Dynamic Melting of the Vortex Lattice

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We investigate analytically and numerically the melting of the vortex lattice moving in an inhomogeneous environment under the applied current j. We predict the existence of a dynamic phase transition at some characteristic current $j = j_t$ (crystallization current) from the motion of the amorphous vortex configuration at $j < j_t$ to the motion of the vortex crystal at $j > j_t$. The crystallization current j_t exceeds essentially the critical current j_c for strongly disordered systems and diverges as temperature approaches the melting temperature T_m of the undisturbed lattice.

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A sudden drop in the resistivity in the vortex state of high temperature superconductors at a certain temperature $T_m(B)$ well below the mean field transition temperature, accompanied by pronounced non-Ohmic effects, is viewed as the hallmark of the expected [1] flux liquid to flux lattice freezing transition [2]. A sharp change in vortex transport reflects, however, a dramatic change of the effectiveness of pinning rather than marks directly the transition between the distinct thermodynamic phases. The comprehension of the freezing transition should be based, therefore, on the consideration of both dynamic and thermodynamic effects involved. Another point to be noted is that the crystalline long-range order of the static lattice is destroyed by the disorder however weak [3]. This implies that the freezing of the static vortex liquid should be a liquid to a glassy solid phase transition, and, as we show below, the freezing into a perfect lattice is possible only for a sufficiently rapidly moving lattice.

The *averaged* effect of disorder on the vortex lattice motion has been considered in [4,5] where the *averaged* pinning force was found within the first order perturbation theory with respect to disorder. In this Letter we address the effect of the *fluctuating* component of the pinning force $F_p(\mathbf{r},t)$ on dynamics of the moving vortex configuration and the melting transition. This force causes additional fluctuations of vortex lines which in the lowest order perturbation expansion over disorder resemble the fluctuations due to the thermal Langevin force. Therefore statistical properties of F_p can be characterized by the effective "shaking" temperature, $T_{\rm sh} \propto v^{-1}$. At $T < T_m$ a dynamic phase transition occurs at a certain current $j_t(T) > j_c(T)$ from the motion of a vortex crystal at $j > j_t$ to the motion of an amorphous configuration at $j < j_t$ [$j_c(T)$ is the pinning critical current]. The transition (crystallization) current diverges near T_m as $j_t \propto (T_m - T)^{-1}$. As a result the vortex liquid moving with the velocity v solidifies at $T_m(v) < T_m$, where T_m is the melting temperature of the undisturbed lattice. The possibility of the additional heating due to collisions of the vortices with the pinning centers had been proposed by Worthington *et al.* [6]. However, they assumed that the disorder heating effect grows with increasing current opposite to our result $T_{\rm sh} \propto j^{-1}$ at $j > j_c$.

Ordering of the vortex lattice at large applied currents was first demonstrated by Thorel *et al.* [7] by neutron diffraction. The jumplike transition between pinned static state and homogeneously moving lattice was later observed by Wördenweber and Kes in thick NbGe films [8]. The field dependence of the transition current separating different regimes of vortex motion in the layered superconductor $2H$-$\mathrm{NbSe_2}$ has been recently studied by Bhattacharya and Higgins [9].

We study the 2D system of pancake vortices subject to a disorder potential $U(r)$ generated by the randomly distributed pointlike pinning centers. Such systems were experimentally realized in thin films of low-temperature superconductors with large Ginzburg-Landau parameters [10,11]. Consider the regime of fast vortex motion under the large currents, $j \gg j_c$. In the zero approximation with respect to disorder, the mean vortex velocity is $v = F_{\rm ext}/\eta$, $F_{\rm ext} = (B/c)j$, and η is the friction coefficient. To investigate the *fluctuations* of the vortex motion, we find the correlation function

$$\aleph_{\alpha\alpha'}(\mathbf{r},t) = \langle F_{p\alpha}(0,0)F_{p\alpha'}(\mathbf{r},t)\rangle \tag{1}$$

of the fluctuating pinning force $\mathbf{F}_p = -\sum_\nu \delta(\mathbf{r} - \mathbf{R}_\nu(t))\nabla U(\mathbf{r} - \mathbf{v}t)$ within the first order perturbation theory. Here subscripts $\alpha, \alpha' = (x,y)$ denote the force components, and $\mathbf{R}_\nu(t)$ are the vortex coordinates. Taking the correlation function of the random potential $U(\mathbf{r})$ as $\langle U(\mathbf{r})U(\mathbf{r}')\rangle = \gamma_U f(\mathbf{r} - \mathbf{r}')$, where $f(\mathbf{r})$ drops rapidly at distances $r > r_p$ and is normalized by the condition $\int d\mathbf{r}\, f(\mathbf{r}) = 1$, we get

$$\aleph_{\alpha\alpha'}(\mathbf{r},t) = n_v\gamma_U \sum_{\nu'}\langle\delta(\mathbf{r} - \mathbf{R}_\nu(t) + \mathbf{R}_{\nu'}(0))\rangle\nabla_\alpha\nabla_{\alpha'}f(\mathbf{r} - \mathbf{v}t), \tag{2}$$

