QCD Corrections to Decays of Polarized Charm and Bottom Quarks

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Distributions of leptons in semileptonic Λ_c and Λ_b decays can be used as spin analyzers for the corresponding heavy quarks. QCD corrected charged lepton spectra are known for polarized up-type quarks. The analogous formulas are derived for polarized down-type quarks. These results are applied to the decays of polarized charm and bottom quarks. For charged leptons in charm decays the corrections to asymmetries are small. For bottom decays they exhibit a nontrivial dependence on the energy of the charged lepton.

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Charmed and bottom Λ baryons offer a unique opportunity to measure polarizations of heavy quarks produced in Z boson decays. Distributions of leptons from semileptonic decays of Λ_c and Λ_b can be used to this end.

It has been proposed long ago that weak hadronic [1,2] and semileptonic [3–5] decays of charmed and bottom Λ baryons can be used as spin analyzers for the corresponding heavy quarks. Recently there has been considerable progress in the theory of the inclusive semileptonic decays of heavy flavor hadrons. In the framework of heavy quark effective theory (HQET) and $1/m_Q$ expansion it has been shown that the leading order spectra for hadrons coincide with those for the decays of free heavy quarks [6]. There are no $\Lambda_{\rm QCD}/m_Q$ corrections to this result [6] and $\Lambda^2_{\rm QCD}/m_Q^2$ corrections have been calculated in [7,8] for B mesons and in [8] for polarized Λ_b and Λ_c baryons. For some decays the results are similar to those of the well-known ACCMM model [9]. In the present Letter, order-$\alpha_s(\mu)$ perturbative QCD corrections are described to semileptonic decays of polarized heavy quarks. This completes the theoretical description of inclusive lepton spectra for the decays of polarized heavy flavor hadrons. The main remaining theoretical uncertainty originates from the next order α_s^2 contribution and/or the choice of the scale μ for α_s. As usual, it can be estimated by varying the scale μ around the heavy quark mass m_Q. Since m_c is rather low this problem is more serious for charm. It can be eased when m_c and $\alpha_s(\mu)$ are fixed using unpolarized decay modes. Charged leptons are considered massless; i.e., our results can be used for electrons and muons from decays of charm and bottom quarks. Calculations are in progress for τ in the final state.

The best way to use our results for the QCD corrected distributions of leptons is to evaluate their moments. An example is the charged lepton average energy in the laboratory frame at the CERN e^+e^- collider LEP, discussed in [5]. These moments are integrals of some analytic functions related to the current-current correlator for the weak hadronic current. As explained in [6] these integrals can be evaluated along a deformed contour where perturbative QCD provides a valid description [6,8]. For the moments the contributions from some dangerous corners of phase space are integrated and do not pose problems. The total decay rate is an example. For the distributions other procedures are possible, e.g., exponentiation of the large logarithm [9] and/or smearing near the lepton energy end points [10], but we think that the moments are better.

Bottom and charmed Λ baryons from Z^0 decays can be viewed as sources of highly polarized heavy quarks. According to the standard model the degree of longitudinal polarization is fairly large, amounting to $\langle P_b\rangle = -0.94$ for b and $\langle P_c\rangle = -0.68$ for c quarks [11]. The polarizations depend weakly on the production angle. QCD corrections to the Born results have been calculated recently [12]. Polarization transfer from a heavy quark Q to the corresponding Λ_Q baryon is 100% [13], at least in the limit $m_Q \to \infty$. Thus, a large net polarization is expected for heavy quarks in samples enriched with these heavy baryons. Since semileptonic decays are under control it is possible to measure these polarizations. Many new opportunities arise, polarization studies for other decay channels among them. One of the most interesting may be studies of nonperturbative effects in fragmentation of bottom and charm quarks. Comparison of polarizations for Λ_b and Λ_c baryons can be instrumental in studying nonperturbative corrections to the spin transfer in fragmentation. This will be possible only if experimentalists are able to separate directly produced baryons from those from resonances. Assuming that this is possible and anticipating further progress in HQET as well as in perturbative QCD calculations we believe that polarization studies for b systems at LEP will offer new opportunities to test the standard model.

