Effective Lowering of Dimensionality in the Strongly Correlated Two Dimensional Electron Gas

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We study a system of fermions interacting with a gauge field which can be used to describe either a spin liquid or the $\nu = 1/2$ quantum Hall state. We propose a generalized model with a dimensionless parameter N. We evaluate the properties of the model in both limits $N \gg 1$ and $N \ll 1$ and deduce the properties of the model in the most physically interesting case of $N = 1, 2$. At $N \ll 1$ the motion of the fermions becomes one dimensional. By applying the bosonization method in this limit, we obtain the fermion Green function and response functions.

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Recently the problem of two dimensional (2D) fermions interacting with a gauge field has become a subject of extensive research. The low energy behavior of this model is often believed to describe the properties of two strongly interacting electron systems: the normal state of the high T_c cuprates [1–3] and the $\nu = 1/2$ state in the quantum Hall (QH) effect [4,5]. We begin by formulating the problem, then we describe our results, and finally we sketch their derivation.

The localized spins in high T_c are assumed to form a gapless spin liquid [6] which does not break any symmetry [the so-called resonating valence bond (RVB) state]. It is convenient to describe the excitations of this state in terms of chargeless fermions with spin 1/2 (spinons): $\mathbf{S} = f^\dagger_\beta \sigma_{\beta\alpha} f_\alpha$, where f_α is the fermion destruction operator [1]. To ensure that each site is occupied by one and only one spin, it is necessary to couple these fermions to the gauge field. Formally, this gauge field appears after a Hubbard-Stratanovich decoupling of the spin-spin interaction [1,2]: $H = \sum \mathbf{S}_i \cdot \mathbf{S}_j$.

The 2D electron gas in the QH regime acquires very special properties in the vicinity of the filling factor $\nu = 1/2$. Attaching two flux quanta to each electron, one maps this problem onto the problem of fermions interacting with a fluctuating "magnetic" field of zero average at $\nu = 1/2$ [4,5]. Neglecting these fluctuations, one ends up with a gapless Fermi liquid. The experimentally observed nonzero conductivity [7] around $\nu = 1/2$ means that this Fermi liquid is a reasonable starting point. To complete the theory one must take into account the fluctuations of the gauge field.

Both problems can be reduced to the Hamiltonian

$$H = -\frac{1}{2m} f^\dagger_\sigma (\nabla - i\mathbf{a})^2 f_\sigma + \frac{\chi}{2}(\nabla \times \mathbf{a})^2, \qquad (1)$$

where $\mathbf{a}$ is a transverse vector potential ($\nabla \cdot \mathbf{a} = 0$), $\sigma = 1, \ldots, N$. For the spin polarized QH state $N = 1$, whereas for the RVB state $N = 2$, corresponding to two possible spin polarizations. The Hamiltonian (1) correctly describes only low energy modes: fermions in the vicinity of the Fermi line and transverse gauge field. The high energy modes (fermions deep in the Fermi sea, longitudinal gauge field) were already integrated out in (1) resulting in the stiffness of the gauge field [last term in (1)] and the fermion mass renormalization. The interaction mediated by the longitudinal gauge field is screened and becomes short ranged, so it can be omitted from the Hamiltonian (1). In the case of the QH effect, there is also a coupling between transverse and longitudinal gauge fields which gives an additional contribution to χ.

The main difficulty associated with (1) is the singularity of the interaction mediated by the gauge field at low energy ω and momentum k transfer. Fermions near the Fermi line result in a Landau damping [8] ($\propto |\omega|/k$), leading to an overdamped dynamics of the gauge field described by the correlator [2] $\langle a_\mu a_\nu \rangle_{k,\omega}$ which we write in Matsubara (Euclidean) formalism:

$$\langle a_\mu a_\nu \rangle_{k,\omega} = (\delta_{\mu\nu} - k_\mu k_\nu / k^2) D(\omega, k) = \frac{\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{\frac{N p_0 |\omega|}{2\pi |k|} + \chi k^2}. \qquad (2)$$

Here p_0 is the inverse curvature of the Fermi line. For a noncircular Fermi line p_0 varies along the Fermi line. In this case one has to evaluate the curvature at a point where the tangent to the Fermi line is parallel to $\mathbf{k}$.

