Rotation Induced Prolate Spheroid above the Critical Temperature

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For temperatures above T_c (where the nuclear equilibrium shape is spherical at rotational frequency $\omega = 0$), a small rotation can generate a prolate equilibrium shape "rotating" around its symmetry axis. This prolate spheroid is caused by a quantum shell effect which exists at positive ω, but not at $\omega = 0$. For $T = 1.5$ MeV ($>T_c$), ^{188}Os is prolate for all positive spins up to $60\hbar$.

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Classical systems like the Earth, the Sun, and a liquid drop would be spherical if they were not rotating. In the seventeenth century there was a debate regarding the effect of the Earth's rotation on its shape [1]. Newton argued that a small rotation would make the shape slightly oblate, where the symmetry axis coincides with the rotation axis. On the basis of current astronomical evidence, Cassini argued for a prolate shape rotating about its symmetry axis. Geodetic measurements by Maupertuis in Lapland (1738) confirmed that the Earth's shape is oblate.

At low temperatures most atomic nuclei have non-spherical equilibrium shapes, even when they are not rotating. These deformed shapes are caused by quantum shell effects, which are amplified by nucleon-nucleon interactions. If a nucleus is not rotating, then it has a critical temperature T_c, above which the deformation producing shell effects become ineffective, and the equilibrium shape is spherical [2–4]. It has been expected that this hot nucleus would resemble a classical liquid drop. The value of T_c has a maximum for isotopes midway between magic numbers, with smaller T_c for other isotopes. For rare earth nuclei the maximum T_c is 1.85 MeV [5].

What effect does a small "rotation" have upon the shape of a spherical nucleus with $T > T_c$ [6]? Since the shell effects which produce deformation are ineffective for $T > T_c$ and $\omega = 0$, one expects this spherical nucleus to respond to a small rotation in the same manner as a classical system, i.e., as a slightly oblate spheroid rotating about its symmetry axis. This expectation is confirmed by previous calculations with the macroscopic Landau theory [7,8] and the microscopic finite-temperature Hartree-Fock-Bogoliubov cranking (FTHFBC) theory [9]. The Landau calculations provide a universal phase diagram, which predicts that any positive rotational frequency ω generates an oblate spheroid rotating about its symmetry axis, if $T > T_c(\omega)$. The previous FTHFBC calculations on ^{166}Er, ^{158}Yb, and ^{148}Sm give the same result.

This Letter reports new FTHFBC calculations for ^{188}Os. The critical temperature is 1.33 MeV. For $T = 1.5$ MeV, rotating the spherical shape produces a *prolate* spheroid rotating around its symmetry axis, for all positive spins up to $I = 60$. This result is in marked contrast with previous calculations, which predict oblate shapes above the critical temperature. For $T = 1.5$ MeV and $I = 40$ the prolate spheroid has a quadrupole deformation $\beta = 0.053$. This value of β is comparable to those obtained in previous FTHFBC calculations [9] for ^{158}Yb and ^{148}Sm which gave oblate spheroids at $T = 1.5$ MeV and $I = 40$ with $\beta = 0.031$ and 0.044, respectively, and for ^{166}Er which gave an oblate spheroid at $T_c = 1.64$ MeV and $I = 40$ with $\beta = 0.042$.

What is the explanation for this rotation induced prolate spheroid (RIPS) above the critical temperature? Is this effect peculiar to ^{188}Os, or can it be expected in other nuclei and other physical systems? For $T > T_c$ and $\omega = 0$ a nucleus has a spherical equilibrium shape. Then the finite-temperature mean field equation is

$$h|jm\rangle = e_j|jm\rangle, \tag{1}$$

where h is the spherically symmetric mean field, m implies m_z, and the single-particle energies e_j are degenerate in m. Consider a small rotation of this sphere around the z axis [6]. (All axes are equivalent.) Then the mean field equation in the rotating frame is

$$(h - \omega J_z)|jm\rangle = (e_j - \omega m)|jm\rangle. \tag{2}$$

Each orbital has an occupation probability

$$f_{jm} = \frac{1}{1 + e^{(e_j - \mu - \omega m)/T}}, \tag{3}$$

where the chemical potential μ is adjusted so that $\sum_{jm} f_{jm}$ equals the particle number. The angular momentum of the nucleus is generated by individual nucleons, not by a collective rotation [6]. The spin is

