Optimized Variables for the Study of Λ_b Polarization

G. Bonvicini
CERN, CH-1211 Geneva, Switzerland

L. Randall
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 15 March 1994)

The value of the b-baryon polarization can be extracted from inclusive data at the CERN e^+e^- collider LEP with better than 10% precision based on current statistics. We present a new variable by which to measure the polarization, which is the ratio of the average electron energy to the average neutrino energy. This variable is both sensitive to polarization and insensitive to fragmentation uncertainties.

PACS numbers: 13.88.+e, 13.38.Dg

The problem of polarization transfer from the (heavy) quark produced in Z decay to the experimentally observed hadron has attracted considerable interest in the last two years [1–6]. The b-quark coupling to the Z is -94% polarized according to the standard model. At the CERN e^+e^- collider LEP b quarks are produced copiously in Z decays with an energy of ~45 GeV.

Because the Λ_b baryon, which accounts for roughly 10% of all b hadrons, retains the initial b-quark spin if produced directly, one expects a highly polarized Λ_b sample. Because higher mass b baryons will almost certainly decay strongly to $\Lambda_b\pi$ [7], possibly with large depolarization [4], some polarization information will very likely be lost.

In a native spin counting model, the Λ and Σ baryons have approximately equal probability of formation [8], with higher mass states being less likely. Thus a (47–94)% polarization of the Λ_b can be reasonably expected at LEP. A measurement of polarization will tell us the degree of fragmentation into states which retain the b-quark polarization relative to those which do not.

Three predictions for exclusive Λ_b decays have been published [1–3]. However, b baryons are best observed inclusively at LEP [9], via an excess of jets containing a hard lepton and a charge-correlated Λ_s, the latter tagging the cascade weak decay of the baryon.

Previous suggestions to measure polarization using inclusive semileptonic Λ_b decay [6] would utilize only the electron spectrum and were not sufficiently sensitive to polarization. (After completion of this work, we were sent Ref. [10], which also suggests the utility of the neutrino energy spectrum to study polarization.) Furthermore, fragmentation uncertainties could compromise the utility of the previous proposals.

Since there is abundant data (of order a few hundred tagged Λ_b's) to study, it is worthwhile investigating whether there exists a variable which is more sensitive to polarization. In this paper, we show that the variable $y = \langle E_l\rangle/\langle E_{\bar\nu}\rangle$ optimizes sensitivity while being remarkably free of theoretical uncertainties.

Experimental cuts reject events with low-energy lepton and Λ_s's, and therefore reject most of the already scarce b hadrons with low ($x \sim 0.2$) fractional energy [9]. While this induces a tiny cut dependence, it helps in two ways. First, above 0.2, the perturbative depolarizing effects are very small [11], and fragmentation and decay are essentially decoupled, a fact we will use later. Second, the relativistic β from rest frame to laboratory is close to unity. The energy in the laboratory can be expressed as follows:

$$\langle E\rangle = \langle\gamma\rangle\langle E^*\rangle + \langle\gamma\beta\rangle\langle p_L^*\rangle \simeq \langle\gamma\rangle(\langle E^*\rangle + \langle p_L^*\rangle),$$

where p_L is the longitudinal momentum, and all starred quantities are in the rest frame. The average of $\langle E^*\rangle$ and $\langle p_L^*\rangle$ depend solely on the decay process, while the quantity $\langle\gamma\rangle$ depends solely on the fragmentation, so they are independent.