The model (1) has been discussed repeatedly in the framework of $1/N$ expansion [2,3,9–12]. At finite temperatures the static part of gauge field fluctuations [$\omega = 0$ in (2)] dominates [10]; i.e., the contributions from $\omega \neq 0$ fields are small in $1/N$. The Green's function of the fermions evaluated in a static approximation shows that coherence effects are suppressed by magnetic field fluctuations; as a result, the particle moves semiclassically: for large times only a small region around the classical trajectory determines the Green's function.

The relevance of $1/N$ expansion for the interesting cases of $N = 1, 2$ was questioned in [12]. Here we shall consider the behavior of the model in both limits $N \to \infty$ and $N \to 0$ and discuss the implications for $N = 1, 2$ [13]. The limit $N \to 0$ is realized in the generalized model with M species of photons. Such a model may describe a spin liquid with M sublattices. Since in this model the effective correlator (2) is multiplied by M, it can also be described by Eqs. (1) and (2) with modified parameters: $\tilde{N} = N/M$, $\tilde{\chi} = \chi/M$. In the following we shall drop the tildes and study the model for general values of N and χ. As will be clear below, the value of the stiffness χ sets the relevant length scale l_0,

$$l_0^{-1} = \left(\frac{1}{3\sqrt{3}}\right)^3 \frac{2v_F^2}{\pi^2 p_0 \chi^2 N}. \qquad (3)$$

At shorter scales the effects of gauge interaction are not important, whereas the qualitative behavior at larger scales is governed by the value of N.

At $N \gg 1$ the static approximation is valid at any temperature T. At $N \ll 1$ the fermion motion becomes qualitatively different: it is strongly nonclassical and even more one dimensional. In this limit the fermion Green's function is given by a 1+1 dimensional integral:

$$G(\mathbf{p}, \epsilon) = \int G(x,t) e^{i\epsilon t - i(|\mathbf{p}| - p_F)x} dt\, dx, \qquad (4)$$

$$G(x,t) = \frac{i}{2\pi(x - iv_F t)} \exp\left(\frac{-\Gamma(2/3) l_0^{-1/3}|x|}{[|x| - i\mathrm{sgn}(x) v_F t]^{2/3}}\right).$$

Clearly, the fermion density of states $\rho = \mathrm{Im}\int G(0,t) \times e^{i\epsilon t} dt$ remains the same as for noninteracting fermions. At the same time, the probability to move along the classical path $x = v_F t$ decays exponentially; the particle is smeared over a distance $r \propto t^{2/3}$ and the velocity of the wave packet vanishes at $t \to \infty$. Therefore, due to gauge field fluctuations, a single fermion cannot propagate.

However, a particle-hole pair (which is neutral with respect to the gauge field) propagates if the momentum of the relative motion is small: $k \ll k_\omega = \left(\frac{N\omega p_0}{\chi}\right)^{1/3}$. In this case the absorption (the imaginary part of the fermion density correlator) at $k \ll k_\omega$ remains the same as in the noninteracting Fermi gas, $\mathrm{Im}R(\omega, k) \propto \omega/|k|$.

The particles with large relative velocities become essentially independent and propagate poorly. This effect suppresses the absorption at large wave vectors $k \gg k_\omega$; we estimate it as $\mathrm{Im}R(\omega, q) \le \omega(p_F l_0)$.

Increasing N results in a crossover to the state where static fluctuations of the gauge field dominate at finite temperatures. At $T = 0$ the most important effect of the $1/N$ corrections is the anomalous power law behavior of the density (and spin) correlators at $k = 2p_F$ and the backward scattering amplitudes; these exponents are proportional to $1/N$. The power laws cross over to an exponential behavior at $N \to 0$.

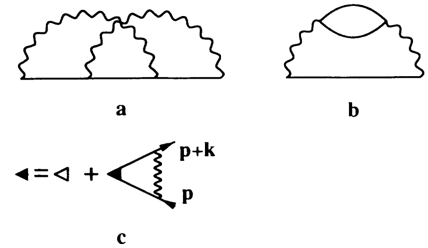

FIG. 1. Typical high order diagrams. (a) Simplest high order diagram for the self-energy with crossings. (b) Loop insertion in the gauge field line that should be excluded from 1D theory. (c) At $N \ll 1$ only ladder diagrams are important for vertex renormalization. $k \approx 0$ corresponds to vertex Γ_D; $k \approx 2p_F$ corresponds to vertex Γ_P. Filled triangle denotes renormalized vertices Γ_D, Γ_P.

