Electroweak String Configurations with Baryon Number

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In the context of electroweak strings, the baryon number anomaly equation may be reinterpreted as a conservation law for baryon number minus helicity. Since the helicity is a sum of the link and twist numbers, linked or twisted loops of electroweak string carry baryon number. We evaluate the change in the baryon number obtained by delinking loops of electroweak Z string and show that twisted electroweak string segments may be regarded as extended sphalerons. We also suggest an alternative scenario for electroweak baryogenesis.

Over the past few years, there has been renewed interest in the study of classical solutions in the standard model of the electroweak interactions. Vortex solutions [1–4] are of particular interest and there is indication [5–7] that they may be the building blocks for other solutions, such as the sphaleron [2,8]. The sphaleron, in turn, is of crucial interest to the study of baryon number violating processes and to the possibility of baryogenesis during the cosmological electroweak phase transition [9,10]. The point of this paper is to show that electroweak string configurations can carry baryon number and play a (sphaleronlike) role in baryon number changing processes.

The starting point for our analysis is the anomaly equation

$$\partial_\mu j_B^\mu = \frac{N_F}{32\pi^2}[-g^2 W_{\mu\nu}^a \tilde{W}^{a\mu\nu} + g'^2 Y_{\mu\nu}\tilde{Y}^{\mu\nu}], \quad (1)$$

in the usual notation (see Ref. [9], for example). The right-hand side of (1) is a total divergence and so the equation can be integrated. If we assume that the baryonic flux through the surface of the three volume of interest vanishes, the result relates the change in the baryon number within the volume, Q_B, to the Chern-Simons numbers of the fields:

$$\Delta Q_B = N_F \Delta(N_{CS} - n_{CS}), \quad (2)$$

where the $SU(2)_L$ Chern-Simons number is

$$N_{CS} \equiv \frac{g^2}{32\pi^2} \int d^3x \epsilon_{ijk}\left[W^{aij} W^{ak} \right.$$
$$\left. -\frac{g}{3} \epsilon_{abc} W^{ai} W^{bj} W^{ck} \right] \quad (3)$$

and the $U(1)_Y$ Chern-Simons number is

$$n_{CS} \equiv \frac{g'^2}{32\pi^2} \int d^3x \epsilon_{ijk}[Y^{ij} Y^k] . \quad (4)$$

The Δ in (2) means that the difference is to be taken between initial and final configurations; i, j, k are spatial indices and a, b, c are group indices.

We will be interested in initial and final field configurations in which $W_\mu^1 = 0 = W_\mu^2$. With this simplification, and the transformation

$$Z_j = \cos\theta_w W_j^3 - \sin\theta_w Y_j,$$
$$A_j = \sin\theta_w W_j^3 + \cos\theta_w Y_j, \quad (5)$$

Eq. (2) gives

$$\Delta Q_B = \Delta\left[N_F \frac{\alpha^2}{32\pi^2} \int d^3x \left\{ \cos(2\theta_w)\mathbf{B}_Z \cdot \mathbf{Z} \right.\right.$$
$$\left.\left. +\frac{1}{2}\sin(2\theta_w)(\mathbf{B}_Z \cdot \mathbf{A} + \mathbf{B}_A \cdot \mathbf{Z}) \right\} \right], \quad (6)$$

where $\tan\theta_w = g'/g$, $\alpha = \sqrt{g^2 + g'^2}$, $\mathbf{B}$ denotes the magnetic field, and the subscripts denote the gauge field for which the magnetic field is to be evaluated.

The terms on the right-hand side have a simple interpretation in terms of helicity [11]. The helicity associated with the Z field is

$$H_Z = \int d^3x \, \mathbf{B}_Z \cdot \mathbf{Z}. \quad (7)$$

If we think in terms of flux tubes of Z magnetic field, H_Z measures the sum of the link and twist number of these tubes [12]:

$$H_Z = L_Z + T_Z. \quad (8)$$

For a pair of untwisted Z flux tubes [13] that are linked once as shown in Fig. 1(a) the helicity is

$$H_Z = 2\Phi_Z^2, \quad (9)$$

where Φ_Z is the magnetic flux in each of the two tubes. Note that the helicity is positive for the strings shown in

Fig. 1(a). If we reversed the direction of the flux in one of the loops, the magnitude of H_Z would be the same but the sign would change. For the Z string, we also know that

$$\Phi_Z = 4\pi/\alpha \tag{10}$$

and so Eq. (6) (ignoring the Δ sign for now) yields

$$\mathcal{N}_{CS}\,(\text{in}) = N_F \cos(2\theta_w). \tag{11}$$

where $\mathcal{N}_{CS}$ denotes the Chern-Simons number of the configuration.

