Composite-Fermion Picture for the Spin-Wave Excitation in the Fractional Quantum Hall System

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The spin-wave excitation mode from the spin-polarized ground state in the fractional quantum Hall system at the Landau level filling of $\nu = 1/(2m + 1)$ is shown, by comparing with the exact numerical result for finite systems, to be accurately described, for wavelengths exceeding the magnetic length, in terms of the composite-fermion picture for the spin-wave (magnon) theory formulated in the spherical geometry. This indicates that the composite-fermion approach extends to excited states and also provides the spin stiffness in terms of exchange interaction with transformed interparticle interactions.

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Although it is more than a decade since the fractional quantum Hall (FQH) effect [1] was discovered and identified as a novel prototype of a strongly correlated electron system, a new way of looking at the problem is now being explored in terms of the composite-particle picture, or the Chern-Simons (CS) gauge field theory. Specifically, much attention is focused on the composite-fermion (CF) picture, which was first proposed by Jain [2] as an alternative to the Haldane-Halperin hierarchy, while the bosonic CS theory is also developed [3].

The CF picture asserts that a quantum Hall liquid of electrons in an external magnetic field B_ν, corresponding to the Landau level filling of $\nu = p/(2mp + 1)$ (m, p: integers), is equivalent, in a mean-field sense, to a liquid of composite fermions each carrying $2m$ flux quanta, immersed in the effective magnetic field $B_{\text{eff}} \equiv B - B_{1/2m}$, which corresponds to $\nu = p$. This was shown by Lopez and Fradkin in terms of the fermionic CS gauge theory, and the response functions and excitation spectra were also calculated within the random-phase approximation (RPA) [4].

The fermionic CS theory has been worked out by Halperin, Lee, and Read to show that the (spinless) system can indeed be thought of as a "Fermi liquid" of composite fermions near $\nu = \frac{1}{2}$ [5]. This was confirmed by some recent experiments on Fermi-liquid-like transport properties around $\nu = \frac{1}{2}$ [6]. Specifically, the transport around $\nu = \frac{1}{2}$ does indeed resemble the usual Shubnikov–de Haas oscillation with the energy gap $\Delta \propto B_{\text{eff}}$ [7], for which the effective mass of the composite fermion was subsequently estimated [8]. These results are rather surprising, since there is no *a priori* reason why the CF picture, starting from a mean-field approximation, should be a good approximation.

There are a number of attempts at justifying the CF mapping. These include a study of the overlap between the exact eigenstates and the projected CF trial wave functions [9], where the overlaps turned out to be reduced for the states having similar energies with the same quantum number, or Chern-Simons calculations in the modified RPA for the excitation spectra, which account for the effective-mass renormalization within a Landau-Fermi-liquid theory approach [10]. Although the modified RPA is expected to be accurate in the large $p(\nu \rightarrow 1/2m)$ limit, where the motion of composite fermions becomes semiclassical, the approximation contains the effective mass of a composite fermion as a free parameter and only gives a qualitatively correct description of the excitation spectra for small p.

Now, we consider that a true test for a many-body theory is its ability to describe excited states. For the filling factor of $\nu = 1/(2m + 1)$, the ground state (Laughlin's quantum liquid) is fully spin polarized due to the electron correlation even when the Zeeman energy is neglected. Then the low-lying excitations are the collective spin-wave mode that restores the broken SU(2) symmetry (rotational symmetry in spin space). Thus a prominent question is: Can we extend the CF picture to ferromagnetic spin-wave excitation spectra? In this Letter we present the first parameter-free calculation of an entire collective mode (spin wave), based on the CF interpretation. This does not need procedures such as the lowest-Landau-level projection of the wave function and is characterized by a *quantitative* agreement with the exact numerical results.

While the usual practice in studying the FQH system is to ignore the spin, the spin degrees of freedom are in fact fascinating, since a most drastic effect of electron correlation in the ordinary correlated system (e.g., the Hubbard model) is the spin state, which is thought of as a manifestation of the exchange interaction in the appropriate basis. This usually involves the competition of kinetic and interaction energies, which causes singular behaviors in the spin-wave dispersion when the ground state is spin polarized due to electron correlation [11]. In the FQH system, by contrast, the intra-Landau-level excitations such as the spin wave are dominated solely by the Coulomb interaction due to the quenched kinetic energy. The spin wave has recently been experimentally observed with inelastic light scattering at $\nu = \frac{1}{3}$ [12].

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Here the spin-wave (magnon) theory for the FQH system is formulated in the spherical geometry to exploit the rotational symmetry [13]. We shall show that the spin-wave excitation spectrum for $\nu = 1/(2m+1)$, when compared with the exact spin-wave excitation numerically obtained from the diagonalization of finite FQH systems, is explained surprisingly accurately by a CF picture with a mean-field approximation unless the wavelength is smaller than the magnetic length. One quantitative outcome is that this approach enables us to look at the spin stiffness in the FQH system in terms of an exchange interaction with transformed interparticle interactions.

