Restoration of Rectangular Pulse Shape after Edge Erosion for a Space-Charge Dominated Electron Bunch

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(Received 17 February 1994)

We present in this Letter a novel experimental method that demonstrates the time reversability of the 1D cold-fluid equations for a space-charge dominated beam. A bunch of charged particles with an initially rectangular line-charge and velocity profile experiences edge erosion during transport through a focusing system due to the nonlinear, longitudinal space-charge forces at the edges. By imposing on the eroded beam pulse a linear velocity gradient at a proper location one can restore the rectangular line-charge profile downstream. This technique could be used to offset edge erosion and assure a rectangular pulse shape for efficient acceleration and uniform power flow in the interaction region for applications of intense, high-power beams.

PACS numbers: 29.27.Bd, 29.17.+w, 41.85.Ja, 52.35−g

It is well known that the line-charge profile of a bunched beam in the low-temperature limit must have a parabolic form to assure that the space-charge forces are linear and hence that the pulses retain their parabolic shape. For some applications of intense charged particle beams, e.g., heavy ion inertial fusion [1] or free electron lasers, however, it is desirable to have bunches with a rectangular current profile. Such square-shaped pulses provide a constant load to the cavities and hence efficient energy gain in the accelerator and uniform power flow in the interaction region. The major drawback is the strong nonlinear, longitudinal space-charge forces at the two ends which lead to rapid edge erosion, as demonstrated experimentally and analytically by Faltens, Lee, and Rosenblum [2]. The conventional remedy for this problem is to use auxiliary acceleration gaps to produce the so-called "ear field" which balances the nonlinear space-charge fields and to keep the beam edges short. This approach complicates the acceleration structure considerably.

We propose a rather simple method to restore the rectangular shape of a high-current beam after edge erosion has occurred. This technique is based on the time reversibility of the cold fluid equations. In Ref. [3], Ho, Brandon, and Lee applied this idea to charged particle beams, and proposed a sophisticated beam compression scheme for heavy ion fusion accelerators. They showed very promising computer simulation results. But, to the best of our knowledge, there has not been an experiment to demonstrate this time reversibility with charged particle beams. In this Letter we report the first experimental demonstration of the time reversibility of the cold-fluid equations with a space-charge dominated electron beam propagating in a periodic solenoid focusing channel. This leads to a simple approach to restore the rectangular pulse shape after edge erosion has taken place. In addition, the experiment verifies in part the theoretical pulse compression scheme of Ho, Brandon, and Lee. We will first briefly introduce the one-dimensional cold-fluid model and analytical formulas for the restoration process. Then we present the experimental results, which show fairly good agreement with the theory.

The longitudinal dynamics of a space-charge dominated beam is governed by the one-dimensional cold-fluid equations, given by

$$\begin{aligned} &\frac{\partial \Lambda}{\partial t} + v\frac{\partial \Lambda}{\partial z} + \Lambda \frac{\partial v}{\partial z} = 0, \\ &\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} \approx \frac{e}{m\gamma^3}E_z \approx -\frac{eg}{4\pi\varepsilon_0 m\gamma^5}\frac{\partial \Lambda}{\partial z}, \end{aligned} \tag{1}$$

where Λ is the line charge density, v is velocity in the beam frame, ε_0 is the permittivity of free space, e/m is the ratio of charge to mass of the particles, γ is the Lorentz factor, and g is a geometry factor in the order of unity. The longitudinal electrical field E_z in Eq. (1) is calculated under the long-wavelength limit, which requires that the beam length is much greater than the beam radius. This set of nonlinear equations can be solved by the method of characteristics [4–6], originally developed to study the unsteady supersonic gas dynamics, and the exact analytical solutions are available in some parameter range. For an initial rectangular beam with a uniform velocity, the analytical solutions of Eq. (1) can be found in Ref. [2, 3]. In the simple wave region, the line charge density and velocity can be put, for the application to the design of the restoration experiment, in the form

$$\frac{\Lambda}{\Lambda_0} = \begin{cases} 1, & \tau > \left(\frac{s}{2s_{\text{cusp}}} - \frac{1}{2}\right)\tau_0, \\ \left[\frac{2}{3} + \frac{s_{\text{cusp}}}{3s}\left(1 + \frac{2\tau}{\tau_0}\right)\right]^2, & \left(-\frac{s}{s_{\text{cusp}}} - \frac{1}{2}\right)\tau_0 < \tau < \left(\frac{s}{2s_{\text{cusp}}} - \frac{1}{2}\right)\tau_0, \end{cases} \tag{2}$$

