Collapse of Quantized Conductance in a Dirty Tomonaga-Luttinger Liquid

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The way the quantized value of conductance in a quantum wire disappears is predicted as a function of temperature and length of the wire by taking account of both impurity scattering and mutual Coulomb interactions, i.e., in the case of a dirty Tomonaga-Luttinger liquid. It is shown that quantization of conductance is realized for short wires, while for longer wires the effect of impurity scattering is important and a deviation from the quantized value becomes apparent especially at low temperatures. In this case the conductance shows a distinctive temperature dependence.

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Recent progress in nanostructure technology has stimulated studies of quantum transport in mesoscopic systems [1]. One of the most remarkable phenomena is the observation of quantization of conductance through a point contact, which reflects the number of active channels [2–4]. This quantization, however, is not robust but appears to be sensitive to the parameters of actual experiments; sometimes it gives away for cases with small carrier density (i.e., for low gate voltages) [5]. Recently it has become possible to study the size dependence of conductance for a fairly large variety of parameters [6, 7]. The system length and the mean free path can be roughly of the same order of magnitude. In such situations, an interesting size dependence is expected.

In this paper we study the way this collapse of the quantization of conductance is described as a function of temperature and length of the system. We consider the case of a single channel in a quantum wire with both impurity scattering and mutual Coulomb interaction, i.e., the case of a dirty Tomonaga-Luttinger liquid [8] in a finite system. We confine ourselves to the case of static ($\omega = 0$) Ohmic conductance. In this case the conductance g is given as $g = \sigma/L$ for the one-dimensional case, where σ is the static Ohmic conductivity and L is the length of the system. For the discussion of quantization of conductance, g is usually evaluated by use of the Landauer formula [9]. First of all, the conductance per channel in the presence of mutual interaction becomes [10]

$$g = \frac{e^2}{\pi\hbar} K_\rho , \tag{1}$$

where K_ρ is a correlation exponent [11] which is related to the interaction strength. $K_\rho = 1$ for noninteracting electrons; when the interaction is repulsive $K_\rho < 1$, and when it is attractive $K_\rho > 1$, respectively. For the repulsive Hubbard model we have $\frac{1}{2} \leq K_\rho \leq 1$, especially $K_\rho \to \frac{1}{2}$ toward half filling, which is a direct consequence of the spin-charge separation near the Mott insulator.

The exponent K_ρ shows up in (1) because the current-density operator in the Tomonaga-Luttinger liquid is strongly renormalized by the interaction [8]. It is to be noted here that, as far as the Tomonaga-Luttinger liquid is realized, quantized conductance (1) is obtained [10, 12]. On the contrary, in point contacts, electrons pass through the contact before they feel the mutual interactions. In this case conductance will be quantized at $g = e^2/\pi\hbar$. Roughly speaking we expect a crossover from $g = e^2 K_\rho/\pi\hbar$ to $g = e^2/\pi\hbar$ when the system size is very short and $2\pi/L > k_F$, where the discreteness of the energy levels invalidates the bosonization. In this paper we do not consider the case of point contacts but study the case of the quantum wires whose lengths are sufficient to make the system the Tomonaga-Luttinger liquid.

The effect of a single impurity on (1) has been discussed [12, 13]. In such calculations, however, the thermodynamic length of the one-dimensional system is implicitly assumed and the impurity density is $n_i = 1/L$. On the other hand, in actual experimental situations which we are interested in, the characteristic length of the potential fluctuation can be much shorter than the system size. In such cases, the effect of finite impurity density is to be investigated. Apel and Rice [10] discussed the conductance g in the presence of finite impurity density to show that the impurity scattering is renormalized to strong coupling for $K_\rho < 2$ and g goes to zero as temperature decreases. Recently Fukuyama *et al.* [14] calculated the explicit temperature dependence of conductivity in the thermodynamic limit ($L \to \infty$). They obtained a finite conductivity which means that the conductance g is proportional to $1/L$. Apparently this result (finite conductivity) and the quantized conductance in Eq. (1) (finite conductance) look incompatible at first glance.