The variable y which we propose is then

$$y = \frac{\langle E_l\rangle}{\langle E_{\bar\nu}\rangle} = \frac{\langle E_l^*\rangle + \langle p_{Ll}^*\rangle}{\langle E_{\bar\nu}^*\rangle + \langle p_{L\bar\nu}^*\rangle}. \quad (1)$$

For an unpolarized particle,

$$y = y_0 \equiv \frac{\langle E_l^*\rangle}{\langle E_{\bar\nu}^*\rangle}. \quad (2)$$

Notice that this ratio is dependent only on the rest frame angular distributions. It is independent of the fragmentation. We measure sensitivity by the deviation from unity of the ratio of a fully polarized quantity to the unpolarized value. That is, for any quantity x, the measure of sensitivity is $(x_{\rm pol} - x_{\rm unpol})/x_{\rm unpol}$. We will show that with this definition, the parameter y is about 5 times more sensitive to polarization than the average electron energy alone. The theoretical uncertainties are at the few percent level. These are the unknown ratio of the charm and beauty quark masses which we will see gives an effect of this order, and the derivations from the parton model, which are less than or of order a few percent. Perturbative QCD corrections which may flip the spin of the quark and alter the distributions are expected to be small.

0031-9007/94/73(3)/392(4)$06.00

This is because [5,12,13] the inclusive differential distribution from semileptonic hadron decay (appropriately averaged) is equal to the parton model (free quark) prediction up to corrections of order $(\Lambda_{\rm QCD}/m_q)^2$ which should be no more than a few percent. Notice that by studying the *inclusive* spectrum and focusing on the lepton system, we do not have the uncertainties due to the poorly known Isgur-Wise function which one has in the study of exclusive decays [1–3].

So the rate for

$$b \to c l \bar{\nu} \tag{3}$$

is proportional to (the p_x here are 4-vetors)

$$d\Gamma \propto (p_c \cdot p_l)(p_b \cdot p_{\bar{\nu}} - m_b s_b \cdot p_{\bar{\nu}}) d\Phi ,$$

where s_b is the spin of the decaying b quark and $d\Phi$ is the phase space factor. In the unpolarized case, the spin-dependent term goes to zero.

With the definitions

$$x_l = 2E_l^*/m_b \tag{4}$$

$$x_{\bar{\nu}} = 2E_{\bar{\nu}}^*/m_b , \tag{5}$$

$$\epsilon = (m_c/m_b)^2 , \tag{6}$$

$$f(\epsilon) = 1 - 8\epsilon + 8\epsilon^3 - \epsilon^4 - 12\epsilon^2 \ln \epsilon, \tag{7}$$

the inclusive differential decay distribution in the b-quark rest frame is

$$\begin{aligned} \frac{1}{\Gamma}\frac{d^2\Gamma}{dx_l d\cos\theta_l} = {} & \frac{1}{f(\epsilon)} \frac{x_l^2(1-\epsilon-x_l)^2}{(1-x_l)^3} \\ & \times \{(1-x_l)(3-2x_l) + \epsilon(3-x_l) \\ & - \cos\theta_l\,[(1-x_l)(1-2x_l) \\ & - \epsilon(1+x_l)]\}. \end{aligned} \tag{8}$$

Here we have defined the θ_l as the angle of the charged lepton with respect to the direction *opposite* that of the spin in anticipation of the application to a decaying left handed b quark, where this will be the boost direction. The inclusive neutrino differential decay distribution is predicted to be

$$\begin{aligned} & \frac{1}{\Gamma}\frac{d^2\Gamma}{dx_{\bar{\nu}} d\cos\theta_{\bar{\nu}}} \\ & \quad = \frac{6}{f(\epsilon)} \frac{x_{\bar{\nu}}^2(1-x_{\bar{\nu}}-\epsilon)^2}{1-x_{\bar{\nu}}}(1-\cos\theta_{\bar{\nu}}). \end{aligned} \tag{9}$$

The angle $\theta_{\bar{\nu}}$ was defined with the same convention as that above for θ_l.