We start from the spin-wave excitation spectrum for the flat geometry at $\nu = 1$, which has been exactly given by Kallin and Halperin [14] as

$$\omega(k) - g\mu_B B = \frac{1}{2\pi}\int_0^\infty dq\, qV(q)[1 - J_0(kql^2)]e^{-(ql)^2/2}, \tag{1}$$

where $V(q) = 2\pi e^2/\epsilon q$ is the Fourier transform of the Coulomb interaction with ϵ being the dielectric constant, $J_0(z)$ Bessel's function, and $l = \sqrt{c\hbar/eB}$ the magnetic length.

If we now turn to the spherical geometry, everything can be written in terms of angular momentum quantum numbers. As we stereographically map the flat system to the spherical one, the translational symmetry is translated into the rotational symmetry, so that the wave number k and total angular momentum L are related by $k = L/R$, where R is the radius of the sphere. When the total magnetic flux going out of the sphere is $2S$ (an integer due to Dirac's condition) times the flux quantum, the radius of the sphere becomes $R = l\sqrt{S}$, while the relation to ν is $2S = \nu^{-1}N$, which is an integer with N the number of electrons. There the creation operator for the magnon with angular momentum L and its z component M is given by $C_{LM}^\dagger = \sum_{j,k}(-1)^{S-k}\langle S, j; S, -k|LM\rangle a_{j\downarrow}^\dagger a_{k\uparrow}$, where $\langle S, j; S, -k|LM\rangle$ is the Clebsch-Gordan coefficient and $a_{j\sigma}^\dagger$ is the creation operator for jth spatial orbit with spin σ.

The spin-wave excitation spectrum ω_L, which is now a function of L, in the spherical geometry requires a tedious calculation, but the result is rather elegant in that it is given in terms of Wigner's $6j$ symbol, familiar in nuclear physics. Namely we have for $\nu = 1$

$$\omega_L - g\mu_B B = \sum_{J=0}^{2S}(2J+1)(-1)^{2S-J}V_J \times\left[\frac{1}{2S+1} - (-1)^{2S-J}\begin{Bmatrix} S & S & L \\ S & S & J\end{Bmatrix}\right], \tag{2}$$

where $L\ (= 0, 1, \ldots, 2S)$ is the total angular momentum, $\{{}^{SSL}_{SSJ}\}$ is the $6j$ symbol arising from products of four Clebsch-Gordan coefficients, and V_J is the Haldane pseudopotential for the relative angular momentum $2S - J$ [15]. Since $\{{}^{SSL}_{SSJ}\} = (-1)^{2S-J}/(2S+1)$, the excitation energy at $k = L/R = 0$ satisfies the relation $\omega_0 = g\mu_B B$ for any interparticle interaction ($\{V_J\}$), which guarantees Larmor's theorem.

Now we apply Jain's composite-fermion picture by attaching $2m$ flux quanta extracted from the external field to each electron in the $\nu = 1/(2m+1)$ state. When the $2m$ flux quanta are attached, the relative angular momentum n between two electrons translates into the relative angular momentum $n - 2m$ between composite fermions [16], since an extra phase factor $e^{2m\theta i}$ appears in the wave function of the relative motion (as often described in terms of the CS theory in the literature). Since the field is reduced to $B_{\rm eff} = B/(2m+1)$, the magnetic length l changes into $\tilde{l} = \sqrt{2m+1}\, l$ in a mean-field sense.

We are now in position to formulate the spin-wave excitation at $\nu = 1/(2m+1)$. The advantage of working in the spherical geometry is that the transformation into the CF picture is simply given by

$$2\tilde{S} = \frac{2S}{2m+1} = N - 1, \qquad \frac{\tilde{V}_{2\tilde{S}-(n-2m)}}{e^2/\epsilon\tilde{l}} = \frac{V_{2S-n}}{e^2/\epsilon l}, \tag{3}$$

where V_{2S-n} is the pseudopotential with the relative angular momentum $n = 2S - J$. If we plug this transformation into the "$6j$ formula," Eq. (2), we finally arrive at the desired expression for the spin-wave excitation spectrum for $\nu = 1/(2m+1)$ in the CF picture as

$$\omega_L - g\mu_B B = \sum_{J=0}^{2\tilde{S}}(2J+1)(-1)^{2\tilde{S}-J}\tilde{V}_J \times\left[\frac{1}{2\tilde{S}+1} - (-1)^{2\tilde{S}-J}\begin{Bmatrix} \tilde{S} & \tilde{S} & L \\ \tilde{S} & \tilde{S} & J\end{Bmatrix}\right], \tag{4}$$

where the range of L is now reduced to $L = 0, 1, \ldots, 2\tilde{S}$.