In this paper we clarify the relation between them and show that the collapse of the quantization is naturally understood by the crossover between the two cases of the quantized conductance and the finite conductivity, by a detailed study of the length and temperature dependences of the conductance. For short wires we expect finite (quantized) *conductance* on one hand, and for longer wires (and at finite temperatures) we will have a finite *conductivity* or $g = \sigma/L \propto L^{-1}$ on the other hand. The possibilities of an experimental check at an accessible

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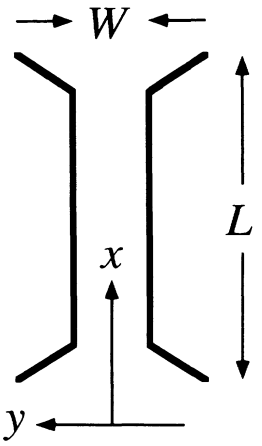

FIG. 1. A schematic geometry of a quantum wire in the present study.

temperature and wire size are also discussed based on experimentally available parameters.

Here we note that the Anderson localization becomes important for samples with short mean free paths and at low temperatures. In this paper, however, we examine the conductance of relatively cleaner samples in the temperature region where the localization effect does not play an important role. The relation between the present calculation and the Anderson localization will be explicitly discussed in the following.

Our model is a quantum wire of length L and width W which is connected to perfect leads on both sides, as shown in Fig. 1. We assume that the carrier density n and W are such that there exists only a single subband along the wire (one-dimensional electrons). The electrons are scattered by impurities and mutually interacting via long-range Coulomb interaction, both of which play important roles in actual systems. The Hamiltonian is

$$H = \sum_{k,s} \epsilon(k)\, a^{\dagger}_{k,s} a_{k,s} + \frac{1}{2}\sum_{q} v(q)\rho_q \rho_{-q} + \sum_{q} u(q)\rho(q), \tag{2}$$

where $\epsilon(k) = \hbar^2 k^2/2m^*$ and $v(q)$ and $u(q)$ are one-dimensional Fourier transforms of the Coulomb interaction and the random impurity potential, respectively. The density operator of electrons, $\rho(q)$, is defined as $\rho(q) = \sum_{k,s} a^{\dagger}_{k+q,s} a_{k,s}$. In Eq. (2) we ignored the electron-phonon scattering since we are interested in the region of relatively low temperatures.

First let us examine the noninteracting case. In a clean system $\sigma(\omega)$ is given as $\sigma(\omega) = (i\omega)^{-1}[\Phi(\omega) - \Phi(0)]$,

$$\Phi(\omega) = -\frac{e^2\hbar^2}{i\pi m^{*2} L}\sum_{k} k^2 \int_0^{\hbar\omega} dz G^R\left(k, \frac{z}{\hbar}\right) \times G^A\left(k, \frac{z}{\hbar} - \omega\right), \tag{3}$$

where $G^R(k,\omega) = [\hbar\omega - \epsilon_k + i\delta]^{-1} = [G^A(k,\omega)]^*$. Taking account of the finite length of the system, we evaluate $\Phi(\omega)$ as follows:

$$\Phi(\omega) = \begin{cases} -\frac{ne^2\omega}{m^*(\omega + i\delta)}, & \text{for } \omega \gg v_F/L, \quad (4a) \\ i\frac{e^2\omega L}{\pi\hbar}, & \text{for } \omega \ll v_F/L. \quad (4b) \end{cases}$$

As is well known, substitution of Eq. (4a) into (3) gives the Drude conductivity, $\sigma = ne^2/m^*(-i\omega + \delta)$. On the other hand, Eq. (4b) being valid in systems with finite length leads to the quantized value of the conductance of a noninteracting one-channel system, $g = e^2/\pi\hbar$. Equation (4) indicates that a replacement $-i\omega \to 2v_F/L$ is valid for systems with finite lengths and the static limit. (Note that $n = 2k_F/\pi = 2m^* v_F/\pi\hbar$.) In mesoscopic systems Eq. (4b), instead of (4a), is relevant in the static limit. In the presence of mutual interaction the extra factor K_ρ is present as in Eq. (1).