Next consider the operation of delinking the loops shown in Fig. 1. The first step is to let the loops self-intersect and intercommute. This process preserves helicity [14,15] as the linking of the loops in Fig. 1(a) changes to a twist of the loop in Fig. 1(b). The twisted loop in Fig. 1(b) can be broken since the Z string is not topological. The result is a Z-string segment that is twisted by 2π and has a monopole (m) at one end and an antimonopole ($\bar{m}$) at the other [Fig. 1(c)]. The field configurations of m and $\bar{m}$ have been written by Nambu [1,16]:

$$\Phi_m = \begin{pmatrix} \cos(\theta_m/2) \\ \sin(\theta_m/2)e^{i\phi} \end{pmatrix}, \quad \Phi_{\bar{m}} = \begin{pmatrix} \sin(\theta_{\bar{m}}/2) \\ \cos(\theta_{\bar{m}}/2)e^{i\phi} \end{pmatrix}. \tag{12}$$

where θ_m and ϕ are spherical coordinates centered on m (and similarly for $\bar{m}$). In the $\theta_w = 0$ case, the gauge fields are given by

$$A_\mu = -i\frac{2}{g}\,\partial_\mu U\,U^{-1}. \tag{13}$$

where U is a 2×2 matrix defined by $\Phi = U(1,0)^T$. (The case of nonzero θ_w requires a more elaborate expression for the gauge field [1] and is treated later.) The important thing to note is that $-\Phi_m$ and $\Phi_{\bar{m}}$ are also valid Higgs field configurations for m and $\bar{m}$ and the gauge fields are unaffected by the overall minus sign. The next step in the delinking process is shown in Fig. 1(d), where the Z-string segment of Fig. 1(c) is broken in the middle with m and $\bar{m}$ in the configurations $-\Phi_m$ and $\Phi_{\bar{m}}$, respectively. Now we have two Z-string segments, each one twisted by π [Fig. 1(d)]. The next step is to perform rotations ($\phi \rightarrow \phi \pm \pi$) of the newly created poles so that the twists are undone [Fig. 1(e)]. The Higgs configurations obtained this way are called $-\Phi_m\,(-\pi)$ and $-\Phi_{\bar{m}}\,(\pi)$ in Fig. 1(e). Now we can write down the Higgs configuration for one of the segments in Fig. 1(e) as

$$\Phi_{m\bar{m}} = \begin{pmatrix} \cos\left(\dfrac{\theta_m + \theta_{\bar{m}}}{2}\right) \\ \sin\left(\dfrac{\theta_m + \theta_{\bar{m}}}{2}\right)e^{i\phi} \end{pmatrix}, \tag{14}$$
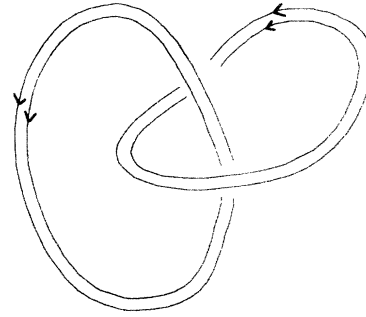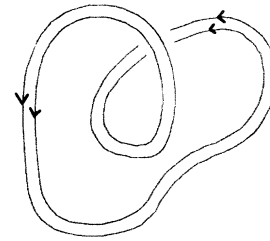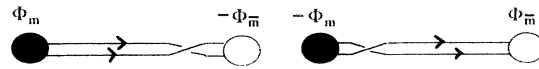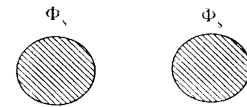

FIG. 1. A linked pair of loops and a delinking process (see text for details). The parallel curves represent Z magnetic field lines for the same string.

