# Instabilities of Short-Pulse Laser Propagation through Plasma Channels

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The stability of short laser pulses propagating through plasma channels is investigated theoretically. Perturbations to the laser pulse are shown to modify the ponderomotive pressure, which distorts the dielectric properties of the plasma channel. The channel perturbation then further distorts the laser pulse. A set of coupled mode equations is derived, and a matrix dispersion relation is obtained analytically for the special case of a quadratic radial density variation. As an example, the spatiotemporal growth of a pure dipole perturbation is evaluated in various parameter regimes. Mechanisms for suppressing the instability are discussed.

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The stable propagation of intense laser pulses in underdense plasmas is critical to the realization of laser wake-field accelerators. For the plasma oscillation to successfully accelerate an electron bunch it must have a high degree of coherence. Nonlinear coupling between the laser and plasma is well known to produce instabilities [1]. If the instabilities are not controlled, they may destroy the desired coherence.

The laser pulses must be focused to a small spot size in order to generate a large amplitude plasma wave, and, thereby, a high accelerating gradient [2]. The laser will, in free space, remain focused over a diffraction length (Rayleigh range) $Z_r = \omega\sigma^2/2c$, where $\omega$ is the laser frequency, $c$ the speed of light, and $\sigma$ the laser spot size at the focus. A homogeneous plasma, which has a dielectric constant $\epsilon = 1 - \omega_{p0}^2/\omega^2$, where $\omega_{p0} = 4\pi e^2 n_0/m$ is the electron plasma frequency, $-e$ the electron charge, $m$ the electron rest mass, and $n_0$ the plasma density, will only enhance the tendency of the light to diffract. To achieve a net acceleration of, say, 10 GeV, will require, with present terawatt lasers, propagation lengths of order 10–20 Rayleigh ranges. For overall efficiency reasons, the propagation lengths must be long enough for a substantial fraction of the laser energy to be converted into plasma oscillation. This will require propagation over many diffraction lengths.

Several schemes have been proposed to overcome diffraction. Relativistic guiding [3–6] relies on the energy dependence of the plasma frequency, $\omega_{p0}^2/\gamma$, where $\gamma = \sqrt{1 + \mathbf{p}\cdot\mathbf{p}/m^2c^2}$. The electron momentum $|\mathbf{p}|$ will be largest where the laser pulse is most intense; therefore the plasma frequency will be lower there, and the pulse will generate a nonlinear index of refraction which is larger at the center of the pulse than at the pulse edges. Analysis has shown [4] that, in steady state, relativistic guiding can focus the pulse whenever the total power is greater than $P_c = 16.2(\omega/\omega_{p0})^2$ GW.

For the short pulses (of order a plasma wavelength) envisioned in many wake-field accelerators, however, relativistic guiding is substantially reduced [7]. This is due to the tendency of the ponderomotive force from the front of the pulse to push plasma electrons forward and generate a density increase which balances the relativistic mass increase. The plasma frequency then has no transverse variation and cannot optically guide the laser pulse. Relativistically guided long laser pulses suffer from Raman forward and sidescatter instabilities [8,9]. The instability leads to the breakup of the pulse into small pulselets of order 1 plasma wavelength. The utility of using these pulselets themselves for acceleration is at present unclear [10].

An alternative scheme which has been investigated [11] envisions guiding the laser pulse with a plasma density channel. The channel should have a higher density on the outside than on the inside, giving it an index of refraction which decreases from the channel axis. A fixed plasma channel is analogous to an optical fiber, and its guiding properties can be similarly analyzed. The plasma channel can be used to guide short pulses and has been studied using axisymmetric models for parabolic density variation [11] and for hollow channels [12].

This Letter considers the dynamic stability of channel-guided pulses in the presence of plasma wakes. A perturbation to the guided equilibrium leads, through the ponderomotive force, to a plasma density perturbation which, in turn, couples back to the perturbed field. Thus, the plasma couples different longitudinal slices of the laser pulse.

For example, a transverse instability of channel-guided pulses occurs when the laser pulse is initially not centered on the channel axis. The underlying physics is straightforward: The off-centered laser produces a ponderomotive force with a dipole component; this causes the surrounding plasma electrons to try to follow the laser pulse. Thus the shape of the channel is distorted, and its guiding properties are perturbed. The result is that the perturbation has exponential spatiotemporal growth. In general, there will be a coupling between higher-order multipoles, so that the back of the laser pulse will widen.

