Low Energy Expansions for Double-Pion Photoproduction

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The leading terms of a power series expansion in $\mu = M_\pi/M_N$ for the threshold amplitude of double-pion photoproduction on a nucleon are obtained by the use of an effective chiral Lagrangian. Contributions due to the $\Delta(1232)$ resonance are included.

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The past few years have seen renewed experimental and theoretical interest in electromagnetic pion production from nucleons. In particular, threshold neutral pion photoproduction from the proton has received much attention in connection with the low energy theorems (LET) based on gauge invariance and chiral symmetry. At threshold, where the $E1$ electromagnetic multipole produces an s-wave pion, the LET gives [1]

$$E_{0+}^{\mathrm{LET}}(\pi^0 p) = -\frac{e g_{\pi N}}{8\pi M_N}\left\{\mu - \frac{\mu^2}{2}(3+\kappa_p)\right\} + O(\mu^3) \simeq -2.3\times 10^{-3}/M_{\pi^+}, \tag{1}$$

where $\mu = M_\pi/M_N$, $\kappa_p = 1.793$ is the anomalous magnetic moment of the proton, e is its charge, and $g_{\pi N}$ is the pion-nucleon coupling constant. Although experiments from Saclay [2] and Mainz [3] originally indicated a strong violation of the LET this disparity subsequently disappeared by a reanalysis [4–6] of the data. The accepted experimental number for $E_{0+}^{\mathrm{LET}}(\pi^0 p)$ is $(-2.0\pm 0.2)\times 10^{-3}/M_{\pi^+}$, in excellent agreement with the LET prediction of Eq. (1). However, calculations based on chiral perturbation theory (CHPT) [7] indicate that there is an extra contribution, not contained in the LET, which is of order μ^2. It was argued that, since the anomalous magnetic moment of the nucleon is generated by pion loops, all remaining one-loop diagrams would have to be included for consistency. It was these loop diagrams which gave rise to the extra contribution mentioned above, thereby modifying the LET to

$$E_{0+}^{\mathrm{CHPT}} = E_{0+}^{\mathrm{LET}} + \frac{e g_{\pi N}}{8\pi M_N}\frac{M_N^2}{16F_\pi^2}\,\mu^2 \simeq 0.89\times 10^{-3}/M_{\pi^+}, \tag{2}$$

where the pion decay constant $F_\pi \simeq 93$ MeV. We note that the effect of the extra loop diagrams has been to change the sign of E_{0+} and generally to give a result in severe disagreement with experiment. However, as was discussed in [7], due to slow convergence the CHPT results are not amenable to an expansion in μ. In fact the total tree level plus one-loop (including counterterms) contribution gives -1.33 ± 0.09 in the same units, which demonstrates the importance of μ^3 and higher terms. This result still deviates substantially from experiment and has inspired recent efforts in estimating effects due to isospin breaking [8], although as shown by Naus [9] such effects appear not to modify the LET. Eventually two-loop effects would have to be included. The reasons for the apparent discrepancy between the LET of Eq. (1) and that of the CHPT calculations have been discussed by Scherer, Koch, and Friar [10] and by Davidson [11]. Similar arguments were given by Naus [12] who examined the effect of the Li-Pagels [13] mechanism on LET's. We remind the reader that the usual LET's for charged pion photoproduction are still valid since the loop contributions as calculated in CHPT are suppressed in comparison to the dominant Kroll-Ruderman term [14].

The situation with respect to LET's for single-pion photoproduction is sufficiently interesting to warrant broadening the discussion to double-pion photoproduction. Moreover, such an analysis is required if one is to determine the nonresonant background contributions to photoproduction and electroproduction of two pions in the resonance region. Such experiments are in progress at Mainz [15,16] and are future possibilites at CEBAF. Earlier theoretical treatments [17–19] based on current algebra and PCAC (partial conservation of axial-vector current) or chiral Lagrangians did not deal in detail with the threshold region. Most recently, Dahm and Drechsel [20] calculated the threshold tree-level Born diagrams without making explicit the low energy expansion as we do here and fail to include the contribution from the $\Delta(1232)$ resonance. Here we use a chiral Lagrangian to calculate threshold ampitudes for the various double-pion photoproduction processes depicted by

$$\gamma(k) + N(p_i) \rightarrow N(p_f) + \pi^a(q_1) + \pi^b(q_2). \tag{3}$$

The four-momentum for each particle is indicated in parentheses and the two outgoing pions have Cartesian isospin indices a and b. One can show that the $\mathcal{T}$ matrix allows a decomposition into six independent isospin amplitudes, e.g.,

$$\mathcal{T} = T^1\,\delta_{ab} + T^2\,\delta_{ab}\tau_3 + T^3\,\delta_{3a}\tau_b + T^4\,\delta_{3b}\tau_a + T^5\,i\epsilon_{abc}\tau_c + T^6\,i\epsilon_{ab3}. \tag{4}$$