$$\langle J_z\rangle = \sum_{jm} m f_{jm}. \tag{4}$$

For $T > T_c$, the quadrupole moment of a slowly rotating nucleus is

$$Q_{20} = \sum_{jm} \langle jm|Q_{20}|jm\rangle f_{jm}. \tag{5}$$

The quadrupole moment of a nucleon harmonic oscillator orbital is

$$\langle jm|Q_{20}|jm\rangle = \frac{1}{4}\left(\frac{5}{4\pi}\right)^{1/2} b^2\left(N + \frac{3}{2}\right)\left(1 - \frac{3m^2}{j(j+1)}\right), \tag{6}$$

where the oscillator length $b = (\hbar/m\omega_0)^{1/2}$ and N is the oscillator quantum number.

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It is instructive to first consider a single j shell, i.e., $j = \frac{13}{2}$. The single-particle energies in the rotating frame $(e_j - \omega m)$ are linear in m, as shown in Fig. 1. From Eq. (6) it follows that the orbitals with the lowest and highest m values have negative quadrupole moments (oblate), whereas orbitals with intermediate m values have positive quadrupole moments (prolate). Consider zero temperature. (For even nuclei at or near the magic numbers, the ground state shape is spherical, so that $T_c = 0$.) For $T = 0$ and $\omega > 0$ orbitals with $(e_j - \omega m) < \mu$ are occupied $(f_{jm} = 1)$ and orbitals with $(e_j - \omega m) > \mu$ are empty $(f_{jm} = 0)$. Summing the quadrupole moments of the occupied orbitals, Eq. (5) gives the nuclear quadrupole moment as a function of μ, or equivalently as a function of the particle number N. Figure 2 gives $Q_{20}(N)$ for $T = 0$ and $\omega > 0$. If the j shell is less (more) than half full, then the quadrupole moment is negative (positive), and the shape is oblate (prolate). A half full shell gives a spherical shape. For any finite temperature, $Q_{20}(N)$ has the same oscillatory shape as at zero temperature, but the amplitude of $Q_{20}(N)$ is significantly reduced as T increases.

This argument shows that for temperatures above T_c (where the equilibrium shape is spherical at $\omega = 0$), a small rotation breaks the degeneracy in the quantum number m, thereby creating a residual shell effect which can generate a prolate spheroid rotating about its symmetry axis. It should be emphasized that this effect is produced solely by the rotation, and does not require any nucleon-nucleon interaction.

Next we include many j shells in the calculation of the quadrupole moment for $T > T_c$. One can numerically evaluate the sum in Eq. (5). However, it is more instructive to consider the Taylor series expansion of $Q_{20}(\omega)$ about $\omega = 0$

$$Q_{20}(\omega) = Q_{20}(0) + \left(\frac{dQ_{20}}{d\omega}\right)_{\omega=0} \omega + \frac{1}{2}\left(\frac{d^2Q_{20}}{d\omega^2}\right)_{\omega=0} \omega^2 + \cdots . \tag{7}$$

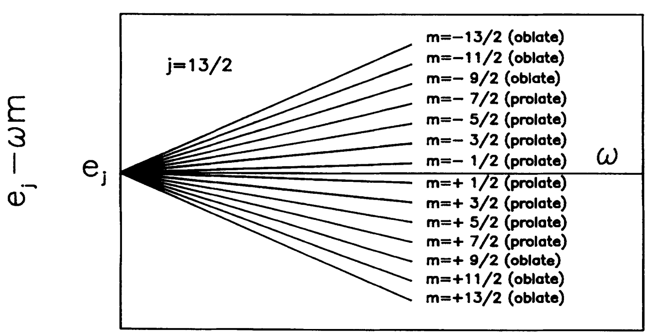

FIG. 1. Single-particle energies in the rotating frame versus the rotational frequency for a $j = \frac{13}{2}$ shell.