We study only average quantities. The distributions in the laboratory are greatly broadened by the fragmentation function and the b-hadron decay. It can be proven that the average of each observable is the most sensitive to small differences independent of the precise knowledge of the fragmentation function and decay. The average energies and longitudinal momenta in the fully polarized b-quark rest frame for 100% polarization ($P = -1$) are

$$\begin{aligned} \langle E_{\bar{\nu}}^* \rangle &= \frac{m_b}{f(\epsilon)}\Big(\frac{3}{10} - 3\epsilon - 2\epsilon^2 + 6\epsilon^3 \\ &\qquad - \frac{3\epsilon^4}{2} + \frac{\epsilon^5}{5} - 6\epsilon^2 \ln\epsilon\Big), \\ \langle E_l^* \rangle &= \frac{m_b}{f(\epsilon)}\Big(\frac{7}{20} - \frac{15\epsilon}{4} - 6\epsilon^2 + 10\epsilon^3 \\ &\qquad - \frac{3\epsilon^4}{4} + \frac{3\epsilon^5}{20} - 9\epsilon^2\ln\epsilon - 3\epsilon^3\ln\epsilon\Big), \\ \langle p_{L\bar{\nu}}^* \rangle &= \frac{m_b}{f(\epsilon)}\Big(-\frac{1}{10} + \epsilon + \frac{2}{3}\epsilon^2 - 2\epsilon^3 \\ &\qquad + \frac{\epsilon^4}{2} - \frac{1}{15}\epsilon^5 + 2\epsilon^2\ln\epsilon\Big), \\ \langle p_{Ll}^* \rangle &= \frac{m_b}{f(\epsilon)}\Big(\frac{1}{20} - \frac{3}{4}\epsilon - 4\epsilon^2 + 4\epsilon^3 \\ &\qquad + \frac{3}{4}\epsilon^4 - \frac{\epsilon^5}{20} - 3\epsilon^2\ln\epsilon - 3\epsilon^3\ln\epsilon\Big). \end{aligned} \tag{10}$$

The average center-of-mass energies are independent of polarization, while for $P \neq -1$, the average longitudinal momenta scale with P, and go to zero in the unpolarized case. This formula is accurate up to corrections of order $(\Lambda_{\rm QCD}/m_q)^2$ and $(m_l/m_q)^2$. From this we can predict the value of y as a function of ϵ.

Comparing the four equations above, one can see that the lepton energy (in the laboratory frame) is about 7/3 times less sensitive to polarization than the neutrino energy. More important, y is about 5 times more sensitive than $\langle E_l \rangle$ alone.

For the purpose of cross-checking the experimental distributions, we construct also a variable which, unlike y, is never singular event-by-event, and can be used to check against the experimental distributions,

$$y' = \frac{E_l - E_{\bar{\nu}}}{E_l + E_{\bar{\nu}}} . \tag{11}$$

Figure 1 shows the distributions for the y' variable. Note that the average of the variable y' is nonzero even in the unpolarized case. From the figure, we see that the realistic cases, Figs. 1(b) and 1(d), are not so well distinguished, other than by their average. The detailed form of the distribution can be useful, however, for distinguishing good from bad events.

Neutrinos can be used at LEP in heavy-flavor analysis. If M_1 is the mass of the jet containing the neutrino, M_2 the mass of the recoiling jet, and $E_{\rm beam}$ the beam energy, the neutrino energy is measured as

$$E_{\bar{\nu}} = \frac{4E_{\rm beam}^2 + M_1^2 - M_2^2}{4E_{\rm beam}} - E_{\rm jet} .$$