We now turn to the numerical results for the low-lying excitations in the spherical geometry for a six-electron system at $\nu = \frac{1}{3}$ and for a five-electron system at $\nu = \frac{1}{5}$ in Fig. 1, where we show both the one-spin-flip excitations (with the change in the total spin $\Delta S_{\rm tot} = -1$) and the charge ($\Delta S_{\rm tot} = 0$) excitations. In the spin-wave mode, which is the lowest branch in the excitation, we immediately recognize that the states in these finite systems appear only in the range $0 \le L \le 2\tilde{S} = N - 1$, while naively there is no reason why the states should not extend for $0 \le L \le 2S = (2m+1)(N-1)$. In fact, higher-energy spin excitations do indeed exist for larger L in Fig. 1. We attribute this truncation from $(2m+1)(N-1)$ to $N-1$ to the fact that the original system at $\nu = 1/(2m+1)$ may be mimicked by a system of composite fermions with $\nu = 1$ for the spin-wave excitation.

If we look at the spin-wave dispersion curve in Fig. 1, the prediction from the composite-fermion mean-field approximation (CFMFA), Eq. (4), exhibits an excellent agreement with the exact result up to the wave number $k \sim l^{-1}$. The exact result starts to deviate from the CFMFA for larger k, which implies that the effect of

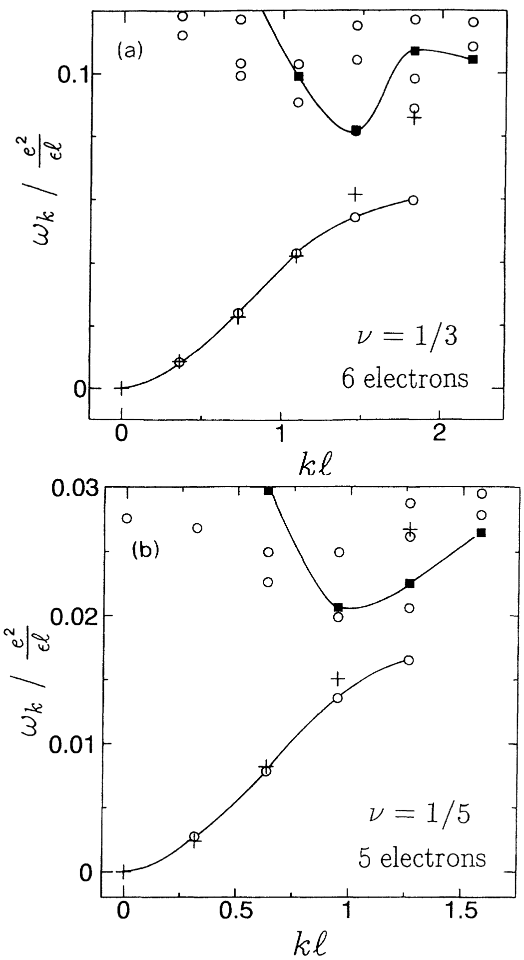

FIG. 1. The excitation spectrum for a FQH system of spin $\frac{1}{2}$ electrons, which comprises one-spin-flip ($\Delta S_{\mathrm{tot}} = -1$) excitations (open circles) and charge ($\Delta S_{\mathrm{tot}} = 0$) excitations (solid squares), is shown for (a) a six-electron system at $\nu = \frac{1}{3}$ and (b) a five-electron system at $\nu = \frac{1}{5}$ (note a difference in vertical scales). The result for the spin-wave excitation in the composite-fermion mean-field approximation for the same number of electrons is shown by crosses. The spin-wave and charge (magnetoroton) excitations are, respectively, connected by a curve as a guide to the eye.

fluctuations from the mean CS field becomes appreciable for large-wave-number excitations. The higher-energy spin excitations above the spin-wave mode, which have no counterparts for $\nu = 1$, should also be dominated by the fluctuations and may be associated with inter-(quasi-)Landau-level excitation of the composite fermion with spin flip.

The result that both the range of discrete modes in a finite system and their dispersion agree with the prediction from the CFMFA confirms the picture that the CF picture is valid not only for the ground state but also for the spin-wave excitation. This is the key message of the present Letter.