It is well known [14] that the QCD corrections modify significantly the lifetimes of heavy quarks. The shapes of the lepton energy distributions, however, are not strongly affected by QCD corrections [9,15–18]. This is a consequence of the fact that the correction function is fairly flat away from the energy end point. Thus, the correction can be absorbed into the overall normalization. However, in the decay of polarized quarks the polarization depen-

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dent and independent parts may be affected in different ways. In such a case the precision of the polarization measurement would be spoiled. Therefore, an evaluation of QCD corrections is necessary.

QCD corrections to the energy spectra of charged leptons in the decays of c and b quarks have been calculated in Refs. [15,16] and [15,17], respectively. In a later calculation [18] errors were traced in [9,16,17]. The results of [18] agree with Monte Carlo simulation for charm and bottom [15] and numerical results for top [19]. In [19] the formulas from [15] were used, and the accuracy achieved was good enough to observe discrepancies with [16] and [17]. After a trivial integration over the angle between the spin vector of the decaying quark and the direction of the charged lepton the results described in the present paper reproduce the results of [18] adding a further cross-check to those already provided there.

The QCD corrections to the joint angular and energy distributions of charged leptons in polarized top quark decays have been calculated in [20]. In the present Letter these results are applied and the angular dependence is evaluated for charged leptons in charm quark decays. The corresponding QCD corrections have been calculated also for the polarized bottom quark. The calculations follow closely the method outlined in Refs. [18] and [20]. In particular the notation is the same and the calculations are performed in the rest frame of the decaying quark of mass m_1. The angle between its polarization vector $\vec{s}$ and the three-momentum of the charged lepton is denoted by θ and $x = 2E_l/m_1$ is the scaled energy of the charged lepton. The variable x varies between 0 and $x_M = 1-\epsilon^2$, where $\epsilon = m_2/m_1$ and m_2 denotes the mass of the quark produced in the decay. $S \equiv |\vec{s}| = 1$ corresponds to fully polarized, $S = 0$ to unpolarized decaying quarks. The square of the invariant mass of the leptons in the final state is denoted by $y = \mu^2/m_1^2$.

In the Born approximation the double differential distribution for polarized charm quark decays is given by the following formula:

$$\frac{d\Gamma_c^{(0)}}{dx\, d\cos\theta} = \frac{G_F^2 m_c^5}{32\pi^3} \frac{x^2(x_M - x)^2}{1-x} (1 + S\cos\theta). \tag{1}$$

For the polarized bottom quark one has (see, e.g., [21])

$$\frac{d\Gamma_b^{(0)}}{dx\, d\cos\theta} = \frac{G_F^2 m_b^5}{32\pi^3} \frac{x^2(x_M - x)^2}{6(1-x)^2} \left[3 - 2x + \epsilon^2 + \frac{2\epsilon^2}{1-x} + S\cos\theta \left(1 - 2x + \epsilon^2 - \frac{2\epsilon^2}{1-x}\right)\right]. \tag{2}$$

The QCD corrections to the above formulas arise from the exchange of virtual gluons and from real gluon radiation. In the calculation of the virtual effects we follow the classic articles on muon decay [22]. The contribution of real gluon radiation is calculated exactly in the same way as in Ref. [20]. The final result is free from infrared divergences and can be expressed as follows:

$$\frac{d\Gamma_{c,b}^{(1)}}{dx\, d\cos\theta} = -\frac{G_F^2 m_{c,b}^5}{32\pi^3} \frac{2\alpha_s}{3\pi} \int_0^{y_m} dy\, (F_1^{c,b} + S\cos\theta J_1^{c,b}), \tag{3}$$