$n_v = B/\Phi_0$ is the vortex density. Further on we assume the Gaussian shape of $f(\mathbf{r})$, $f(\mathbf{r}) = \exp(-r^2/2r_p^2)/2\pi r_p^2$.

In the case of the "incoherent" fluidlike motion (in strongly disordered materials this type of motion takes

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place in a wide region above the critical velocity), one can neglect contributions with $\nu' \neq \nu$ in Eq. (2). Note that the thermal fluctuations of vortices can also be neglected if the fluctuation displacement of the vortex during the time $\approx r_p/v$ is much smaller than r_p. Then $\aleph_{\alpha\alpha'}(\mathbf{r},t)$ reduces to $\aleph_{\alpha\alpha'}(\mathbf{r},t) = n_v \gamma_U \delta(\mathbf{r}) \nabla_\alpha \nabla_{\alpha'} f(\mathbf{v}t)$. This expression resembles the correlation function of the Langevin force $\mathbf{F}_T(\mathbf{r},t)$: $\langle F_{T\alpha}(\mathbf{r},t) F_{T\alpha'}(0,0)\rangle = 2\eta T \delta_{\alpha\alpha'} \delta(\mathbf{r}) \delta(t)$, differing in the anisotropy in subscript dependence of $\aleph_{\alpha\alpha'}$. However, due to the large difference between the transversal and longitudinal rigidities of vortex configuration, its response is mainly determined by the transversal part of $\mathbf{F}_p$, and this anisotropy does not influence much the "thermal" effect of disorder. To characterize the amplitude of $\mathbf{F}_p$, one can then define the effective "shaking" temperature as

$$T_{\rm sh} = \frac{1}{4\eta} \sum_\alpha \int d\mathbf{r} \int dt\, \aleph_{\alpha\alpha}(\mathbf{r},t) = \frac{1}{4\sqrt{2\pi}} \frac{n_v \gamma_U}{F_{\rm ext} r_p^3}. \quad (3)$$

As far as random motion of the vortices is concerned, this shaking temperature should be simply added to the true temperature, giving the effective temperature of the moving vortex system $T_{\rm eff} = T + T_{\rm sh}$. The condition $T_{\rm eff}(F_{\rm ext} = F_t) = T_m$, where $T_m = 0.62 C_{66} a^2/4\pi$ is the melting temperature of the ideal vortex lattice [12,13] and C_{66} is the shear modulus, defines the "crystallization" force F_t above which the system recovers a crystalline order. At $T = 0$

$$F_t = \frac{n_v \gamma_U}{4\sqrt{2\pi} T_m r_p^3}, \quad (4)$$

valid for the strong disorder where $\gamma_U^{1/2}/r_p > T_m$ and F_t exceeds considerably the critical force $F_c \approx 0.2 n_v \gamma_U^{1/2}/r_p^2$. In the interval $F_c < F < F_t$, the motion of the vortex medium is almost homogeneous, but the crystalline long-range order is lost. Near F_c homogeneous motion transforms into a plastic flow regime due to formation of islands of strongly pinned vortices around which flux flow continues [14,15]. F_t grows with temperature and diverges at $T \to T_m$ as

$$F_t = \frac{n_v \gamma_U}{4\sqrt{2\pi}(T_m - T) r_p^3}, \quad (5)$$

contrary to the critical force F_c which decays with temperature due to thermal fluctuations. In the case of weak disorder, the dynamic phase transition at $T = 0$ occurs slightly above F_c and again diverges near T_m according to (5). This kind of behavior of F_t has been recently observed experimentally [9].