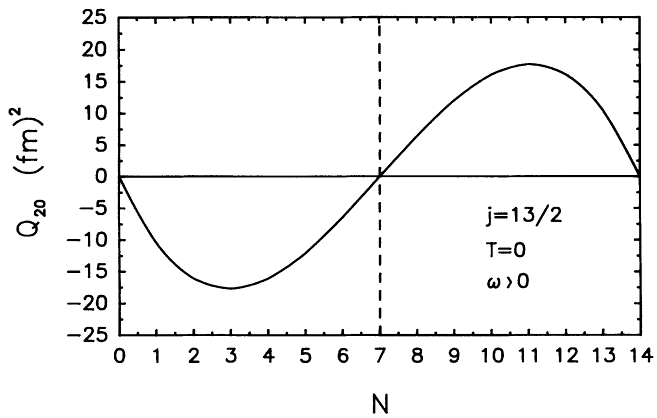

FIG. 2. The quadrupole moment versus the particle number for a $j = \frac{13}{2}$ shell.

From Eqs. (3), (5), and (6) it can be shown that

$$Q_{20}(0) = 0, \tag{8}$$

$$\left(\frac{dQ_{20}}{d\omega}\right)_{\omega=0} = 0, \tag{9}$$

$$\left(\frac{d^2Q_{20}}{d\omega^2}\right)_{\omega=0} = \frac{1}{15}\left(\frac{5}{4\pi}\right)^{1/2}\frac{b^2}{T^2}\sum_j\left(N + \frac{3}{2}\right)(2j + 1) \times \left(j^2 + j - \frac{1}{4}\right)f_j(1 - f_j)(2f_j - 1), \tag{10}$$

where f_j is the orbital occupation at $\omega = 0$,

$$f_j = \frac{1}{1 + e^{(e_j - \mu)/T}} . \tag{11}$$

This quadratic approximation for $Q_{20}(\omega)$ agrees with the exact value [Eq. (5)] within 2% at spin $I = 2$. Simply knowing the sign of $(d^2Q_{20}/d\omega^2)_{\omega=0}$ determines whether the spherical shape at $\omega = 0$ and $T > T_c$ will become oblate or prolate when it is rotated. Observe that the right-hand side of Eq. (10) does not involve ω or m. Since $j \geq \frac{1}{2}$ and $0 \leq f_j \leq 1$, every term in Eq. (10) must be nonnegative except $(2f_j - 1)$. If a j shell is more (less) than half full, i.e., $f_j > \frac{1}{2}$ $\left(f_j < \frac{1}{2}\right)$ at $\omega = 0$, then it induces a prolate (oblate) shape when rotated. From Eq. (11) it follows that if $e_j < \mu$ $(e_j > \mu)$ then rotating a j shell gives a prolate (oblate) contribution to the quadrupole moment. Each j shell gives a maximum contribution to $Q_{20}(\omega)$ when $f_j(1 - f_j)(2f_j - 1)$ has an extremum, i.e., $f_j = 0.21$ or 0.79. Equations (7)–(11) show that for a hot $(T > T_c)$ spherical nonrotating system, a few simple properties (i.e., the temperature, chemical potential, and spherical shell energies e_j) determine whether a small rotation creates an oblate or a prolate equilibrium shape rotating around its symmetry axis.

The quadratic approximation [Eq. (7)] to $Q_{20}(\omega)$ has been evaluated for the model space and spherical shell energies used by Kumar and Baranger [10]. Figure 3 shows

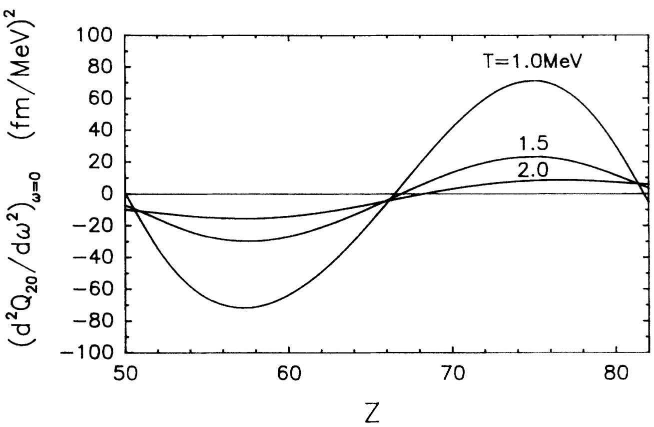

FIG. 3. Quadratic expansion coefficient of the quadrupole moment versus the proton number for rare earth nuclei at various temperatures.