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$$\frac{v}{c_s} = \begin{cases} 0, & \tau > \left(\frac{s}{2s_{\text{cusp}}} - \frac{1}{2}\right)\tau_0, \\ \frac{2}{3}\left[1 - \frac{s_{\text{cusp}}}{s}\left(1 + \frac{2\tau}{\tau_0}\right)\right], & \left(-\frac{s}{s_{\text{cusp}}} - \frac{1}{2}\right)\tau_0 < \tau < \left(\frac{s}{2s_{\text{cusp}}} - \frac{1}{2}\right)\tau_0. \end{cases} \tag{3}$$

Here $\tau = z/v_0$ is the longitudinal beam-frame coordinate from the beam center at which $z = 0$, and $\tau = 0$, v_0 is the beam center velocity in the laboratory frame, τ_0 is the initial beam pulse duration, c_s is the wave velocity given by $c_s = (egI_i/4\pi\varepsilon_0 m v_0 \gamma^5)^{1/2}$ with I_i being the beam current, s is the traveling distance of the beam center, and s_{cusp} at which the two rarefaction waves just meet at the beam center is given by $s_{\text{cusp}} = (v_0)^2\tau_0/2c_s$. Note that the formulas (2) and (3) are only valid for the front half of the beam ($\tau < 0$), including the leading edge. It is symmetric for the other half where $\tau > 0$ except for a sign difference of the velocity v.

A set of schematic plots in Fig. 1 illustrates the expanding process of a rectangular pulse, described by Eqs. (2) and (3). Figure 1(a) shows the initial rectangular pulse in its phase space (current I_i, velocity c_s), while Fig. 1(b) shows a typical edge erosion in the simple wave region. The electrons at the leading edge speed up and gain energy while those at the falling edge slow down. The velocity distribution on the edges is linear. The bottom of the beam current edges moves out at a speed of $2c_s$ in the beam frame, while the top of the edges moves into the flat region of the beam pulse at a speed of c_s. Consequently, the slope of the linear velocity distribution on the edges decreases and the full length of the beam expands at a speed of $4c_s$ as the beam continues to propagate. There comes a distinct moment at $s = s_{\text{cusp}}$, as shown in Fig. 1(c), when the two rarefaction waves just meet at the beam center, the so-called "cusp" point. At this moment the velocity distribution is linear along the entire bunch.

Utilizing the time reversability of the fluid equations we can restore and accelerate rectangular bunches by imparting a linear velocity tilt in the longitudinal phase space at the cusp location. The restoration and acceleration process is schematically illustrated in Fig. 2. Suppose that an initial rectangular beam pulse as shown in Fig. 1(a) reaches, due to drift expansion, the cusp point as shown in Fig. 1(c). According to the time reversibility, if an appropriate linear velocity tilt is imparted onto the beam while the density remains the same as shown in Fig. 2(e), the beam current profile will be restored with the same rectangular shape, but a higher constant energy, downstream as shown in Fig. 2(g). At this point the beam may be accelerated by a constant field to achieve efficient energy gain and uniform power flow. After that the beam starts expanding again just like that from Figs. 1(a) to 1(c). This restoration and acceleration process can be repeated as desired, though the intervals between cusp points are not really constant due