Now we sketch the derivation of these results. To the first order in the interaction with the gauge field the Green's function is $G^{(1)-1}(\epsilon, \mathbf{p}) = i\epsilon - v_F(|p| - p_F) - \Sigma^{(1)}(\epsilon)$,

$$\Sigma^{(1)}(\epsilon) = v_F^2 \int \frac{D(\omega, k) d\omega d^2k}{i(\epsilon + i\omega) - v_F(|p+k| - p_F)} = -i\left|\frac{\omega_0}{\epsilon}\right|^{1/3} \epsilon, \qquad (5)$$

$$\omega_0 = \left(\frac{3}{2}\right)^3 \frac{v_F}{l_0} \propto \frac{1}{N}.$$

For the idea of the derivation see [3,14].

At $N \gg 1$ diagrams with crossings are negligible. Consider, e.g., the diagram for the self-energy shown in Fig. 1(a). To evaluate it we introduce the components $k_\parallel$ ($k_\perp$) of the momenta k parallel (perpendicular) to $\mathbf{p}$. The product of the three Green's functions in (10) decreases rapidly at $v_F k_\parallel \gg \Sigma(\epsilon)$, so the main contribution to (10) comes from the range of small $[k_\parallel \lesssim l_0^{-1}(\frac{\epsilon}{\omega_0})^{2/3}]$ momenta. Both G and D decrease only when $k_\perp \gtrsim k_\epsilon$, so the main contribution to (10) comes from the range $k_\perp \lesssim k_\epsilon \sim k_\parallel \left(\frac{\omega_0}{\epsilon}\right)^{1/3} \gg k_\parallel$. Neglecting the $k_\parallel$ dependence of the gauge field propagator we get

$$\Sigma^{(2)}(\epsilon) = v_F^2 \int \frac{A(\omega_1, \omega_2)(d\omega_1 d\omega_2 dk_{\perp 1} dk_{\perp 2})}{A^2(\omega_1, \omega_2) - k_{\perp 1}^2 k_{\perp 2}^2/m^2} D(\omega_1, k_{\perp 1}) D(\omega_2, k_{\perp 2}), \qquad (6)$$

$$A(\omega_1, \omega_2) = v_F(p_F - p) + i\omega_0^{1/3}(|\epsilon + \omega_1 + \omega_2|^{2/3} + |\epsilon + \omega_1|^{2/3} + |\epsilon + \omega_2|^{2/3}). \qquad (7)$$

In the limit $N \gg 1$ we find

$$\Sigma^{(2)} = -ic\left(\frac{\ln N + O(1)}{4\pi N}\right)^2 \epsilon \left|\frac{\omega_0}{\epsilon}\right|^{1/3}, \qquad c \approx 2.16. \qquad (8)$$

Note that the ratio of $\Sigma^{(2)}/\Sigma^{(1)}$ is governed only by N, and that at $N \gg 1$, $\Sigma^{(2)} \ll \Sigma^{(1)}$. This smallness can be traced back to the term $k_{\perp 1}^2 k_{\perp 2}^2$ in the denominator of (6) caused by the crossing in this diagram.

Since each crossing results in an additional factor $1/N^2$, only the diagrams with the minimal numbers of crossings are important at $N \gg 1$. From the theory of localization such diagrams are known to include the ladder-like pieces in particle-hole and particle-particle channels and lead to potentially dangerous singularities. These diagrams make $1/N$ expansion subtle: to find the subleading terms one cannot consider only one diagram with single crossings but needs to renormalize vertices by the whole ladder series. The details of such calculations will be presented elsewhere [15]; here we only summarize their conclusions: (i) the self-energy preserves its form at $p = p_F$ and allows expansion in series of $1/N$: $\Sigma(\epsilon, p = p_F) = f(1/N)\epsilon|\omega_0/\epsilon|^{1/3}$, but it acquires some momentum dependence in higher orders of $1/N$ expansion; (ii) $1/N$ corrections for polarization bubble become very small ($\propto |\omega|$ in the infrared limit), and can be completely neglected.