where θ_m and $\theta_{\bar{m}}$ are measured from the z axis defined by the line from $\bar{m}$ to m. This configuration has the right properties since it reduces to Φ_m when $\theta_{\bar{m}} \rightarrow 0$ and it reduces to $-\Phi_{\bar{m}}(\pi)$ when $\theta_m \rightarrow \pi$. It also exhibits an untwisted Z-string segment between the poles. Now consider what happens when the segment shrinks to zero

size. Then $\theta_m \rightarrow \theta_{\bar{m}} = \theta$, and we are left with

$$\Phi_{m\bar{m}} = \begin{pmatrix} \cos\theta \\ \sin\theta e^{i\phi} \end{pmatrix}. \tag{15}$$

But this, with the gauge fields given by (13), is precisely the Higgs field configuration for a sphaleron (Φ_s) in the $\theta_w \rightarrow 0$ limit [17]. Therefore the two segments of Fig. 1(e) shrink down to two sphalerons [Fig. 1(f)], each carrying $\mathcal{N}_{CS} = N_F/2(\theta_w = 0)$—which can then decay into different vacuua with $\mathcal{N}_{CS} = 0$, N_F, or $2N_F$. If we consider decay into the $\mathcal{N}_{CS} = 0$ vacuum, we must conclude that the linked loops of Fig. 1(a) (with unit linkage) carry baryon number

$$Q_B = N_F \cos(2\theta_w). \tag{16}$$

Note that we have explicitly shown the equivalence of twisted electroweak strings with the sphaleron only in the $\theta_w = 0$ case; however, the result in (16) is true for any θ_w because $\mathcal{N}_{CS}$ (in) is given by (11) and the loops can always decay into the vacuum with zero Chern-Simons number.

For general θ_w, one can construct Z-string segments with arbitrary twist and, equivalently, with arbitrary Chern-Simons number. The secret to this construction is the realization that the configurations $e^{i\gamma}\Phi_m$ and $e^{i\gamma}\Phi_{\bar{m}}$ also describe a monopole and an antimonopole for any constant γ since these are simply global gauge transformations of Φ_m and $\Phi_{\bar{m}}$. [For $\theta_w = 0$, we had chosen $\gamma = \pi$ to arrive at (14).] Now consider the Higgs field configuration

$$\Phi_{m\bar{m}}(\gamma) = \begin{pmatrix} \sin(\theta_m/2)\sin(\theta_{\bar{m}}/2)e^{i\gamma} + \cos(\theta_m/2)\cos(\theta_{\bar{m}}/2) \\ \sin(\theta_m/2)\cos(\theta_{\bar{m}}/2)e^{i\phi} - \cos(\theta_m/2)\sin(\theta_{\bar{m}}/2)e^{i(\phi-\gamma)} \end{pmatrix}. \tag{17}$$

This reduces to Φ_m when $\theta_{\bar{m}} \rightarrow 0$ and to $e^{i\gamma}\Phi_{\bar{m}}$ when $\theta_m \rightarrow \pi$ and, in addition, we perform the rotation $\phi \rightarrow \phi + \gamma$. So the Higgs field configuration in (17) will describe a monopole and antimonopole connected by a Z string that is twisted through an angle γ provided we choose the gauge fields suitably. The gauge fields can be written down using the general formalism developed by Nambu [1]:

$$gW_\mu^a = -\epsilon^{abc}n^b\partial_\mu n^c$$
$$\quad - i\cos^2\theta_w n^a(\Phi^\dagger\partial_\mu\Phi - \partial_\mu\Phi^\dagger\Phi), \tag{18a}$$

$$g'Y_\mu = -i\sin^2\theta_w(\Phi^\dagger\partial_\mu\Phi - \partial_\mu\Phi^\dagger\Phi), \tag{18b}$$

up to "external" electromagnetic potentials [1], and where

$$n^a \equiv \Phi^\dagger\tau^a\Phi \tag{19}$$

is a unit vector.