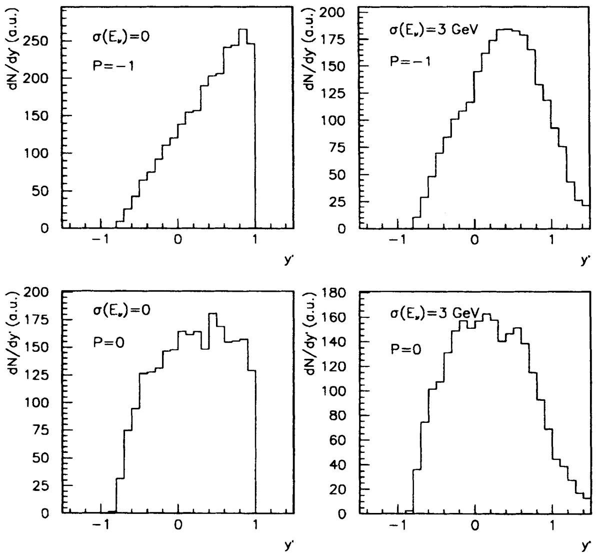

FIG. 1. The y' distribution, with minimal cuts $E_l > 3$ GeV, $(E_l + E_{\bar{\nu}}) > 1$ GeV, obtained with 10^4 Monte Carlo events using the theoretical decay spectrum. (a) $P = -1$, perfect neutrino energy resolution; (b) $P = -1, 3$ GeV neutrino energy resolution; (c) $P = 0$ perfect neutrino energy resolution; (d) $P = 0, 3$ GeV neutrino energy resolution.

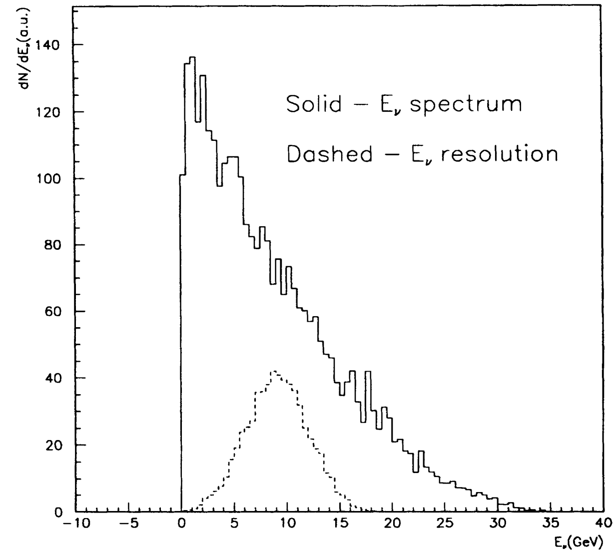

FIG. 2. Comparison of the expected neutrino spectrum from unpolarized b hadrons and neutrino energy resolution at LEP, as obtained from 10^4 Monte Carlo events. Solid: neutrino energy spectrum; dashed: expected neutrino energy resolution (3 GeV) for a 9 GeV neutrino.

Using a Monte Carlo with 10^4 events, we have modeled the expected neutrino spectrum based on the theoretical cross section. While the above formula is valid only for neutrinos collinear with the jet, the formula above exhibits a resolution of 3 GeV [14] (entirely due to the jet energy resolution of the detector), which is substantially less than the neutrino energy spectrum (Fig. 2), obtained from the most recent measurements of the b fragmentation at LEP. The y resolution is dominated by the neutrino energy resolution. Comparing 3 GeV with the neutrino energy spectrum yields an expected y resolution of (N is the number of events)

$$\sigma_y \sim 0.4/\sqrt{N},$$

where the small coefficient is a nontrivial consequence of the independence of fragmentation and should allow a better than 10% error in the determination of P with the existing data.

Practically, it will be easier to take the difference or ratio between a b-meson sample and a b-baryon tagged sample, as this eliminates a plethora of systematics. The ratio and difference of the baryon and meson samples,

$$R_y = y_{\text{meson}}/y_{\text{baron}}, \qquad D_y = y_{\text{meson}} - y_{\text{baron}} = K_D P,$$

are well calibrated with respect to polarization and can be calculated with Eq. (10). The dependence of R on the polarization is nonlinear with $R = 1.66 \pm 0.02$ for $P = -1$. D is proportional to the polarization P. With the definition above, $K_D = 0.75 \pm 0.03$. Both numbers are for ϵ ranging between 0.06 and 0.14, and before cuts. The difference from 1 to 0, respectively, is a direct measure of polarization. Neither QCD perturbative corrections, nor m_c mass uncertainties (which enter in the error purely due to kinematics), nor the kinematical approximations used above contribute more than 0.02 to the calibration error. (In Ref. [10], the QCD perturbative corrections have been calculated. Using their results, we estimate these corrections to be at most a few percent (in the variable y). Hence perturbative QCD corrections are less than the uncertainties due to the quark masses.)