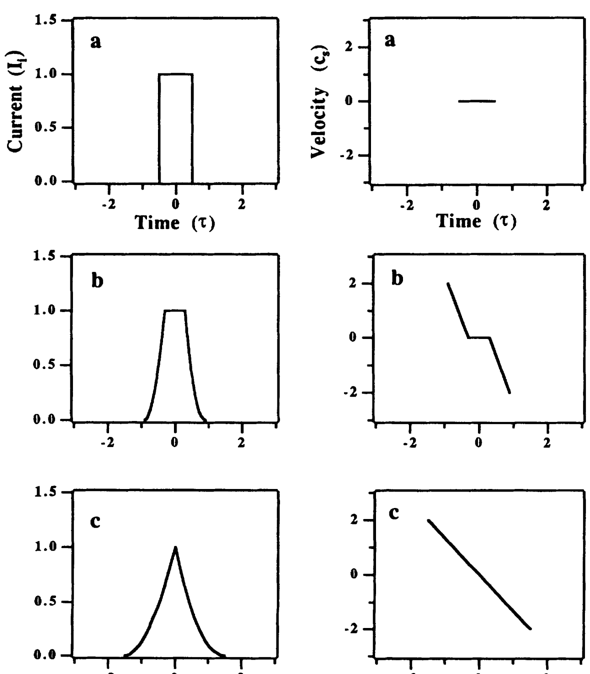


FIG. 1. Expansion of a rectangular bunch in the "simple wave" region; (a) an initial rectangular pulse in phase space; (b) typical edge erosion; (c) "cusp" point.

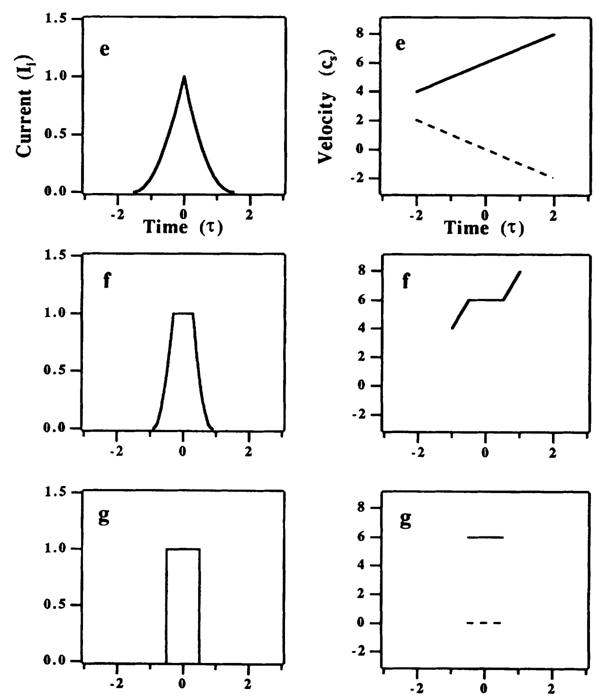


FIG. 2. Illustration of pulse restoration process: (e) an external linear velocity tilt applied at the cusp point; (f) an intermediate stage during restoration process; (g) restored rectangular pulse with uniform current profile but higher energy.

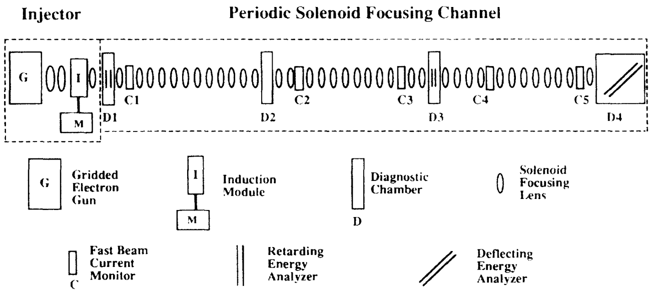


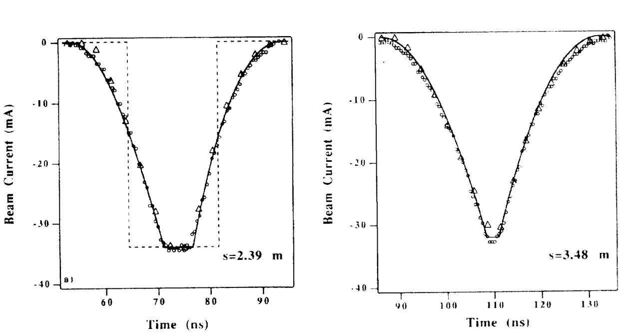
FIG. 3. Experimental configuration.