In the presence of both randomness and mutual interaction we evaluate the conductivity based on the Mori formula [15, 16] which reads as follows:

$$\sigma = \frac{ne^2 K_\rho}{m^*}\left[\frac{2v_F}{L} + \frac{1}{\tau_{\rm tr}}\right]^{-1}, \tag{5}$$

$$\frac{1}{\tau_{\rm tr}} \equiv n_i \sum_{k,k'} u^2(k - k')\left(\frac{\partial\epsilon}{\partial k} - \frac{\partial\epsilon}{\partial k'}\right)^2 \times \frac{{\rm Im}N(k - k', \omega)}{\omega}\bigg|_{\omega\to 0} \equiv \frac{F(T,L)}{\tau^0_{\rm tr}}, \tag{6}$$

where n_i is the density of impurities and $N(k,\omega)$ is the density-density correlation function which takes full account of the mutual interactions. We have K_ρ in the numerator of (5) for the same reason as in (1). In Eq. (6) $\tau^0_{\rm tr}/\hbar = [2\pi D(0)\, n_i \bar{u}^2]^{-1}$ is the transport relaxation time for noninteracting electrons at absolute zero with $\bar{u} = u(2k_F)$ and $D(0) = (2\pi\hbar v_F)^{-1}$ is the half of density of states per spin. Note that the conductivity (5) is obtained from a usual Drude formula with the replacement $-i\omega \to 2v_F/L$. Usually in discussing the static conductivity, the term $-i\omega$ is neglected, but in the present case it is important to keep this term, replacing it with the size-dependent term. The Mori formula is seen to be very suited for observing the importance of this term. Equation (5) naturally interpolates the quantization of conductance (1) for small systems with $2v_F/L \gg 1/\tau_{\rm tr}$, and the usual conductivity for longer systems with $2v_F/L \ll 1/\tau_{\rm tr}$.

The quantity $F(T,L)$ represents the effect of the impurity backward scattering. In the limit of a long wire ($L \gg \hbar v_F/k_B T$), $F(T,L)$ is simply related to the $2k_F$ component of density-density correlation function in the thermodynamic limit, and will be denoted as $\tilde{F}(T)$. If the interaction is short ranged, it is well known that $N(2k_F,\omega)$ has a power-law behavior as ω and T go to zero in the Tomonaga-Luttinger liquid [8]. This gives $\tilde{F}(T) \sim (k_B T/\omega_F)^{K_\rho - 1}$, where the temperature is normalized by the cutoff frequency ω_F, which is of the order of the Fermi energy. This temperature dependence is, however, a critical behavior only for $k_B T \ll \omega_F$. The global temperature dependence has been calculated recently [14]

for the realistic long-range Coulomb interactions, based on the phase Hamiltonian [17–19]. In this case $\tilde{F}(T)$ is also a rapidly increasing function of T as the temperature decreases, which is consistent with $(k_BT/\omega_F)^{K_\rho - 1}$ for $K_\rho < 1$. In the following we utilize the result for $\tilde{F}(T)$ obtained by Fukuyama, Kohno, and Shirasaki [20].

For the case of the short wires, or when L is comparable with $\hbar v_F/k_BT$, we have to take account of the effect of the finite length L. This effect is introduced in $\tilde{F}(T)$ by replacing the temperature T by $(T^2 + T_L^2)^{1/2}$ where $k_BT_L = \hbar v_F/L$, i.e., $F(T,L) = \tilde{F}((T^2 + T_L^2)^{1/2})$. Below the temperature T_L, where the thermal length becomes longer than the system size, the divergence of $\tilde{F}(T)$ as $T \to 0$ is terminated and $F(T,L)$ becomes T independent.