Now we estimate the "shaking action" of the random force for the opposite limit of the crystal moving along a symmetry direction. The equation of motion for the vortex lattice, which we treat as an elastic medium is now $\eta \partial_t \mathbf{u} = (C_{11} - C_{66}) \nabla(\nabla \mathbf{u}) + C_{66} \Delta \mathbf{u} + \mathbf{F}_p(\mathbf{r},t)$, where C_{11} and C_{66} are the elastic moduli. This description holds until the disorder-induced lattice deformations are small. For such a "coherent" type of motion the above Langevin analogy generally does not work because $F_p(\mathbf{r},t)$ has long-range correlations in time and space. The amplitude of the random displacement $\mathbf{u}_{\rm ran}(\mathbf{q})$ at the wave vector $\mathbf{q}$ induced by $F_p(\mathbf{r},t)$ is now given by

$$\langle |\mathbf{u}_{\rm ran}(\mathbf{q})|^2\rangle = \frac{2T_{\rm sh} q_x^2}{C_{66} q^4} \frac{\sinh(C_{66} q^2 a/\eta v)}{\cosh(C_{66} q^2 a/\eta v) - \cos(q_x a)} \quad (6)$$

(this was derived under the condition $C_{11} \gg C_{66}$ valid at $H \gg H_{c1}$). One can see that the behavior of $\mathbf{u}_{\rm ran}(\mathbf{q})$ in the most part of the 2D Brillouin zone is controlled by the relation between the typical period a/v and the phonon relaxation time $\tau_{\rm rel} = \eta/C_{66} n_v$ or, equivalently, between v and the typical velocity $v_{\rm rel} = C_{66}/\eta a$. At low velocities, $v \ll v_{\rm rel}$, the periodicity is not relevant and the shaking effect of pinning force is again characterized by the shaking temperature (3); i.e., $\langle |\mathbf{u}_{\rm ran}(\mathbf{q})|^2\rangle = 2q_x^2 T_{\rm sh}/C_{66} q^4$. In the limit $v \gg v_{\rm rel}$ the q dependence of the displacement amplitude occurs to be identical to the thermal one, $\langle |\mathbf{u}_{\rm ran}(\mathbf{q})|^2\rangle \propto 1/q^2$, which means that the effect of pinning can again be described by some fictive temperature $T_{\rm sh}^{\rm coh}$ introduced by the relation $\langle |\mathbf{u}_{\rm ran}(\mathbf{q})|^2\rangle = T_{\rm sh}^{\rm coh}/C_{66} q^2$, where

$$T_{\rm sh}^{\rm coh} = \left(\frac{\sqrt{3}}{4\pi}\right)^{1/2} \frac{n_v^{3/2} \gamma_U C_{66}}{(\eta v)^2 r_p^3} \approx \frac{v_{\rm rel}}{v} T_{\rm sh}. \quad (7)$$

However, the analogy to the thermal noise should be taken with reservations since the dynamic behavior of the random displacements is very different from that of the thermal displacements. The shaking effect of the collisions is reduced for the coherently moving crystal as compared to that for incoherently moving vortices. Note that for a strongly disordered system the "crystallization" velocity $v_t = F_t/\eta$ exceeds $v_{\rm rel}$: $v_t/v_{\rm rel} \approx 2a\gamma_U/(4\pi T_m)^2 r_p^3 > 1$. This observation combined with Eq. (7) implies that the incoherent motion cannot transform smoothly into the coherent one and that the transition between these two states should be the *first* order transition. Moving crystal can be easily "superheated"; i.e., it can move coherently at forces smaller than F_t, or, equivalently, at temperatures higher than the melting temperature at given force $T_m(F)$. This should lead to hysteretic behavior of the nonlinear resistivity in the vicinity of F_t $[T_m(F)]$. On the other hand, any preexisting defects in the moving crystal strongly facilitate its transformation into the amorphous state due to the local increase of the effective shaking temperature. This suggests that the transition in a real system occurs at $F = F_t$ and that the vortex crystal moving under force $F < F_t$ is in a metastable state.

The very similar description applies to the moving system of 3D vortex lines. In particular, for a layered superconductor with uncorrelated disorder in the layers, the effective shaking temperature is again given by Eq. (3) where $F_{\rm ext}$ has to be replaced by the external force per layer. This means that the transition force between crystallike and fluidlike motion has the same behavior as in the 2D case, i.e., diverges as temperature approaches the melting point. We expect also that the transition line F_t can merge the line of the critical current $F_c(T)$ at some finite temperature if the disorder is weak. An important feature of the 3D case is the existence of the distinct liquid-glass transition for the static lattice. The influence of lattice motion on this transition will be discussed in detail elsewhere.