Amplitudes for the various physical channels are given in terms of these isospin components, i.e., $\mathcal{T}(\gamma p \rightarrow p\pi^+\pi^-) = T^1 + T^2 - T^5 - T^6$. Furthermore, the Lorentz

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structure of each of the T^r's can be written as $T^r = \epsilon^\mu \mathcal{A}^r_\mu$, where ϵ_μ is the incident photon polarization vector. Then the amplitudes $\mathcal{A}^r_\mu$ admit of the decomposition

$$\begin{aligned}\mathcal{A}^r_\mu = & A^r_{1\,\mu} + A^r_2 \gamma_\mu + A^r_{3\,\mu} \gamma \cdot k + A^r_{4\,\mu} \gamma \cdot q_1 \\ & + A^r_{5\,\mu} \gamma \cdot k \gamma \cdot q_1 + A^r_6 \gamma_\mu \gamma \cdot q_1 + A^r_7 \gamma \cdot k \gamma_\mu \\ & + A^r_8 \gamma \cdot k \gamma_\mu \gamma \cdot q_1 ,\end{aligned} \tag{5}$$

with $A^r_{j\,\mu} = A^r_{ji} p_{i\,\mu} + A^r_{jf} p_{f\,\mu} + A^r_{j1} q_{1\,\mu}$ for $j = 1, 3, 4, 5$. For convenience, we have left out the Dirac spinors of the initial and final nucleon. The number of independent invariants is reduced to four by imposing four gauge conditions: $k \cdot A^r_1 = 0$, $A^r_2 + k \cdot A^r_3 = 0$, $k \cdot A^r_4 = 0$, and $A^r_6 + k \cdot A^r_5 = 0$. At threshold, the amplitude takes a simple form in the γN center-of-mass frame,

$$\begin{aligned}\mathcal{M}_{fi} = \mathcal{A}_\mu \varepsilon^\mu = \frac{-i}{N_i} \{ & A_2 - M_\pi A_6 + (E_i + M_N + k) A_7 \\ & + (E_i + M_N - k) M_\pi A_8 \} \boldsymbol{\sigma} \cdot (\boldsymbol{k} \times \boldsymbol{\epsilon}),\end{aligned} \tag{6}$$

where $N_i = \sqrt{2M_N(E_i + M_N)}$ with E_i the target nucleon energy, $k = |\boldsymbol{k}|$ is the photon energy, and A_j is the appropriate combination of the A^r_j's. We note that the threshold amplitude is of magnetic dipole character unlike the case of single-pion photoproduction where it is electric dipole ($\boldsymbol{\sigma} \cdot \boldsymbol{\epsilon}$). Summing over spins and polarizations, the square of the threshold matrix element reduces to

$$\begin{aligned}\sum_{\text{spins},\epsilon} |\mathcal{M}_{fi}|^2 = \frac{4\mu^2}{1+2\mu} \bigg| & A_2 - M_\pi A_6 + \frac{2(1+\mu)}{\mu} M_\pi A_7 \\ & + \frac{2(1+\mu)}{\mu(1+2\mu)} M_\pi^2 A_8 \bigg|^2 ,\end{aligned} \tag{7}$$

where we have used $k = 2M_\pi(1+\mu)/(1+2\mu)$ and $E_i + M_N = 2M_N(1+\mu)^2/(1+2\mu)$. Finally, the differential cross section may be expressed in the form

$$d\sigma = \frac{\mathcal{S}}{16kW} \frac{q_1^3 q_2 d\Omega_1 d\Omega_2 d\omega_2}{q_1^2(W - \omega_2) + \omega_1 \boldsymbol{q}_1 \cdot \boldsymbol{q}_2} X, \tag{8}$$