By Eqs. (5) and (6) we obtain the formula for the conductance as

$$g = \frac{2e^2K_\rho}{h}\frac{1}{1 + (L/2l)F(T,L)} \equiv \frac{2e^2K_\rho}{h}G(T,L), \quad (7)$$

where $l = v_F\tau_{\rm tr}^0$ is the elastic mean free path. The dimensionless conductance, $G(T,L)$ thus obtained is shown in Fig. 2 for several choices of L/l and T_L. The temperature is normalized by the cutoff frequency ω_F. Let us first see the typical experimental values of relevant parameters in the case of GaAs quantum wires, with $n \sim 2 \times 10^{11}/\text{cm}^2$ and $W = 400$ Å as studied in Ref. [14]. The Fermi velocity is $v_F \sim 2 \times 10^5$ m/sec ($m^* = 0.067m_0$, $k_F \sim 1.2 \times 10^8\ \text{m}^{-1}$), and then $\omega_F \sim 100$ K. With these values, $L = 0.1$ μm (0.5, 2, and 10 μm) corresponds to $T_L \sim 10$ K (2, 0.6, and 0.1 K), respectively. The other parameter L/l can be 10–0.03, where the low value is realized in recent high-mobility samples [6] with $l \sim 67$ μm and $L = 2$ to 60 μm.

From Fig. 2 one can see that the conductance has a quantized value in the high temperature region where $F(T,L)$ is still small. However, as the temperature is lowered, the effect of impurity scattering becomes stronger [i.e., $F(T,L)$ becomes larger] and the quantization collapses. Of course if the ratio L/l is small enough, the degree of the collapse is small. On the other hand, if the ratio L/l is large, the conductance is mainly determined from $(L/2l)F(T,L)$. A recent experiment on long wires [7] ($L \sim 20$ μm, $l \sim 1$ μm) shows that the resistance $[1/G(T,L)]$ increases as the temperature is lowered and has a strong dependence on T characterized by a power law, which is qualitatively consistent with the behavior of $F(T,L)$.

Here we remark on the effect of Anderson localization. Formulas (6) and (7) are obtained by taking account of the impurity effect up to second order perturbation. The multiple impurity scattering effect which leads to the Anderson localization at low temperatures is not taken into account. The regime where this effect becomes dominant will be for short mean free paths, $L/l \gg 1$, and at low temperatures, $T \ll v_F/l = T_LL/l$. In Fig. 2 we have indicated such regions where the effect of Anderson localization is to be taken into account (mostly in the lower left part of the figure). Apparently in such samples

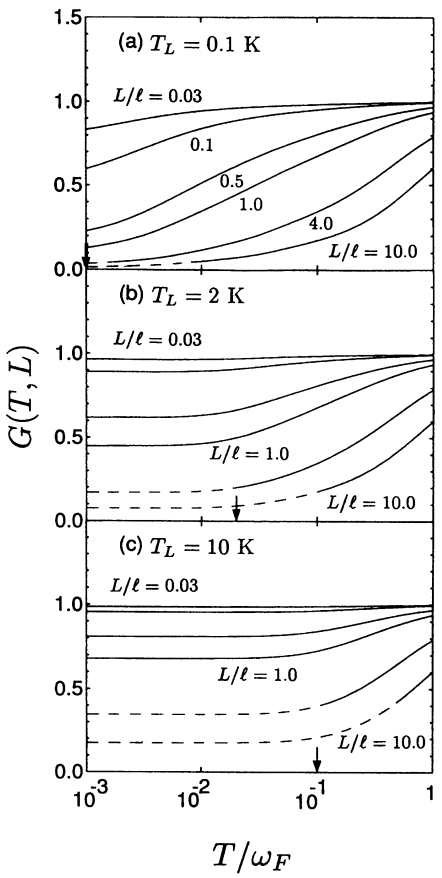

FIG. 2. The temperature dependences of the dimensionless conductance, $G(T,L)$, for several choices of L/l and $T_L = v_F/L$. Each corresponds to $L = 10$, 0.5, and 0.1 μm, respectively. The temperature is normalized by a cutoff frequency $\omega_F \sim 100$ K and the crossover temperature T_L is shown by an arrow. The dashed lines indicate the region where the Anderson localization will play important roles.

the conductance quantization is already destroyed, which is the region we are not interested in here. What we have examined in this paper is the conductance in relatively cleaner samples, where the quantized conductance and its collapse at lower temperatures can be observed. In such cleaner samples the Anderson localization starts at much lower temperatures.