To verify the above predictions, we performed direct numerical simulations of the motion of N_v point vortices in inhomogeneous environment by numerical solution of the set of Langevin equations for the vortex coordinates $\mathbf{R}_i$:

$$\frac{d\mathbf{R}_i}{dt} = \sum_{i\neq j} \mathbf{f}_v(\mathbf{R}_i - \mathbf{R}_j) - \sum_s \nabla_i U_p(\mathbf{R}_i - \mathbf{r}_s) + \mathbf{f}_{\mathrm{Ti}}(t) + \mathbf{f}_{\mathrm{ext}} . \qquad (8)$$

Here $\mathbf{f}_v(\mathbf{r})$ is the intervortex interaction force, $U_p(\mathbf{r})$ is the potential of the pinning center, $\mathbf{r}_s$ are the coordinates of the pinning centers, $\mathbf{f}_{\mathrm{ext}}$ is the external force, and $\mathbf{f}_{\mathrm{Ti}}(t)$ is the Langevin force. The quantity $2s\Phi_0^2/(4\pi\lambda)^2$ and the lattice spacing a_0 of the ideal lattice are taken as units of energy and length . We model the interaction force by $f_v(r) = (1/r)(1 - r^2/r_{\mathrm{cut}}^2)^2$ with $r_{\mathrm{cut}} = 3.33$. The disorder potential is modeled by N_p randomly distributed Gaussian potentials $U_{ps}(r) = A\exp(-|\mathbf{r} - \mathbf{r}_s|^2/r_p^2)$. One then finds the parameter γ_U reads as $\gamma_U = n_p(\pi A r_p^2)^2$, where n_p is the concentration of pins. Simulations have been performed for two sets of parameters: $A = 0.006$, $N_p = 10^4$, $N_v = 400$, $r_p = 0.2$ (*set 1*) and $A = 0.004$, $N_p = 2.5 \times 10^4$, $N_v = 900$, and $r_p = 0.2$ (*set 2*). The *set 1* corresponds to a "dirty" system and individual pinning. The *set 2* describes a moderately disordered system which falls into the regime of the plastic-elastic crossover [15,16].

We solve the Langevin equations (8) at different values of $\mathbf{f}_{\mathrm{ext}}$ and T and calculate the average concentration of vortices n_{def} with coordination numbers not equal to 6 using the triangulation procedure [17]. We use n_{def} as a parameter characterizing the degree of irregularity in the vortex configuration. The melting point corresponds to a sharp increase of n_{def} in the lattice.

Simulations of the clean system give melting transition at $T_m = 0.007$ in agreement with the value of melting point for particles with an infinite logarithmic interaction [13]. To investigate the dynamic melting for the dirty system, we calculate the dependencies of the defect concentration n_{def} and the average velocity v on f_{ext} at different T (Fig. 1). We start from high velocities where the motion of the lattice is homogeneous and the effect of quenched disorder is small. At any temperature below T_m we find a very well-defined force f_t at which the concentration of defects increases sharply, and then saturates. The phase diagram obtained from simulations is shown in Fig. 2. At $T = 0$ the crystallization force $f_t \approx 0.034$ is approximately 2.4 times larger than the critical force $f_c \approx 0.014$. The found value of f_t is in good agreement with the one estimated from Eq. (4), $f_t = F_t/n_v$, $f_t \approx 0.029$. At low temperatures the transition is accompanied by the increase of the dynamic friction force. This kind of correlation between the shapes

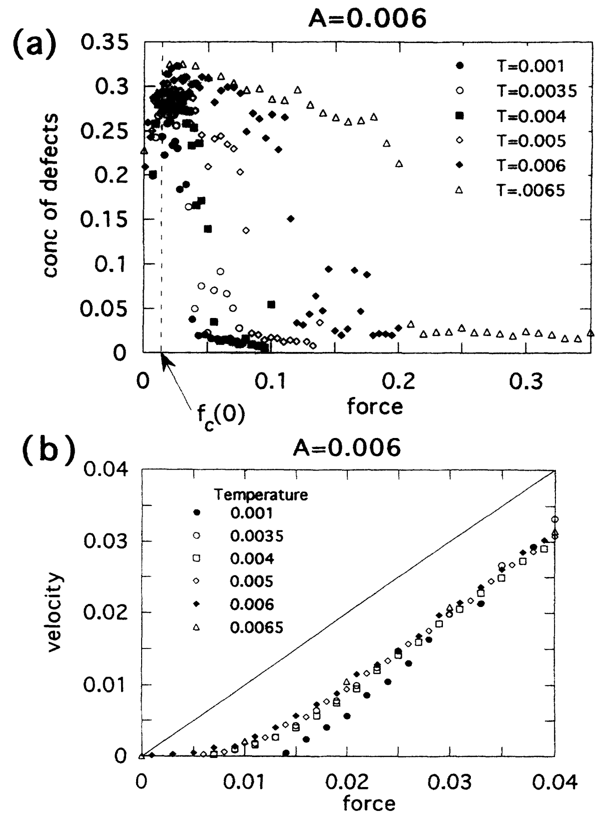

FIG. 1. The dependencies of defect concentration (a) and velocity (b) on the external force at different temperatures for the strong pinning system ($A = 0.006$).

of I-V curves and disorder in the vortex medium was studied in previous simulations [18] and was used to detect the transition point experimentally [9]. However, we observe that the anomaly in the I-V curve is quickly smeared out with an increase of temperature. This means that in general I-V characteristics cannot be used for the reliable detection of the transition.