to the change of the beam velocity. The parameter relations for a restoration process is $D = s_{\text{cusp}} + (v_f)^2\tau_0/2c_{sf}$ and $dv/dt = 4c_{sf}/3\tau_0$. Here s_{cusp} is the location of the acceleration gap to generate the linear ramp field, D is the location where the rectangular pulse shape is restored, and $D > 2s_{\text{cusp}}$, v_f is the final beam center velocity after restoration ($v_f > v_0$), dv/dt is the linear velocity gradient or "tilt" to be appropriately tuned to just balance the space-charge force in order to achieve a perfect rectangular beam with a constant velocity, and c_{sf} is calculated as c_s except that v_0 and γ are replaced by v_f and γ_f.

The experiment for demonstrating the principle of pulse-shape restoration was carried out with the existing experimental facility originally developed for a longitudinal compression experiment [7]. The experimental configuration is illustrated schematically in Fig. 3. The electron bunch was produced by a short-pulse electron beam injector which consisted of a variable-perveance electron gun, an induction acceleration module, and three matching lenses [8]. A proper velocity gradient could be imparted to the beam by the time-varying acceleration voltage of the induction gap. Then, the beam was matched into a 5-m long periodic focusing channel consisting of 36 short solenoid lenses. Five fast wall-current monitors with more than 1.2 GHz bandwidth and three energy analyzers with a time-resolution of 0.5 ns and an energy resolution of a couple of electronvolts were installed to measure the beam current profiles and energy distributions, respectively, at the different locations along the channel.

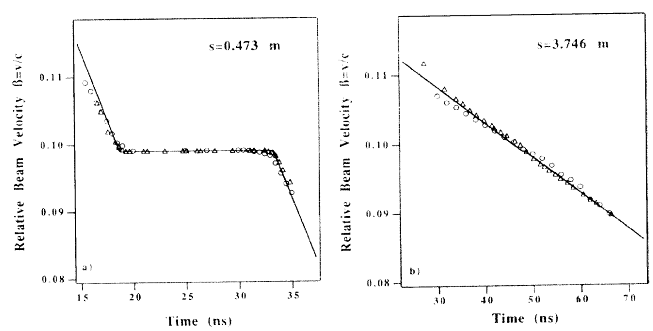
The first experiment was to verify the linear velocity distribution at the cusp point predicted by the method of characteristics. An electron beam with 2.5 keV in energy, 35 mA in current, and 17.5 ns in pulse duration was produced and matched into the 5-m long channel without an induction acceleration. The beam pulse length was carefully chosen so that the cusp point should show up right at the second energy analyzer which was located at a distance of $s = 3.746$ m from the electron gun. The beam current and energy were measured at different locations along the channel. Some of the results are shown in Figs. 4 and 5, where the abscissa is the relative time scale of the signals along the channel, the circles are from the experiment, the triangles are from computer simulation, while the solid curves are from the calculation of Eqs. (2) and (3). Figures 4(a) and 4(b) show the beam current profiles with edge erosion in the simple wave region. Figures 5(a) and 5(b) show the velocity distribution along the beam bunch in two different channel locations. It is evident that the experimental data, the simulation results, and the analytical curves from the method of characteristics are in a very good agreement. Particularly, the beam velocity distribution at the second energy analyzer, i.e., the cusp point, is very linear as shown in Fig. 5(b).

Then, a preliminary pulse restoration experiment was performed. In the experimental setup the distance between the electron gun and the induction gap was about 0.34 m which was rather short. In order to make the cusp point appear at the induction gap, a short beam pulse, high wave velocity and slow beam velocity were desired, that required low energy and high current beams. A 300 eV, 3.3 mA, and 7 ns electron beam was generated and matched into the 5-m channel. Without application of an induction acceleration voltage the beam rapidly expanded and disappeared. The beam current profile could only be measured at the first and second current monitor shown in Fig. 6(a). The expanding bunch length was consistent with the calculation, though the noise became significant due to the very low current level. With a proper induction acceleration to impart a linear velocity tilt, the beam pulse shape was restored downstream and expanded again afterwards.