We can see in Fig. 2 that the crossover temperature T_L also plays an important role. When T_L is large, $F(T,L)$ stops increasing below the temperature T_L and remains small; as a result the quantization is more or less maintained. This reflects the fact that, when the system size is small, the Tomonaga-Luttinger liquid behavior is suppressed at low temperatures because the long-distance power-law behavior, which is essential for the Tomonaga-Luttinger liquid behavior, is cut off by L.

In Fig. 3 we show the resistance, $1/G(T,L) = 1 + (L/2l)F(T,L)$, as a function of the system size for several choices of temperature. The deviation from the quantized value is almost linear in L at high temperatures (for example, $T = 10$ K). This is because $F(T,L)$ is small at

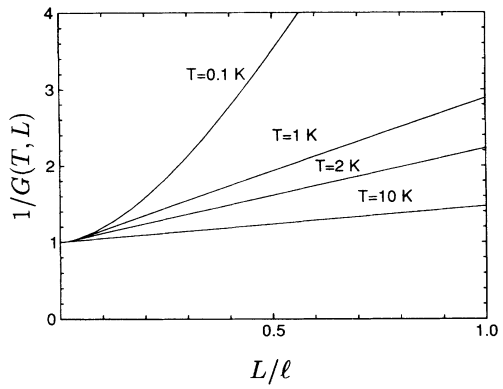

FIG. 3. The dimensionless resistance, $1/G(T,L)$, as a function of the system size for several choices of temperature. A parameter $\hbar v_F/l$ is chosen to be 0.1 K which corresponds to $l = 10$ μm. The resistance changes its behavior at a crossover length (thermal length) at which $T = T_L$ (see the text).

high temperatures and does not depend on L so much. As the temperature is lowered and for a small size, the size dependence of $F(T,L)$ becomes apparent. In the region of small L when $T_L > T$, $F(T,L)$ gradually increases as L increases, starting from a small value. Above a crossover length at which $T = T_L$, or $L_T = \hbar v_F/k_B T$ (i.e., the thermal length), $F(T,L)$ stops increasing and the deviation from the quantized value becomes proportional to L again. This crossover length is $L_T/l = 0.01$, 0.05, 0.1, and 1 for $T = 10$, 2, 1, and 0.1 K, respectively. The size dependence of the resistance observed experimentally [6] is in accordance with this behavior, but more experimental studies are needed to make a detailed comparison [21].

Recently Anderson has suggested that the spin-charge separation in the Tomonaga-Luttinger liquid leads to the linear-T resistivity [22]. In that case, instead of the replacement $-i\omega \to 2v_F/L$, we have $-i\omega \to (v_c - v_s)k_B T/\hbar v_F$ [23]. It is argued, however, that at low temperatures the Anderson localization dominates and the linear-T behavior disappears. At higher temperatures, on the other hand, it will be possible to realize the linear-T resistivity if there are some relaxation processes for spinons and holons, although the mechanism of relaxation is not yet clarified. In our formalism, we have not taken into account such an effect and instead employed the conventional Kubo formula and the density-density correlation function derived in the thermodynamic limit of the Tomonaga-Luttinger liquid.

In summary we have seen that the conductance gets suppressed below the quantized value for various values of L/l and at low temperatures as a consequence of the combined effects of randomness and mutual Coulomb interactions. Several experiments already report such a tendency of the suppression of the conductance at low gate voltage [5–7] but to our knowledge there are not yet systematic investigations of their dependences on the temperature and length of the system, which are of considerable interest.

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