where $y_m = x(x_M - x)/(1-x)$. The lengthy expressions for the coefficient functions $F_1^{c,b}$ can be found in [18], and even more lengthy ones for J_1^c can be extracted from [20]. (The formulas for the neutrino spectrum in up-type quark decay describe the charged lepton spectrum for a down-type quark.) The coefficient functions J_1^b as well as technical details of the calculation for the polarized b quark will be given in a separate publication [23]. After numerical integration the correction to the double differential distribution can be cast into the following form:

$$\frac{d\Gamma_{c,b}^{(1)}}{dx\, d\cos\theta} = \frac{d\Gamma_{c,b}^{(0)}}{dx} \left(-\frac{2\alpha_s}{3\pi}\right) \frac{1}{2} \left[G^{c,b}(x) + S\cos\theta\, G_s^{c,b}(x)\right]. \tag{4}$$

The distributions $d\Gamma_{c,b}^{(0)}/dx$ are well known and follow trivially from Eqs. (1) and (2). The functions $G^{c,b}$ and $G_s^{c,b}$ are shown in Figs. 1 and 2 for charm and bottom, respectively.

For charm the numerical values $\epsilon = 0$, 0.1, and 0.3 are adopted to bracket the range of mass values conceivable for Cabibbo allowed and suppressed decays. For bottom the values $\epsilon = 0$ and 0.35 have been used corresponding to $b \to u$ and $b \to c$ transitions. The functions $G^{c,b}$ have been given in [18]. They are plotted in Figs. 1(a) and 2(a) for completeness. In Figs. 1(b) and 2(b) the corresponding results are shown for the functions $G_s^{c,b}$.

Both $G^{c,b}$ and $G_s^{c,b}$ exhibit the characteristic end point singularity. This singularity is closely related to the infrared divergences and consequently follows the behavior of the Born matrix element; cf. the discussion in [9,18]. Thus the end point singularities will cancel in the ratio relevant for the correction to the asymmetry parameter discussed below. Away from the end point the QCD corrections to the angular dependent and independent parts are typically of the order of 20%. However, for charm

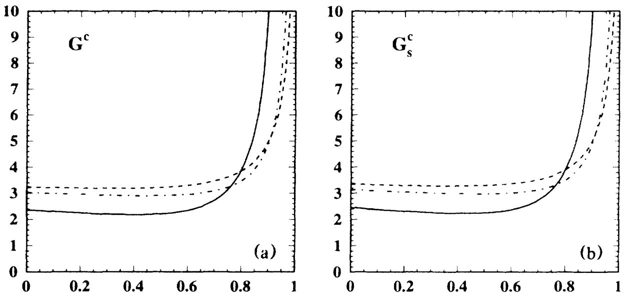

FIG. 1. The correction functions (a) G^c and (b) G^c_s for charm quark: ϵ=0.3, 0.1, and 0.0 (solid, dash-dotted, and dashed lines).

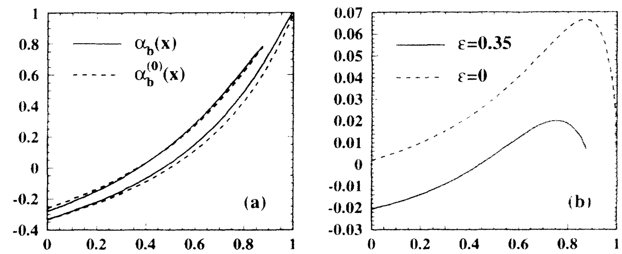

FIG. 3. The asymmetry functions for bottom quark, α_s=0.2, ϵ=0 and 0.35: (a) QCD corrected $\alpha_b(x)$ (solid line) and Born $\alpha_b^{(0)}(x)$ (dashed line); (b) $\alpha_b(x) - \alpha_b^{(0)}(x)$ for the transitions $b \to c$ ($\epsilon = 0.35$) (solid line) and $b \to u$ ($\epsilon = 0.0$) (dashed line).

decays as well as for $b \to c$ transitions they also cancel to a large extent in the ratio. This cancellation, which we believe to be a nontrivial and important result of the present calculation, has been also observed in the case of top quark decays [20].