Raman scattering of laser pulses in plasma has been investigated by numerous authors. The stability of relativistically guided pulses was studied [8,9] using a mildly

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relativistic fluid model. A dispersion relation was found assuming that the perpendicular wave number is large compared to the inverse of the transverse spot size. In this Letter the plasma channel externally guides the laser pulse. Scattering, for the quadratic density channel, is restricted to a discrete set of transverse modes. Our analysis explicitly includes the finite transverse size of the pump, which leads to coupling between different modes and is not restricted to perturbed wave numbers which are large compared to the inverse of the equilibrium mode size. Valeo investigated [13] the dynamic stability of an equilibrium consisting of a long laser pulse guided by a self-induced (by the pulse) channel. His analysis is to the temporal single mode growth on a hydrodynamic time scale and without relativistic effects. The analysis here is of the short pulse (plasma oscillation time scale) dynamics, including mildly relativistic electron motion. Other papers [14,15] have examined the consequences of self-induced channels and filamentation on Raman instabilities. Their theoretical model is based on pressure balance equilibrium and nonrelativistic election motion, and they derive only purely temporal growth rates. A nonrelativistic analysis of the temporal evolution of Raman scattering in a sinusiodally modulated plasma density has been made by Barr *et al.* [16]. These investigations do not yield results for the short time scales and cold plasmas appropriate for high intensity laser accelerators.

With a quadratic density variation, the problem can be solved exactly. The physical model consists of a performed neutral plasma channel with an unperturbed density given by

$$n_0(\mathbf{x}_\perp) = \bar{n}_0(1 + r^2/W^2), \tag{1}$$

where $r^2 = x^2 + y^2$. Since the duration of the laser pulse is assumed to be short compared to $2\pi/\omega_{pi}$, where $\omega_{pi}^2 = 4\pi e^2 \bar{n}_0/m_i$, the ions can be considered immobile. Furthermore, the laser frequency is much larger than the plasma frequency, so that the evolution of the laser pulse, caused by the electron density wake, occurs on a time scale much longer then the laser period. Thus, we consider an averaged (over a laser period), slow time scale, weakly relativistic equation of motion for plasma electrons under the influence of the ponderomotive force of the laser field.

A fluid model, which is applicable in a cold plasma before wave breaking [17] has occurred, is adequate to describe the plasma evolution for the short pulse duration of interest here. For the plasma to be considered cold the thermal velocity of an electron must be less than the oscillatory velocity imparted to it by the laser field.

The channel density is taken to vary over a distance much larger than collisionless plasma skin depth, so that $K = c/\bar{\omega}_p W \ll 1$, where $\bar{\omega}_p = 4\pi e^2 \bar{n}_0/m$ is the plasma frequency at the channel center. Then, as shown below, the unperturbed laser pulse has a spot size $w = \sqrt{Wc/\bar{\omega}_p} \ll W$, so that the density does not vary appreciably in the region where the ponderomotive force is nonzero. We also assume that, in the region where the laser amplitude is non-negligible, $\omega_p^2 \ll \omega^2$, where $\omega_p^2 = 4\pi e^2 n(\mathbf{x}_\perp, z, t)$. The plasma density $n = n_0 + \delta n$, where $n_0$ is given by Eq. (1) and $\delta n \ll n_0$.

We approximate the radiation field $\mathbf{A} = mc^2\mathbf{a}/e$, as

$$\mathbf{a} = \tfrac{1}{2} a(x_\perp, z, t)(\hat{e}_x + i\hat{e}_y)\exp(i(k_0 z - \omega_0 t)) + \text{c.c.} \tag{2}$$

The field equation is, in the weakly relativistic limit ($|\mathbf{a}|^2 < 1$),

$$\left[-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2 - \frac{\omega_p^2(x_\perp, z, t)}{c^2}\left(1 - \frac{|a|^2}{2}\right)\right]\mathbf{a} = 0. \tag{3}$$