As a cross check, we consider the average neutrino and lepton energies themselves rather than the ratio or difference. Here one can consider qualitative equations such as whether the spectrum of the leptons from baryons or mesons is harder. To address more quantitative questions one needs an accurate measurement of the baryon and meson fragmentation spectrum, since this affects the overall energy scale. Recall that the mean value of the hadron energy, $\langle z \rangle = \langle E \rangle / E_{\text{beam}}$, factorizes into a perturbative and nonperturbative contribution, that is,

$$\langle z \rangle = \langle z \rangle_{\text{pert}} \langle z \rangle_{\text{nonpert}} \,. \tag{12}$$

Of course the exact factorization is renormalization scale dependent. If one takes the renormalization scale to be of order the heavy quark mass, $\langle z \rangle_{\text{nonpert}}$ is intrinsically nonperturbative, but for sufficiently large m_q can be expanded as

$$\langle z \rangle_{\text{nonpert}} = 1 - a\left(\frac{\Lambda_{\text{QCD}}}{m_q}\right) + O\left(\left(\frac{\Lambda_{\text{QCD}}}{m_q}\right)^2\right), \tag{13}$$

where $a > 0$ depends on the hadron type [15,16]. The first factor, $\langle z\rangle_{\rm pert}$ is independent of the hadron type and is determined by Altarelli-Parisi evolution, but $\langle z\rangle_{\rm nonpert}$ depends on hadron type. In principle, $\langle z\rangle_{\rm nonpert}$ for the different hadron types can differ by an amount of order $\Lambda_{\rm QCD}/m_q$ which could disguise polarization effects such as a difference between the lepton spectra in the two samples.

In principle, one can use perturbative QCD and the heavy quark expansion to predict the b-quark fragmentation parameters, but with fairly large QCD uncertainties [15–17]. However, one can use directly the measured fragmentation functions of b hadrons from LEP. This is justified because the mean $\langle z\rangle$ for the meson and baryon can be shown to be very nearly the same, based on the ARGUS charmed particle results [18], just below B threshold, which are in agreement with preliminary results from CLEO [19].

Within a 2.5% error, the ARGUS results indicate that the $\langle z\rangle$ of the D, D^*, and Λ_c are the same. The measured value is about 0.65.

Therefore,

$$\Big(\langle z\rangle_{\rm baryon} - \langle z\rangle_{\rm meson}\Big) < \frac{a_b - a_m}{m_b} \le \frac{0.025}{0.65}\frac{m_c}{m_b} \approx 1.25 \times 10^{-2}. \qquad (14)$$

Here we have taken the difference between c-meson and c-baryon mean $\langle z\rangle$ to be of order 2.5% and have taken the maximum perturbative contribution to be the value which is measured (since a is positive) which is approximately 0.65. Finally, we have scaled the nonperturbative correction by the ratio of quark masses, since we have measured c quark fragmentation but need to predict that for the b quark. We conclude that a 2.5% agreement in the D system must translate conservatively into $\langle z_B\rangle \approx \langle z_{\Lambda_b}\rangle$ at the percent level which is well within the theoretical uncertainty.

With this result in hand, one can predict also that the left handedness of the Λ_b will manifest itself in a slightly higher $\langle E_l\rangle$ and a substantially lower $\langle E_{\bar\nu}\rangle$ compared to the meson sample, which is a useful cross check.