In analogy to the description of muon decay we define the asymmetries $\alpha_{c,b}(x)$ which characterize the angular distributions of leptons in the semileptonic c and b quark decays:

$$\frac{d\Gamma_{c,b}}{dx\, d\cos\theta} = \frac{d\Gamma_{c,b}}{dx} \left[1 \pm \alpha_{c,b}(x)\cos\theta\right]/2. \tag{5}$$

The $\pm$ sign in the definition (5) corresponds to the weak isospin $I_3 = \pm 1/2$ of the decaying quark. (The minus sign for b decays follows the convention for muon decays, see, e.g., Ref. [24].) In the Born approximation the asymmetry function for charm is independent of x:

$$\alpha_c^{(0)}(x) = S. \tag{6}$$

The corresponding asymmetry function for bottom is given by

$$\alpha_b^{(0)}(x) = -S\,\frac{(1-x)(1-2x+\epsilon^2) - 2\epsilon^2}{(1-x)(3-2x+\epsilon^2) + 2\epsilon^2}. \tag{7}$$

In the following we assume that the decaying quark is completely polarized, i.e., $S = 1$. The QCD correction to $\alpha_c(x)$ is small. For $\alpha_s = 0.3$ and $\epsilon \leq 0.3$ it decreases the Born value $\alpha_c^{(0)}(x) = 1$ by less than 1% . The case of b quark is more complicated. The QCD corrected asymmetry function $\alpha_b(x)$ [see solid lines in Fig. 3(a)] and its Born approximation $\alpha_b^{(0)}(x)$ (dashed lines) are nontrivial functions of x. The correction to the asymmetry $\alpha_b(x) - \alpha_b^{(0)}(x)$ [see Fig. 3(b)] is important in the region where the Born asymmetry $\alpha_b^{(0)}(x)$ is small. For $\epsilon = 0$ this correction is non-negligible in a much broader range of x. However, in the most interesting case of $b \to c$ transitions ($\epsilon = 0.35$) the effect of the QCD correction to the asymmetry function $\alpha_b(x)$ is fairly small.

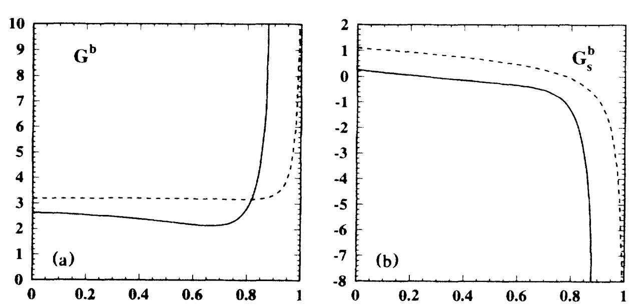

FIG. 2. The correction functions (a) G^b and (b) G^b_s for bottom quark: ϵ=0.35 (solid) and ϵ=0.0 (dashed).

The degree of polarization of charmed quarks from Z decays is smaller than for bottom quarks. On the other hand the original polarization of c quarks is reflected in the experimentally accessible angular distributions of charged leptons practically without any loss. This is not the case for bottom quarks. The angular asymmetry for charged leptons from b decays is smaller than 1 and even changes its sign as a function of the charged lepton energy. Hence both charm and bottom modes are worth pursuing. In contrast to the charged leptons the double differential angular and energy distributions of neutrinos reflect the original polarization of bottom quarks without any loss of information. Thus, the use of neutrinos as spin analyzers may be worth further experimental considerations.

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Note added.—(1) Recently much simpler expressions have been obtained [23] for the functions $F_1^{c,b}(x,y)$ and $J_1^{c,b}(x,y)$. (2) After this paper was first written, our suggestion was confirmed and further developed in [25], that in addition to charged leptons neutrinos can also be useful in polarization studies.

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