the proton quadrupole moment for rare earth nuclei. The proton magic numbers are 50 and 82. For $T \geq 1.0$ MeV each j shell loses its individual oscillatory signature. Instead there is only one oscillation in $Q_{20}(Z)$ between $Z = 50$ and 82. At lower temperatures there are oscillations in $Q_{20}(Z)$ for each j shell. There are similar results for the neutron quadrupole moment between the neutron magic numbers 82 and 126. For ^{188}Os this calculation gives proton and neutron quadrupole moments which are both positive (prolate) for $T > T_c$. This calculation indicates that many other nuclei should exhibit rotation induced prolate spheroids above their critical temperatures.

Since we intend to describe rare earth nuclei with small deformations at low spins, we have used the Kumar-Baranger model space, which does not include the $j_{15/2}$ neutron shell and $i_{13/2}$ proton shell. Including these shells tends to suppress the prolate spheroids for $T = 2.0$ MeV. If these shells are included, then at $T = 1.5$ MeV proton shapes are prolate for $Z = 71-75$ and neutron shapes are prolate for $N = 107-118$; for $T = 1.0$ MeV these ranges are $Z = 67-80$ and $N = 104-122$. The nucleus ^{188}Os remains prolate at $T = 1.5$ MeV and $\omega > 0$, even when these shells are included.

Fully self-consistent FTHFBC calculations in the Kumar-Baranger model space, which include quadrupole-quadrupole nucleon-nucleon interactions, have been done for the isotope chain $^{166-180}$Er at $T = 2.0$ MeV and $I = 2$. These FTHFBC calculations have been compared to the simple expressions given here for quadrupole moments [Eqs. (5) and (7)]. For the Kumar-Baranger model space, all methods predict the same mass number ($A = 174$) for the onset of prolate spheroids above the critical temperature.

All calculations described above refer to the equilibrium shape of the nucleus, and do not include shape fluctuations. The fractional fluctuation in the mass quadrupole moment is

TABLE I. The fractional fluctuations in the quadrupole moments are δQ_{20} and δQ_{22}. The spin is I and the critical temperature is T_c.

I	T_c (MeV)	δQ_{20}	δQ_{22}
20	1.06	0.63	0.36
40	0.57	0.40	0.22
60	0.27	0.43	0.22

$$\delta Q_{2M} = \left[\frac{\langle Q_{2M}^2 \rangle - \langle Q_{2M} \rangle^2}{\langle Q_{2M} \rangle^2} \right]^{1/2}, \qquad (12)$$

where $M = 0$ and 2, and the expectation values are calculated with respect to the FTHFBC equilibrium density operator. Egido [11] has shown that Eq. (12) includes both quantum and statistical fluctuations. For each spin I, there is a different critical temperature T_c, where the FTHFBC equilibrium phase is prolate noncollective rotation for $T \geq T_c$ [12]. Table I gives the fluctuations δQ_{2M} for several values of (I, T_c). These fractional fluctuations are all considerably less than unity. Consequently the equilibrium prolate shape is physically significant and is not washed out by the fluctuations.

In conclusion, for temperatures above T_c, the deformation producing shell effects are ineffective at $\omega = 0$ and the equilibrium shape is spherical. It had been expected that this hot nucleus would behave as a classical liquid drop, so that a small rotation of the spherical nucleus would produce an oblate equilibrium shape. However, this Letter shows that for $T > T_c$, a small rotation of the spherical nucleus unmasks a residual quantum shell effect which can generate a prolate equilibrium shape "rotating" around its symmetry axis. No nucleon-nucleon interactions are needed to create this effect. It would be interesting to search for this phenomenon in other systems, such as metallic clusters.

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[6] Of course a quantum mechanical system cannot rotate collectively around a symmetry axis. However, there can be a noncollective "rotation" about a symmetry axis, where the nuclear spin is generated by components of individual nucleon spins aligned with the symmetry axis. See Eq. (4).

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