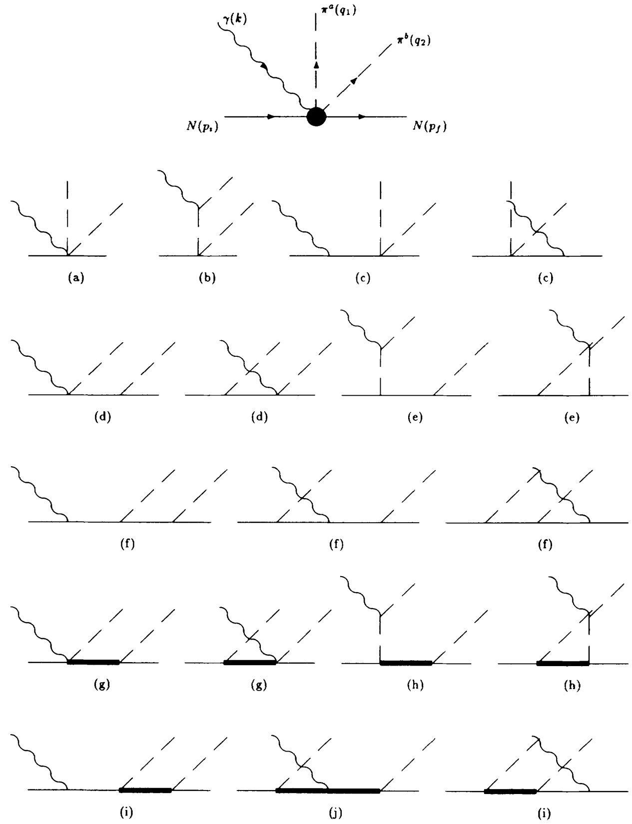

FIG. 1. Tree-level contributions to $\gamma N \to \pi\pi N$ reaction. Born diagrams (a)–(f) and Δ diagrams (g)–(j).

with

$$X = \frac{\alpha M_N^2}{(2\pi)^4} \sum_{\text{spins},\epsilon} |\mathcal{M}_{fi}|^2, \tag{9}$$

and $\alpha = e^2/4\pi$. Here $\mathcal{S}$ is a statistical factor and $W = \sqrt{s}$ the center-of-mass energy. Our amplitudes are calculated in tree level by the use of a chiral effective Lagrangian [21,22] which includes pseudovector (PV) pion-nucleon coupling:

$$\begin{aligned}\mathcal{L}_N = \overline{\psi} \bigg[& \frac{g_{\pi N}}{2M_N} \gamma_\mu \gamma_5 \tau_i \partial^\mu \phi_i - \frac{1}{4F_\pi^2} \gamma_\mu \varepsilon_{ijk} \tau_i \phi_j \partial^\mu \phi_k + \frac{e g_{\pi N}}{2M_N} \gamma_\mu \gamma_5 \varepsilon_{3ij} \tau_i \phi_j A^\mu - \frac{e}{4F_\pi^2} \gamma_\mu (\tau_i \delta_{3j} - \tau_3 \delta_{ij}) \phi_i \phi_j A^\mu \\ & - \frac{e}{2} (1 + \tau_3) \gamma_\mu A^\mu + \frac{e}{4M_N} (k_s + k_v \tau_3) \sigma_{\mu\nu} F^{\mu\nu} \bigg] \psi + e\varepsilon_{i3j} \phi_i \partial_\mu \phi_j A^\mu .\end{aligned} \tag{10}$$

Here $F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu$ is the electromagnetic field tensor, ψ is the nucleon field, and $\boldsymbol{\phi}$ is the pion field. The second and third terms in the Lagrangian are the Weinberg (pion-nucleon scattering) [21] and Kroll-Ruderman (single-pion photoproduction) [14] contact terms, respectively. The fourth term arises from minimal substitution in the Weinberg term and is unique to double-pion photoproduction. The relevant diagrams at the tree level are illustrated in Fig. 1. They can be divided into two sets of diagrams, (a)–(c) and (d)–(f), which are separately gauge invariant. At threshold, only diagrams (a), (d), and (f) are nonvanishing. Table I displays numerical values for each diagram: from this table one observes that the anomalous magnetic moment contributions are only important for photoproduction of two neutral pions. In fact, as in the case of $\gamma n \to \pi^0 n$, it is only these terms which drive the threshold amplitude for $\gamma n \to \pi^0 \pi^0 n$.