The vertex parts at the momentum transfer $2p_F$ are more interesting. Their anomalous energy dependence determines the dependence of spin and density correlators. The first order correction is proportional to $\ln \epsilon$ [9]. The series of logarithms can be summed similarly to the derivation of doubly logarithmic asymptotics in quantum electrodynamics [16]; at small energies and momenta close to $2p_F$ the vertex has a power law singularity:

$$\Gamma_P(\epsilon, 2p_F) \sim \left(\frac{\epsilon_F}{|\epsilon|}\right)^{\frac{1}{2N}}, \quad \Gamma_P(0,k) \sim \left(\frac{p_F}{|k| - 2p_F}\right)^{\frac{3}{2N}}. \tag{9}$$

Now we turn to the limit $N \to 0$. We can neglect the dependence of the fermionic Green's function on the transverse momenta even in higher order diagrams with crossings; e.g., we can neglect the term $k_{\perp 1}^2 k_{\perp 2}^2$ in the denominator of (7) for the diagram in Fig. 1(a). Then $k_\perp$ enters only via $D(\omega, k)$; integrals over them factorize and can be performed independently for each gauge field propagator. The resulting expression for the self-energy diagrams takes a form of integrals over $k_\parallel$ and ω; e.g., the diagram shown in Fig. 1(a) is

$$\Sigma_p^{(2)}(\epsilon) = \int G^{(1)}(\epsilon+\omega_1, p+k_1) G^{(1)}(\epsilon+\omega_2, p+k_2) \times G^{(1)}(\epsilon+\omega_1+\omega_2, p+k_1+k_2)\tilde{D}(\omega_1) \times \tilde{D}(\omega_2)(d\omega_1 d\omega_2 dk_1 dk_2). \tag{10}$$

Here

$$\tilde{D}(\omega) = \frac{v_F g}{|\omega|^{1/3}} \tag{11}$$

is the new propagator governed by the interaction parameter $g = \frac{4\pi}{3}\omega_0^{1/3}$. Note that due to the infrared singularity of the gauge field propagator the main contribution to the self-energy comes from the interaction between fermions that move almost in the same direction. Thus, we can consider only fermions on a small patch of the Fermi surface around momentum $\mathbf{p}$. The absence of $p_\perp$ dependence of the Fermi Green's functions implies that motion in this patch is essentially one dimensional. Moreover, similar arguments applied to higher order diagrams show that the whole series of diagrams coincides, order by order, with a perturbative expansion of the 1D theory with the action

$$S = \int \left(\bar{\Psi}^a_{-\omega,-k}(i\omega - v_F k)\Psi^a_{\omega,k} + \frac{v_F g|\rho_{\omega,k}|^2}{|\omega|^{1/3}} \right) dk\, d\omega, \tag{12}$$

where $\rho_{t,x} = \bar{\Psi}^a_{t,x}\Psi^a_{t,x}$ is the density operator.

The diagrams with Fermi loops [e.g., in Fig. 1(b)] are already taken into account in the gauge field propagator (2). As we show below, the higher order insertions in the loop of the Fermi field do not change the propagator (2) at typical ω and k. However, a perturbative treatment of model (12) generates the loops. To get rid of them we use the replica trick—take the limit of zero number of components of the Fermi field: $a = 1, \ldots, s$, $s \to 0$.

Finally, the retarded interaction between the fermions provides a natural cutoff (ω_0) of the ultraviolet divergencies: regularization of the 1D theory in the time direction is more natural here than the conventional regularization in the space direction.

The 1D model (12) can be solved by bosonization [17]. As usual, we introduce the Bose field ϕ with the action quadratic in Bose fields

$$S_B = \frac{1}{2}\int \left((\omega^2 + k^2)(\phi_a)^2 + \frac{g}{\pi}|\omega|^{5/3}\phi_a\phi_b \right) d\omega\, dk. \tag{13}$$

This allows us to evaluate the Green's function:

$$F_{ab}(\omega, k) = \langle \phi_a \phi_b \rangle = \frac{1}{\omega^2 + k^2}\delta_{ab} - \frac{g|\omega|^{5/3}}{\pi(\omega^2 + k^2)^2}. \tag{14}$$

The original fermions are related to the Bose field by

$$\Psi = \frac{1}{\sqrt{2\pi a}} \exp \sqrt{\pi}(\phi^* + i\phi), \tag{15}$$

where ϕ^* is the dual field: $\partial_t \phi^* = \partial_x \phi$ and $a \sim v_F/\omega_0$. All correlators of the original Fermi fields are reduced to the Gaussian integrals over the Bose fields. Evaluating these integrals we get the fermion Green's function (4).