Introducing the variables

$$s = t - z/v_{g0}, \quad z = z, \tag{4}$$

and using the eikonal approximation results in

$$\left[\frac{\omega_0^2}{c^2} - k_0^2 + \nabla_\perp^2 - \frac{\omega_p^2}{c^2}\left(1 - \frac{|a|^2}{2}\right) + 2ik\frac{\partial}{\partial z}\right]a = 0, \tag{5}$$

where $v_{g0}/c = k_0 c/\omega_0$. The amplitude is expanded as a sum of the unperturbed guided laser pulse and a perturbation driven by the generation of plasma density modulations: $a(\mathbf{x}_\perp, s, z) = a_0(\mathbf{x}_\perp, s) + a_1(\mathbf{x}_\perp, s, z)$. Note that the equilibrium is chosen to be independent of distance $z$, so that it travels through the plasma undistorted with velocity $v_{g0}$. Retaining leading order terms in Eq. (5) and using the dimensionless transverse coordinates $\bar{x} = x/w$, $\bar{y} = y/w$, $\bar{\nabla}_\perp = w\nabla_\perp$, results in

$$(-\bar{\nabla}_\perp^2 + \bar{r}^2)a_0 = w^2\left[\frac{\omega_0^2}{c^2} - k_0^2 - \frac{\bar{\omega}_p^2}{c^2}\right]a_0. \tag{6}$$

Since the primary concern in this paper is guiding by a channel we neglected the nonlinear terms in Eq. (6). In our notation, this is equivalent to requiring

$$|a_0^2|/8K = |a_0^2|\, w^2 \bar{\omega}_p^2/8c^2 \ll 1.$$

In deriving Eq. (6) we have also neglected the wakes in the unperturbed field $a_0$, created by the nonadiabaticity in the unperturbed $a_0(s)$ dependence. Wakes occur in the perturbed field $a_1$, of course.

The calculation proceeds by linearizing Eq. (5) and expanding the perturbation $a_1$ in a complete set of transverse eigenfunctions which $\psi_n^m$ can be expressed, in cylindrical coordinates $(\bar{r}, \theta)$ as

$$\psi_n^m(\bar{r}, \theta) = \exp(-\bar{r}^2/2)\bar{r}^m L_n^m(\bar{r}^2)\exp(im\theta), \tag{7}$$

where $L_n^m$ are the modified Laguerre polynomials.

The unperturbed equilibrium profile $a_0$, consistent with the assumption that the field is small enough so that self-guiding is unimportant, can be taken as

$$a_0(s, z, \bar{\mathbf{x}}_\perp) = \bar{a}_0(s)\exp(-\bar{r}^2/2), \tag{8}$$

which corresponds to the lowest ($m = 0, n = 0$) eigenfunction. The corresponding dispersion relation is

$$\left(\frac{\omega_0^2}{c^2} - k_0^2 - \frac{\bar{\omega}_p^2}{c^2}\right) = \frac{2}{w^2}. \tag{9}$$

Using Eqs. (5) and (9), with $\chi = (\delta n/n_0 - a_0^* a_1/2 - a_0 a_1^*/2)$ the equation for the perturbed field is

$$\left[2 + \bar{\nabla}_\perp^2 - \bar{r}^2 + 2ikw^2 \frac{\partial}{\partial z}\right] a_1 = \frac{\bar{\omega}_p^2 w^2 a_0}{c^2} \chi . \tag{10}$$

We have neglected the relativistic guiding term, proportional to $(\bar{\omega}_p^2/c^2) w^2 a_0^2$, within the bracket on the left hand side (LHS) of Eq. (10). This is consistent with our assumption that the relativistic guiding corrections to the unperturbed profile can be neglected.

The equation for the density modulation $\delta n$ is given by [8]

$$\left(\frac{\partial^2}{\partial s^2} + \omega_{p0}^2\right) \frac{\delta n}{n_0} = \frac{c^2 \nabla^2}{2} (a_0^* a_1 + a_0 a_1^*) . \tag{11}$$

Note that the terms on the right hand side (RHS) of Eq. (10), while of the same order as the relativistic guiding term, are the ponderomotive driving terms for the density perturbation on the LHS of Eq. (11). Equation (11) for $\delta n/n_0$ can be broken into a longitudinal piece and a transverse piece by the use of the quasistatic approximation, $\nabla_\parallel^2 \approx (1/c^2)\partial^2/\partial s^2$.