In conclusion, this paper has addressed the problem of observing polarization transfer in hadronic Z decays. It was found that sensitive, model-independent variables can be extracted from the lepton-neutrino system. The fragmentation problem was solved in two different ways. Our proposed variable y is almost free of theoretical error and increases the sensitivity to polarization by at least a factor of 5 compared to previous proposals. With existing data, the polarization should be measured to better than 10%.

Once the b polarization is measured, new information will be obtained about the relative fragmentation into the Σ_b and Λ_b baryons. Later, it should be possible to tackle c polarization, and both measurements together should provide information on the spin structure of QCD in the nonperturbative regime.

We thank Bob Cahn, Cristi Diaconu, Mike Luke, Paolo Nason, Marcello Quattromini, Nuria Rius, Mossadek Talby, and Barbara Mele for helpful discussions. We thank M. Jezabek for sending us Ref. [10]. L. R. thanks the CERN theory group for their hospitality when this work was initiated. This work is supported in part by funds provided by the U.S. Department of Energy (DOE) under Contract No. DE-AC02–76ER03069 and in part by the Texas National Research Laboratory Commission under Grant No. RGFY92C6. L. R. acknowledges support of the following: National Science Foundation Young Investigator Award, Alfred P. Sloan Foundation Research Fellowship, and Department of Energy Outstanding Junior Investigator Award.

[1] F.E. Close, J. Korner, R.J.N. Phillips, and D.J. Summers, J. Phys. G **18**, 1716 (1992).
[2] J.G. Korner and M. Kramer, Phys. Lett. B **275**, 495 (1992).
[3] T. Mannel and G.A. Schuler, Phys. Lett. B **279**, 194 (1992).
[4] A.F. Falk and M.E. Peskin, Phys. Rev. D **49**, 3320 (1994).
[5] A.V. Manohar and M.B. Wise, Phys. Rev. D **49**, 1310 (1994).
[6] G. Altarelli and B. Mele, Phys. Lett. B **299**, 345 (1993); J.F. Amundson, J.L. Rosner, M. Worah, and M.B. Wise, Phys. Rev. D **47**, 1260 (1993).
[7] W. Kwong, J. Rosner, and C. Quigg, Annu. Rev. Nucl. Sci. **37**, 325 (1987).
[8] T. Sjostrand, Comput. Phys. Commun. **39**, 347 (1986).
[9] Aleph Collaboration, D. Decamp *et al.*, Phys. Lett. B **257**, 492 (1991); L3 Collaboration, B. Adeva *et al.*, Phys. Lett. B **270**, 111 (1991); Delphi Collaboration, P. Abreu *et al.*, Z. Phys. C **53**, 567 (1991); Opal Collaboration, P.D. Acton *et al.*, Phys. Lett. B **274**, 513 (1992).
[10] A. Czarnecki, M. Jezabek, J. Korner, and J. Kuhn, Phys. Rev. Lett. (to be published).
[11] Z. Was, Acta Phys. Pol. B **18**, 1099 (1987), Eq. 19, first and third formula.
[12] J. Chay, H. Georgi, and B. Grinstein, Phys. Lett. B **247**, 399 (1990).
[13] I.G. Bigi, M. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993); B. Blok, L. Koryakh, M. Shifman, and A.I. Vainshtein, Report No. NSF-ITP-93-68 (to be published).
[14] M. Talby (private communication).
[15] R. Jaffe and L. Randall, Nucl. Phys. **B412**, 79 (1994).
[16] G. Colangelo and P. Nason, Phys. Lett. B **285**, 167 (1992).
[17] B. Mele and P. Nason, Phys. Lett. B **245**, 635 (1990); Nucl. Phys. **B357**, 409 (1991).
[18] P.C. Kim, Ph.D. thesis, ARGUS Collaboration, Tables 3.1, 3.4, and 3.5 (we use the fits in Tables 3.3 and 3.6 to correct for the first missing bin); H.C.J. Seywerd, Ph.D. thesis, ARGUS Collaboration, Table 6.2.
[19] P. Drell and D. Cassel (private communication).