The configuration in (17) and (18) describes a twisted segment of string with twist angle γ. If we assume that $\gamma = 2\pi n/m$, where n and m are integers, we can join together m of these twisted segments and form a loop of Z string that is twisted by an angle $2\pi n$. The Chern-Simons numbers of this twisted loop of string are easy to calculate using (7) and (8)—it is $nN_F\cos2\theta_w$. Now, dividing by the number of segments we had joined together to form the loop, this yields the Chern-Simons number of a segment twisted by an angle γ:

$$\mathcal{N}_{CS} = \frac{\gamma}{2\pi}N_F\cos2\theta_w. \tag{20}$$

This shows that we can find string configurations with arbitrary Chern-Simons number by putting in a suitable amount of twist. In particular, if we take $\gamma = \pi/\cos2\theta_w$, the Chern-Simons number is $N_F/2$—the believed value for the sphaleron [18]. This leads us to conjecture that the string with twist $\pi/\cos2\theta_w$ will collapse ($\theta_m \rightarrow \theta_{\bar{m}} = \theta$) into the sphaleron for any Weinberg angle.

We would like to point out that the above arguments are independent of the existence and stability [19] of the Z-string solution. In our analysis we only needed *configurations* that look like linked loops of Z flux. However, if we were to consider the formation of such linked fluxes during the electroweak phase transition, the existence of Z-string solutions and the issue of their instability would become important.

It has been suggested in the past that the sphaleron might secretly be a configuration of electroweak strings [2,5–7]. Our result that linked loops of string can carry baryon number and the deformations outlined above give substance to this belief. It may also be that other knotted string configurations are equivalent to the sequence of solutions conjectured in Ref. [20] and Klinkhamer's S^* solution [21] could have an interpretation in terms of electroweak string knots with zero linkage [22] since it is known that S^* carries zero baryon number.

Next consider the possibility that electroweak strings were produced during the electroweak phase transition. When such strings form, they will be produced with some helicity. The question is: What is the helicity density?

The answer to this question is likely to be very difficult and here we will only attempt to answer a simpler question: What is the probability for getting linked loops

in a phase transition where U(1) is broken completely and topological strings are produced? It is simplest to think in terms of a simulation with the algorithm used in Ref. [23]. In this algorithm, one throws down one of three U(1) phases called 0,1,2 (corresponding to the angles $0, 2\pi/3, 4\pi/3$) on the vertices of a lattice. Now in traversing the links of the lattice, the phase increases or decreases and this corresponds to traversing some segment of the U(1) circle. If on traversing the perimeter of a plaquette and returning to the starting point, we traverse an angle 2π on the U(1) circle, we must have a string passing through the plaquette. In this way, one can construct a whole network of strings on a lattice. (For details see Ref. [23].)

This algorithm allows for the possibility of forming linked loops of string. We have evaluated the probability for a small loop to be threaded by another string within this algorithm and find it to be $\sim 10^{-4}$. Therefore the helicity per unit volume is $\sim 10^{-4}/\xi^3$ where ξ is the correlation length at the time of string formation.

Finally, we wish to point out that the above results suggest a scenario for the generation of baryon number in the early Universe [24]. Suppose that a network of electroweak strings was produced at the electroweak phase transition which then survived long enough to fall out of thermal equilibrium. The network would consist of loops and segments of electroweak string of which some would be linked and twisted. The network will evolve and the helicity will change with time. Every change in the helicity results in a baryon number change—somewhere positive, somewhere negative. Now the evolution of the system is governed by the full electroweak Lagrangian which is CP violating. The CP violating terms would favor a change of helicity in one direction over the other and hence baryon number would be produced. (Remember that the change in the baryon number not only depends on the initial helicity but also on the Chern-Simons number of the final vacuum.) If we use the U(1) string network results to estimate the number density of helicity (n_h), we have $n_h(t_i) \sim 10^{-4}/\xi^3$. If the (model dependent) CP violation bias parameter that preferentially drives baryon number change in one direction is denoted by ϵ, the baryon number density produced will be $\sim 10^{-4}\epsilon/\xi^3$. For $\xi \sim T^{-1}$, the baryon to photon ratio will be $\sim 10^{-4}\epsilon$. Granting all the assumptions we have had to make, this estimate would agree with observations only in particle physics models that give $\epsilon \sim 10^{-6}$.

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