Changing Eq. (11) into an integral equation, integrating twice by parts, and inserting the resulting expression for $(\delta n/n_0)$ into Eq. (10), yields

$$\left(2 + \bar{\nabla}_\perp^2 - \bar{r}^2 + 2ikw^2 \frac{\partial}{\partial z}\right) a_1(s,z) = \frac{a_0(s)}{2} \int_{-\infty}^{s} \bar{\omega}_p \, ds' \sin \bar{\omega}_p (s - s') \left(\bar{\nabla}_\perp^2 - \frac{1}{K}\right) [a_0^*(s') a_1(s',z) + a_0(s') a_1^*(s',z)] . \tag{12}$$

Here both $a_0$ and $a_1$ are implicitly assumed to depend on $\bar{\mathbf{x}}_\perp$.

The fundamental has $m = 0$, so that perturbed modes with differing azimuthal numbers are decoupled. Thus, we concentrate on the evolution of a particular azimuthal mode, $a_1^m$, with an arbitrary radial profile:

$$a_1^m(\bar{\mathbf{x}}_\perp, z, s) = \sum_{n=0}^{\infty} \bar{a}_n^m(z,s) \psi_n^m(\bar{r}, \theta) . \tag{13}$$

It is convenient to introduce dimensionless time and space coordinates:

$$\bar{s} = \bar{\omega}_p s , \quad \bar{z} = z/kw^2 . \tag{14}$$

Inserting Eq. (13) into Eq. (12) and integrating over the transverse dimensions, gives

$$\left(2 - \lambda_{n_1}^m + 2i \frac{\partial}{\partial \bar{z}}\right) \bar{a}_{n_1}^m(\bar{s}, \bar{z}) \frac{(n_1 + m)!}{n_1!} = \frac{\bar{a}_0(s)}{2} \sum_{n_2} G_{n_1 n_2}^m \int_{-\infty}^{\bar{s}} d\bar{s}' \sin(\bar{s} - \bar{s}') \left[\bar{a}_0^*(\bar{s}') \bar{a}_{n_2}^m(\bar{s}', \bar{z}) + \bar{a}_{n_2}^{m*}(\bar{s}', \bar{z}) \bar{a}_0(\bar{s}')\right] . \tag{15}$$

and its complex conjugate. Here $\lambda_n^m = 2 + 2m + 4n$, where $m$ is the azimuthal number, $n$ is the radial number, and

$$G_{n_1 n_2}^m = -\frac{N!}{n_1! \, n_2! \, 2^{N+1}} \left(\frac{1}{K} + \frac{N+1}{2}\right), \tag{16}$$

with $N = n_1 + n_2 + m$. Equation (15) can be used to find the evolution of the instability for a finite duration pulse. If $\bar{a}_0$ has an arbitrary longitudinal profile, these questions need to be solved numerically.

Further analytical progress may be achieved by assuming that either $\bar{a}_0$ varies slowly on the time scales of interest, so that we regard $a_0$ to be independent of $s$. Equation (15) can then be solved by Fourier transform in $s$. With

$$b_n^m = \bar{a}_0 \bar{a}_n^{m*} + \bar{a}_0^* \bar{a}_n^m , \tag{17}$$

after Fourier transform in $\bar{s}$, we find

$$\left[\frac{\partial^2}{\partial \bar{z}^2} + l^2\right] b_{n_1}^m = \frac{|\bar{a}_0^2| \, l}{2^{m+2}(1 - \bar{\omega}^2)} \sum_{n_2} A_{n_1 n_2}^m b_{n_2}^m , \tag{18}$$

where $l = m + 2n_1$, and

$$A_{n_1 n_2}^m = \frac{n_1! 2^{m+1}}{(m + n_1)} G_{n_1 n_2}^m . \tag{19}$$

The dimensionless frequency $\bar{\omega}$ is the Fourier transform variable.

Equation (18) constitutes one of the main results of this paper. It describes the evolution of any initial perturbation to a flattop laser pulse propagating in plasma channel. As it is obvious from Eq. (18), all the radial modes are coupled to each other. This is a consequence of both the nonlinear nature of the laser-plasma interaction and the finite transverse size of the unperturbed equilibrium. This formalism can describe various perturbations to a laser pulse, such as hosing, breathing, and other modulational instabilities, such as quadrupole distortion.