To look into the size dependence of the results for the spin-wave excitation, we have calculated the CFMFA result for a larger 51-electron spherical system at $\nu = \frac{1}{3}$ (Fig. 2). According to the single-mode approximation (SMA) for the spin-wave excitation spectrum for $\nu =$ odd fractions [17], the spectrum is given as

$$\omega(k) - g\mu_B B = \frac{1}{2\pi} \int_0^\infty dq\, qV(q)[1 - J_0(kql^2)] \times [1 - S(q)], \qquad (5)$$

with $S(q)$ being the static structure factor for the fully polarized odd-fraction ground state. The SMA is known to agree with numerical results for finite systems for small wave numbers within the finite-size correction [18]. Here we have plotted for comparison the SMA result for an infinite (flat) system [19] along with the exact diagonalization result for a five- (six-) electron system. We can see that all of the exact CFMFA and SMA results agree with each other for wave numbers up to $k \sim l^{-1}$. The exact result for the finite system for $k < l^{-1}$ is slightly larger than the other ones. This we consider comes partly from a finite-size correction: The Haldane pseudopotential becomes larger for finite systems than that for an infinite system (by about 5% for the present size) [15]. Thus the agreement of the CFMFA result persists for larger systems. For $k > l^{-1}$, the SMA result which is intended for long wavelengths [17], the finite-system result, and the CFMFA result start to deviate from each other.

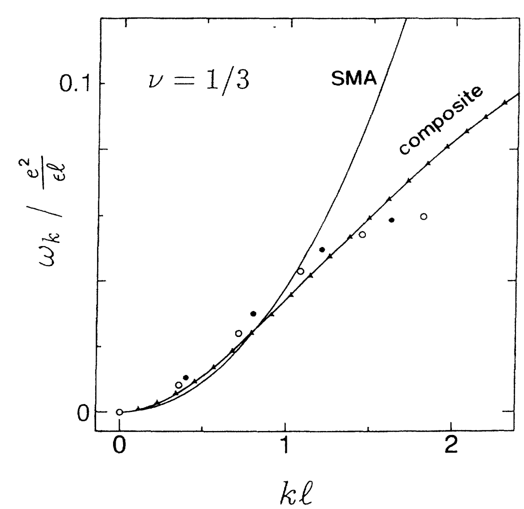

FIG. 2. The spin-wave excitation spectrum for a FQH system of spin $\frac{1}{2}$ electrons at $\nu = \frac{1}{3}$ is shown. The exact result for a five- (six-) electron system is indicated by solid (open) circles. The results for a 51-electron system in the composite-fermion mean-field approximation in the spherical geometry (solid triangles, connected by a curve as a guide to the eye) and the single-mode approximation for an infinite (flat) system (solid curve) are also indicated.

As for the stiffness of the spin wave [D in $\omega(k)/(e^2/\epsilon l) = D(kl)^2$ for small k], this can be estimated either numerically from the $6j$ formula for large systems or by transforming the electron-electron interaction via Eq. (3) in the Kallin-Halperin formula for an infinite system, Eq. (1), to have

$$V(q) = 2\pi l^2 \sum_{n=0}^{\infty} 2V_n L_n(q^2 l^2) \longrightarrow 2\pi l^2 \sum_{n=0}^{\infty} \frac{2V_{n+2m}}{\sqrt{2m+1}} L_n(q^2 l^2), \quad (6)$$

where $L_n(z)$ is Laguerre's polynomial and V_n is the pseudopotential for the relative angular momentum n [15]. The stiffness is thus expressed in terms of a peculiar "exchange interaction" for the composite fermions with the shifted relative angular momenta. Specifically, we can see the spin stiffness significantly decreases as we go from $\nu = 1$ down to $\frac{1}{3}, \frac{1}{5}, \ldots$, since $V(q)$ becomes progressively reduced. The numerical values of the spin stiffness at these fillings are of interest as an input into the effective magnetic actions due to Sondhi *et al.* [20] and Yang *et al.* [21] and will be given elsewhere [22].

We also note that, while in the SMA the effect of going from $\nu = 1$ to $\frac{1}{3}$ appears only via $S(q)$ [or equivalently the radial distribution function $g(r)$] in Eq. (5), our approach is to make the flux attachment directly affect the interparticle interaction [Eq. (6)], where an exact $g(r)$ is used in the system after the transformation [Eq. (1)].

As for the spin excitations other than one-spin flips, we can show that all the low-lying excitations at $\nu = 1$ can be entirely interpreted in terms of the multiplets of weakly interacting magnons, where the intermagnon interaction is attractive at short distances [13]. This is consistent with Rezayi's observation that the two-spin-flip mode has a lower energy than that of the one-spin-flip mode at $\nu = 1$ [23]. Sondhi *et al.* went on to discuss the many-spin flips in analogy with a Skyrmion [20]. It is an interesting problem to ask the applicability of the composite-particle picture to these multispin-flip excitations.

We also notice in Fig. 1 that the rotonlike charge-excitation mode [24] exists as well, identified as a $\Delta S_{\rm tot} = 0$ dispersion with an energy gap. Extension of the composite-particle picture to charge-excitation modes provides another interesting future problem.

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