TABLE I. Numerical values for the threshold amplitude [expression in the curly braces of Eq. (6)] from diagrams depicted in Fig. 1. E and M denote charge and anomalous magnetic moment couplings at the γNN vertex, respectively. Λ is the off-shell parameter at the $\pi N\Delta$ vertex. The units are GeV^{-2}.

	a	d	$f(E)$	$f(M)$	g	i	j
$p\pi^+\pi^-$	57.8	−214.6	−14.4	−0.2	$38.3+21.6\,\Lambda+64.3\,\Lambda^2$	$1,8-8.4\,\Lambda-12.0\,\Lambda^2$	$5.7+11.1\,\Lambda-4.1\,\Lambda^2$
$n\pi^+\pi^0$	−40.9	151.7	−10.2	4.2	$-27.1-15.3\,\Lambda-45.5\,\Lambda^2$	0	$-1.3-2.6\,\Lambda+1.0\,\Lambda^2$
$p\pi^0\pi^0$	0	0	−28.8	5.7	0	$1.8-8.4\,\Lambda-12.0\,\Lambda^2$	$3.8+7.4\,\Lambda-2.7\,\Lambda^2$
$n\pi^+\pi^-$	−57.8	214.6	−14.4	−0.2	$-38.3-21.6\,\Lambda-64.3\,\Lambda^2$	0	$-1.9-3.7\,\Lambda+1.4\,\Lambda^2$
$p\pi^-\pi^0$	−40.9	151.7	−10.2	4.2	$-27.1-15.3\,\Lambda-45.5\,\Lambda^2$	0	$-1.3-2.6\,\Lambda+1.0\,\Lambda^2$
$n\pi^0\pi^0$	0	0	0	−6.1	0	0	0

We note that at threshold the dominant diagram is (d) and not the two-pion contact term (a).

One would expect the $\Delta(1232)$ contribution to play a major role in two-pion production, especially away from threshold. We include this effect by considering the most general form of spin-3/2 interaction Lagrangian and propagator as described in [23,24]

$$\mathcal{L}_\Delta = e\,\overline{\chi}_\alpha\, J^{\alpha\beta}_\mu\, T_3^\Delta\, \chi_\beta\, A^\mu + \frac{g_{\pi N\Delta}}{M_{\pi^+}}\,\overline{\chi}^\nu\, \Theta_{\nu\mu}(\Lambda) T_i\, \psi\, \partial^\mu \phi_i + \frac{e g_{\pi N\Delta}}{M_{\pi^+}}\,\overline{\chi}^\nu\, \Theta_{\nu\mu}(\Lambda)\, \varepsilon_{3ij} T_i\, \psi\, A^\mu\, \phi_j + \text{H.c.}, \tag{11}$$

where χ_μ represents the vector spinor field of the delta, T_i and T_3^Δ are isospin transition matrices, and

$$J^{\alpha\beta}_\mu = g^\alpha_\mu \gamma^\beta - g^\beta_\mu \gamma^\alpha - g^{\alpha\beta}\gamma_\mu + \gamma^\alpha\gamma^\beta\gamma_\mu,$$

$$\Theta_{\alpha\beta}(\Lambda) = g_{\alpha\beta} + \Lambda\gamma_\alpha\gamma_\beta. \tag{12}$$

We use the parameter values [25] $g_{\pi N\Delta} = 2.16$ and $M_\Delta = 1231.7$ MeV. The remaining Δ parameter, the off-shell parameter Λ, is not well established so that our results in Table I will depend explicitly on it. Analytic expressions for the threshold delta contributions are too lengthy to include here. Instead we give in Table I only the numerical values in terms of Λ.

The main results of our analysis can be divided into two parts: (1) the leading order contributions (tree-level diagrams) from the PV effective Lagrangian, and (2) the contributions from the intermediate state resonance $\Delta(1232)$.