It is instructive to examine the evolution of a laser pulse which is initially displaced from the center of the channel as a "rigid body" [the $(m,n) = (1,0)$ mode]. The dispersion relation for this mode, obtained by keeping only the diagonal element, is

$$k^2 = 1 - \mu/(1 - \omega^2) , \tag{20}$$

where $\mu = |\bar{a}_0^2|/8(1 + 1/K) \approx |\bar{a}_0^2|/8K$. A more accurate treatment would involve keeping a finite number of modes and diagonalizing the dispersion matrix.

Asymptotic behavior of the solution for $\bar{z} \geq 1$, $\bar{s} \geq 1$ can be obtained by the steepest descent integration (see, for example, Ref. [18]) in regimes that are delineated by relations between the length of the pulse $s$, the interaction length $z$, and the coupling parameter, $\mu \ll 1$. With $z_R = k_0 w^2$ and in dimensional variables, the asymptotic amplitudes are:

(i) Long pulses $\bar{\omega}_p s \gg 1/\mu(z/z_R)$,

$$\bar{a}_1 \sim \bar{a}_{10} \exp\left[\left(\sqrt{3}/4\right)|a_0^2/8K|^{1/3} (z/z_R)^{2/3} (\bar{\omega}_p s)^{1/3}\right]. \quad (21)$$

(ii) Intermediate pulses $\mu(z/z_r) \ll \bar{\omega}_p s \ll 1/\mu(z/z_R)$,

$$\bar{a}_1 \sim \bar{a}_{10} \exp\left[|a_0^2/8K|^{1/2} (z/z_R)^{1/2} (\bar{\omega}_p s)^{1/2}\right]. \quad (22)$$

(iii) Short pulses $\bar{\omega}_p s \ll \mu(z/z_R)$,

$$\bar{a}_1 \sim \bar{a}_{10} \exp\left[\left(3\sqrt{3}/4\right)|a_0^2/8K|^{1/3} (z/z_R)^{1/3} (\bar{\omega}_p s)^{2/3}\right]. \quad (23)$$

Equation (20) for these parameters was inversely Fourier-Laplace transformed and solved numerically for $10\mu$ radiation propagating through a plasma with $n_0 = 4 \times 10^{16}$ cm$^{-3}$, focused to a 0.1 mm spot, with $a_0 = 0.4$. The perturbation, at a distance $s = \lambda_p/2$ behind the onset of the instability (at $s = 0$), grows tenfold after 20 Rayleigh ranges.

The perturbation exponentiates as $(z/z_R)^q (s\bar{\omega}_p)^r$, where $q + r = 1$. This exponentiation is typical of beam breakup instabilities [19] in conventional accelerators where a charged particle bunch interacts with its electromagnetic environment. Here the laser pulse is somewhat analogous to the particle beam and the plasma channel to the metallic structures through which the beam propagates. While the analogy fails in detail, it is useful in suggesting cures for the instability. In linear electron accelerators, a technique known as BNS [20] damping has been employed to mitigate the effect of beam breakup instabilities. The BNS damping works by introducing a head-to-tail variation in the transverse oscillation frequencies of the accelerated particles. A similar technique may be used here—head-to-tail variation of the diffraction length ($1/z_r$ corresponds to the transverse oscillation wave number for the guided laser pulse). This variation can be imposed through a frequency chirp or may occur naturally, for sufficiently intense pulses, from relativistic guiding. Since the transverse modes in the system are coupled and have different phase velocities, phase mixing can occur as, say, a growing dipole mode generates higher-order modes which then dephase. Plasma accelerators will require stringent control of the phase and amplitude of the accelerating mode—the wake generated in the plasma. The extent to which plasma instabilities limit the performance of a plasma accelerator is under investigation.

The hollow plasma channel [12], which has superior accelerating properties, can, for some parameters, support only a single eigenmode. This may greatly reduce the instability.

Future investigations need to examine in more detail the influence of the nonlinearities in the unperturbed equation, especially wakes generated by the equilibrium mode, relativistic electron velocities, coupling between modes, and different density profiles and pulse shapes.

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