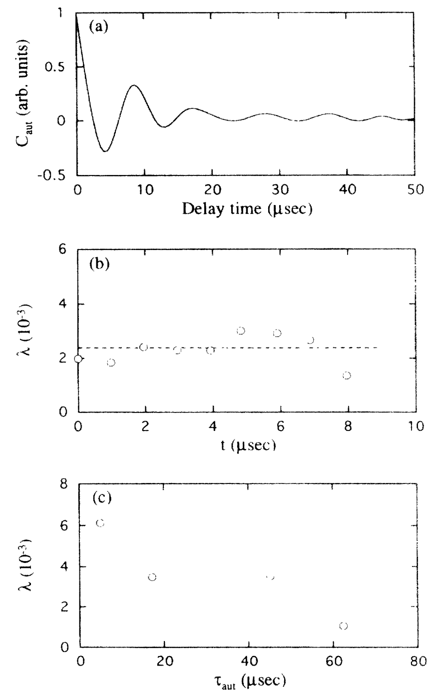

FIG. 4. (a) Autocorrelation function of I. (b) The largest Lyapunov exponent for I with $p = 9$. (c) Relation between λ and τ_{aut}.

analyzed here are those measured in the RF plasma that gives the lowest ν among our three types of plasmas. The obtained values of τ_{aut} are found to differ shot by shot, although the experiment was performed under the same conditions. Other parameters such as power spectrum, ν, and so on are almost the same, respectively, independently of the shot. The reason for the wide range of τ_{aut} is not clear at this stage, but it is considered that a set of imperceptibly different initial states of the plasma leads, in an unpredictable way, to many final states, that is, to different τ_{aut}'s. This property is a kind of sensitivity to initial conditions in a chaotic regime [4].

The Lyapunov exponent is calculated for the time series measured in the RF plasma. Figure 4(b) shows an example of calculation of the Lyapunov exponent, and we can find a plateau that gives an estimation of the largest Lyapunov exponent λ [14]. Apparently, the value of λ is positive, indicating that the distance of neighboring trajectories on the attractor grows exponentially with time. Figure 4(c) shows that there is an interesting relation between λ and τ_{aut}; λ is a decreasing function of τ_{aut}. This result is understandable since, in a chaotic regime, two trajectories that are initially very close will diverge, resulting in loss of all resemblance after a finite time, as mentioned before, that is, the faster the two trajectories diverge due to the large λ, the shorter τ_{aut} becomes. The largest Lyapunov exponent is found ~ 0 for the time series, whose τ_{aut} is $\gtrsim 150$ μsec.

In summary, we have studied a correlation dimension and the largest Lyapunov exponent for broadband electron density fluctuations in the CHS edge plasma. It is concluded that the broadband turbulence is essentially determined by relatively small number of variables, which spans a low dimensional subspace for chaos. An interesting finding is that the plasma heated by two kinds of heating sources yields the larger correlation dimension than that heated by one kind, and hence, the correlation dimension can be surely used as a tool to study the mechanism of edge turbulence. The largest Lyapunov exponent also confirms that the chaos observed in our experiments is deterministic and the attractor is strange. The change of the largest Lyapunov exponent under the same experimental conditions may reflect variations in the nature of the edge fluctuation itself, due to a set of imperceptibly different initial plasma parameters. Finally, further studies will aim at an effect of chaos on particle transport, so that the relation between plasma confinement and turbulent structures in an edge plasma can be better understood.

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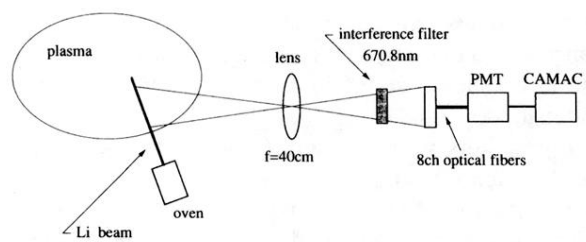

FIG. 1. Schematic of experimental apparatus.