The main contribution to the absorption at small momenta $k \leq k_\omega$ comes from fermion hole pairs close to the Fermi surface with $v_F k_\parallel \sim \Sigma(\omega)$ and perpendicular $k_\perp \approx k \ll k_\omega$. The momentum perpendicular to $\mathbf{p}$ can be ignored and the interaction between fermion and hole can be described within the 1D theory. Effects of the interaction cancel exactly in the density correlator $\langle \rho\rho \rangle = \langle (\bar{\Psi}_a \Psi_b)(\bar{\Psi}_b \Psi_a) \rangle$: the absorption at these wave vectors is exactly the same as for noninteracting fermions.

This also proves that the interaction does not renormalize $D(\omega, k)$ in the important range of ω and k. Thus, we have shown that the gauge field propagator (2) is not renormalized in both limits $N \gg 1$ and $N \ll 1$, so we conjecture that it is not renormalized for any N.

Straightforward analysis [15] shows that fermions with a large angle between their momenta interact weakly. The angle close to π makes an important exception because the gauge field leads to a strong interaction between fermions moving in almost opposite directions, i.e., fermions on two small patches of the Fermi surface around momenta $\mathbf{p}$ and $-\mathbf{p}$. Repeating the arguments that led us to a 1D action (12) we see that the effect of the gauge field on the scattering by angle π can be similarly described in the framework of 1D theory which contains both right and left movers. Moreover, it can be described in the framework of the same bosonization scheme characterized by the action (13): from the Bose fields we may construct left and right moving fermions $\Psi_{L,R}\frac{1}{\sqrt{2\pi a}} \exp \sqrt{\pi}(\phi^* \pm i\phi)$ representing fermions before and after collision. As before, one can check that this 1D theory reproduces correctly all terms in the diagrammatic expansion for the original fermions.

The effect of the interaction between fermions moving with opposite momenta is revealed in its effect on the scattering amplitude of the fermion by impurity with momentum transfer $2p_F$. The backward scattering is known to be renormalized to infinity in a case of a Luttinger liquid with repulsion [18]. In a 1D model (12) the enhancement is much stronger than in a canonical Luttinger liquid discussed in [18]; namely, the bosonization approach gives the backscattering probability

$$\tau_B^{-1} \sim \tau^{(0)-1} \exp\left(\frac{3g}{2\pi|\omega|^{1/3}}\right), \tag{16}$$

where $1/\tau^{(0)}$ is the backscattering probability of free electrons. The difference from the Luttinger liquid originates from the power singularity in (12). The exponentially large amplitude of the backscattering is due to a cancellation of the diagrams in the 1D theory which follows from a generalized Ward identity [19]. This cancellation is exact only at $N \to 0$, so we expect that the result (16) crosses over to a power law behavior (9) at finite N with an exponent that tends to infinity when $N \to 0$.

We believe that the effect of backscattering enhancement was observed experimentally in [20] where it was found that for $\nu = 1/2$ the transport relaxation rate is about 2 orders of magnitude larger than in the state without magnetic field while the total relaxation rate (determined from Shubnikov–de Haas oscillations) remained of the same order. With no magnetic field the total scattering rate was much larger than the transport rate, implying a very small probability of backscattering. This probability can be dramatically enhanced by the mechanism we have proposed in this paper.

When the magnetic field varies away from $\nu = 1/2$, the fermions see an incremental uniform field; its influence on the dimension reduction is still an open question.

In conclusion, we have shown that two dimensional fermions interacting with a gauge field may exhibit one dimensional behavior. This conclusion seems consistent with a qualitative picture proposed by Anderson [21]. This approach may lead to the quantitative description of both the RVB and $\nu = 1/2$ states.

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