(a) (b)

FIG. 4. Current profiles due to drift expansion: (a) at $s = 2.39$ m where the dashed lines indicate the initial rectangular bunch; (b) at $s = 3.48$ m.



(a) (b)

FIG. 5. Velocity distribution due to drift expansion: (a) at $s = 0.473$ m; (b) at $s = 3.746$ m which is the cusp point and shows a linear velocity distribution.

The beam profiles were measured by the five current monitors shown in Fig. 6(b). The distinguishing feature of the pulse restoration are that during the process the peak beam current remains the same, the leading and falling edges become steep, and the flat top maintains a constant velocity or energy. One notices that the time intervals between the pulses are significantly shorter in Fig. 6(b) than that in Fig. 6(a). This is because the induction module raised the beam center energy by more than 700 eV. In the experiment we also varied the beam pulse duration, the slope of the induction gap voltage and beam center energy after the gap. In each case the beam behavior was consistent with the theory. The energy measurement for the restored beam pulse was not available due to the very low beam current signal level at the energy analyzers.

The time reversibility of the fluid equations is very general, valid both in the simple wave region and the nonsimple wave region. In principle, the restoration process can be performed not only from the cusp point, but also from any other point during beam edge erosion as long as an appropriate external field variation can be generated in the acceleration gaps. In practice, however, it is easiest to generate a linear ramp of the acceleration field in the gaps. Thus, a restoration process starting at the cusp point is most practical.

In the analysis and experiment, a unipotential gap is considered and employed. Thus, the energy of the beam pulse is always increased after a restoration process, and the next cusp point will appear in a longer distance. This makes it impractical to perform a periodic restoration in transport channels and accelerators. The remedy for this problem is to use bipotential gaps. In this case the energy transferred from the external linear field to the beam pulse center at the cusp point can be set to zero. A complete restoration, both in current profile and energy distribution, of an initially rectangular beam pulse can be achieved. This will provide a way to form a periodic restoration and acceleration structure for both linear and circular high-current machines.

In summary a simple restoration scheme for space-charge dominated beams was proposed and was experimentally demonstrated for the first time, to the best of our knowledge. The experiment showed the linear velocity distribution along the beam at the cusp point. The experiment also achieved the restoration of a short rectangular pulse. The results agree reasonably well with the theoretical predictions, based on the one-dimensional cold-fluid model. The outcome of this study suggests a possible new scheme to accelerate high-current beams, though there are many practical problems to be solved.

The authors would like to acknowledge I. Haber for his help in computer simulation of beam edge erosion. This research was supported by the U.S. Department of Energy.

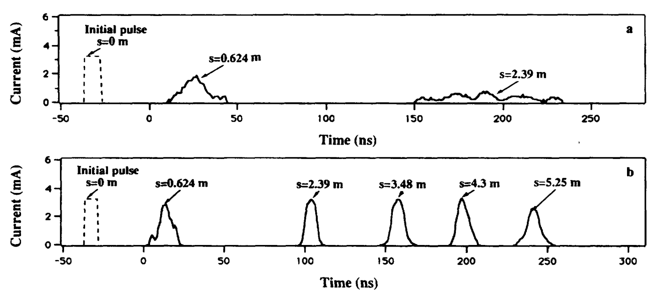


FIG. 6. Comparison between drift expansion and rectangular pulse restoration: (a) free expansion of a rectangular 300 eV, 3.3 mA, and 7 ns bunch, as measured at $s = 0.624$ m and $s = 2.39$ m; (b) restoration of a rectangular bunch initially with 300 eV, 3.3 mA, and 7 ns, as measured by the five current monitors, where the abscissa is the relative time scale of the signals with $t = 0$ ns corresponding to the induction gap location and $s = 0$ m corresponding to the cathode location, and the ideal restored rectangular pulse should appear in between the second ($s = 2.39$ m) and third ($s = 3.48$ m) current monitors.

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