It is well known that this model gives LET's for single-pion photoproduction which are in excellent agreement with experiment. With this model we have computed threshold expansions for all physical channels in double-pion photoproduction from a nucleon. Denoting the content of the curly braces in Eq. (6) by $M_{1/2}$, we summarize our results for the nonresonant diagrams as follows:

$$M_{1/2}(\gamma p \to p\pi^+\pi^-) = -2\left(\frac{g_{\pi N}}{2M_N}\right)^2 \left\{\left(2-\frac{1}{g_A^2}\right) + 2\,\mu - \frac{1}{4}[9+3\,(\kappa_p+\kappa_n)]\,\mu^2\right\} + O(\mu^3), \tag{13}$$

$$M_{1/2}(\gamma p \to n\pi^+\pi^0) = \sqrt{2}\left(\frac{g_{\pi N}}{2M_N}\right)^2 \left\{\left(2-\frac{1}{g_A^2}\right) - \frac{1}{4}[7-3\,(\kappa_p-\kappa_n)]\,\mu^2\right\} + O(\mu^3), \tag{14}$$

$$M_{1/2}(\gamma p \to p\pi^0\pi^0) = -4\left(\frac{g_{\pi N}}{2M_N}\right)^2 \left\{\mu - \frac{1}{4}[1+3\,\kappa_p]\,\mu^2\right\} + O(\mu^3), \tag{15}$$

$$M_{1/2}(\gamma n \to n\pi^+\pi^-) = 2\left(\frac{g_{\pi N}}{2M_N}\right)^2 \left\{\left(2-\frac{1}{g_A^2}\right) - \frac{1}{4}[7-3\,(\kappa_p+\kappa_n)]\,\mu^2\right\} + O(\mu^3), \tag{16}$$

$$M_{1/2}(\gamma n \to p\pi^-\pi^0) = \sqrt{2}\left(\frac{g_{\pi N}}{2M_N}\right)^2 \left\{\left(2-\frac{1}{g_A^2}\right) - \frac{1}{4}[7-3\,(\kappa_p-\kappa_n)]\,\mu^2\right\} + O(\mu^3), \tag{17}$$

$$M_{1/2}(\gamma n \to n\pi^0\pi^0) = 3\left(\frac{g_{\pi N}}{2M_N}\right)^2 \kappa_n\,\mu^2 + O(\mu^3), \tag{18}$$

where we used the Goldberger-Treiman $F_\pi = M_N g_A/g_{\pi N}$ with g_A being the axial-vector coupling constant. It is interesting to observe that in the chiral limit the only nonvanishing amplitudes are those for which one of the pions is charged. This leading term which arises from diagrams (a) and (b) in Fig. 1 is large in compari-

TABLE II. Contributions to the quantity X as defined by Eq. (8) in μbarn.

	Born	Born+Δ(1232)		
		$\Lambda = 0$	$\Lambda = -0.250$	$\Lambda = -0.175$
$p\pi^+\pi^-$	3.234	1.739	1.824	1.816
$n\pi^+\pi^0$	1.211	0.644	0.673	0.674
$p\pi^0\pi^0$	0.059	0.034	0.036	0.035
$n\pi^+\pi^-$	2.226	1.146	1.201	1.203
$p\pi^-\pi^0$	1.211	0.644	0.673	0.674
$n\pi^0\pi^0$	0.004	0.004	0.004	0.004

son to higher order terms in the μ expansion. This is the equivalent of the Kroll-Ruderman theorem extended to double-pion photoproduction. However, in the case of $2\pi^0$ production, chiral symmetry breaking is seen to be important, a result which holds also for single neutral pion photoproduction. The anomalous magnetic moment contribution appears at the μ^2 level and it is the leading order (in μ) contribution to $\gamma n \to n\pi^0\pi^0$. It is expected that a loop calculation in CHPT would modify these terms but not terms of lower order in μ. Hence the double-pion photoproduction process should provide an interesting example for study within the CHPT formalism.

The effect of the delta contribution is due mostly to diagram (g). For $p\pi^0\pi^0$, only diagrams (i) and (j) contribute while none of the Δ diagrams considered here contribute to $n\pi^0\pi^0$. However, we did neglect anomalous magnetic moment terms in diagrams (i) and (j) as well as diagrams corresponding to electromagnetic excitation of the Δ. The effect of the off-shell parameter present at Δ interaction vertices is shown in Table II. For this table we list the quantity X which is related to the slope of the cross section as q_1 and q_2 tend to zero. We have chosen three values of Λ to illustrate the results: $\Lambda = 0$ which corresponds to the absence of the off-shell term, $\Lambda = -1/4$ as suggested by Peccei [22], and finally, $\Lambda = -0.175$ which arises from a fit to single-pion photoproduction in the delta region by Davidson, Mukhopadhyay, and Wittman [25]. One can see that the Λ dependence is minor. Moreover, whereas one observes that although the delta excitation can be substantial it clearly does not dominate the cross section at threshold.

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