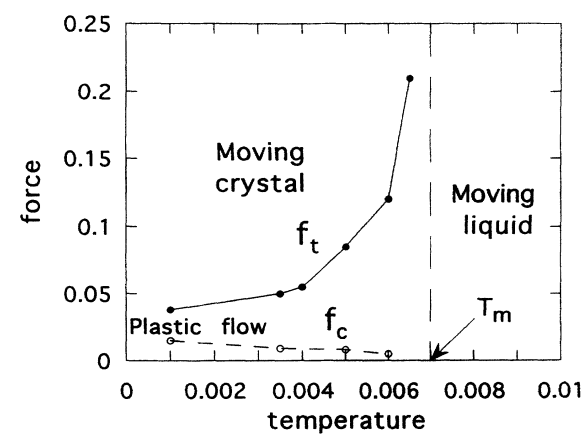

FIG. 2 Numerically obtained phase diagram for the strong pinning system. The line $f_t(T)$ corresponds to the dynamic phase transition between the moving crystal and the incoherent fluidlike motion. $f_c(T)$ is the critical force corresponding to the crossover between flux flow and thermally assisted creep.

Interestingly, at small velocities $n_{\mathrm{def}}(f)$ decreases again. This new ordering starts at $f = f_c$, i.e., in the region where quasihomogeneous flow of the amorphous phase transforms into a thermally activated motion. Therefore, the onset of viscous motion at the critical force is accompanied by a considerable increase of stresses in the lattice which increases its defectiveness. This phenomenon was also observed experimentally [7].

In Fig. 3 we show the force dependencies of n_{def} and v at low temperature for the system at the plastic-elastic crossover (*set 2*). For this system we found a more pronounced increase of the friction force at the transition point. Moreover, if the system has enough time to arrange itself into a coherently moving perfect crystal at large velocities, then such type of motion is preserved well below the transition point, and the transformation to the amorphous state is accompanied by a jump in I-V curve. No such hysteresis is observed if nonequilibrium defects exist in the moving crystal.

In conclusion, we have shown that the fluctuating component of the pinning force may effect the driven vortex similar to the thermal Langevin force. We have proposed the existence of the dynamic crystallization transition at the distinct transition force and found the dynamic phase diagram of the vortex system.

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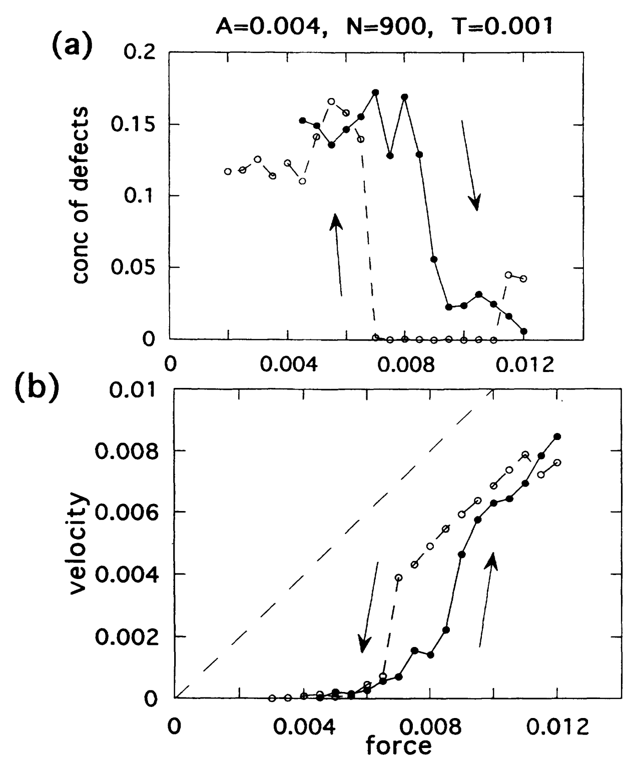

FIG. 3. The hysteretic dependencies of defect concentration (a) and velocity (b) on external force at small temperature for the system at the "plastic-elastic" crossover ($A = 0.004$). "Superheating" of the coherently moving crystal below the transition force is observed. For this system the dynamic transition is accompanied by a sharp increase of the friction force.

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