

Catalysis of Dynamical Flavor Symmetry Breaking by a Magnetic Field in 2 + 1 Dimensions

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It is shown that in 2 + 1 dimensions, a constant magnetic field is a strong catalyst of dynamical flavor symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. The effect is illustrated in the Nambu–Jona-Lasinio model in a magnetic field. The low-energy effective action in this model is derived, and the thermodynamic properties of the model are established.

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Relativistic field models in (2 + 1)-dimensional space-time have been a subject of considerable interest: Their sophisticated dynamics is interesting in itself; they also serve as effective theories for the description of long wavelength excitations in planar condensed matter systems [1,2]; also, their dynamics imitates the dynamics of (3 + 1)-dimensional theories at high temperature.

In this Letter, we will show that a constant magnetic field acts as a strong catalyst of dynamical flavor symmetry breaking in 2 + 1 dimensions, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. We would like to stress that this effect is universal, i.e., model independent, in 2 + 1 dimensions. This point may be important in connection with considering this effect in such condensed matter phenomena as the quantum Hall effect [1] and high temperature superconductivity [2] and in (3 + 1)-dimensional theories at high temperature.

Let us begin by considering the basic points in the problem of a relativistic fermion in a constant magnetic field B in 2 + 1 dimensions. The Lagrangian density is

$$\mathcal{L} = \frac{1}{2} [\bar{\Psi}, (i\gamma^\mu D_\mu - m)\Psi], \quad (1)$$

where the covariant derivative D_μ is

$$xD_\mu = \partial_\mu - ieA_\mu^{\text{ext}}, \quad A_\mu^{\text{ext}} = -Bx_2\delta_{\mu 1}. \quad (2)$$

We use four-component spinors corresponding to a reducible representation of the Dirac algebra [3]:

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, & \gamma^1 &= \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix}, \\ \gamma^2 &= \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}. \end{aligned} \quad (3)$$

At $m = 0$, the Lagrangian (1) is invariant under the flavor $U(2)$ transformations with generators $T_0 = I$, $T_1 = \gamma^5$, $T_2 = \frac{1}{i}\gamma^3$, and $T_3 = \gamma^3\gamma^5$, where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. The mass term breaks $U(2)$ down to $U(1) \times U(1)$ with generators T_0 and T_3 .

Our basic observation is that in 2 + 1 dimensions, as $m \rightarrow 0$, spontaneous $U(2)$ symmetry breaking takes place for fermions in a constant magnetic field. In order to prove this, we will show that in the limit $m \rightarrow 0$, the flavor condensate $\langle 0|\bar{\Psi}\Psi|0\rangle$ is nonzero: $\langle 0|\bar{\Psi}\Psi|0\rangle = -|eB|/2\pi$.

The condensate is expressed through the fermion propagator $S(x, y) = \langle 0|T[\Psi(x)\bar{\Psi}(y)]|0\rangle$. The propagator S can be calculated by using the Schwinger (proper time) approach [4]. It is

$$S(x, y) = \exp\left(ie \int_y^x A_\lambda^{\text{ext}} dz^\lambda\right) \tilde{S}(x - y), \quad (4)$$

where the integral is calculated along the straight line, and the Fourier transform of $\tilde{S}(x)$ (in Euclidean space) is

$$\begin{aligned} \tilde{S}(k) &= -i \int_0^\infty ds \exp\left[-s\left(m^2 + k_3^2 + \mathbf{k}^2 \frac{\tanh(eBs)}{eBs}\right)\right] \\ &\times \left(-k_\mu \gamma_\mu + m + \frac{1}{i}(k_2 \gamma_1 - k_1 \gamma_2) \tanh(eBs)\right) \\ &\times \left(1 + \frac{1}{i} \gamma_1 \gamma_2 \tanh(eBs)\right) \end{aligned} \quad (5)$$

($k_3 = -ik^0$ and γ_μ are anti-Hermitian matrices). The condensate is

$$\begin{aligned} \langle 0|\bar{\Psi}\Psi|0\rangle &= -\lim_{x \rightarrow y} \text{tr} S(x, y) = -\frac{i}{(2\pi)^3} \text{tr} \int d^3k \tilde{S}(k) \\ &= -\lim_{\Lambda \rightarrow \infty} \frac{4m}{(2\pi)^3} \int d^3k \int_{1/\Lambda^2}^\infty ds \\ &\times \exp\left[-s\left(m^2 + k_3^2 + \mathbf{k}^2 \frac{\tanh(eBs)}{eBs}\right)\right] \\ &\xrightarrow{m \rightarrow 0} -\frac{|eB|}{2\pi} \end{aligned} \quad (6)$$

(Λ is an ultraviolet cutoff). Thus in a constant magnetic field, spontaneous breakdown of the $U(2)$ symmetry takes place even though fermions do not acquire a dynamical mass. We note that in 3 + 1 dimensions, the result would be $\langle 0|\bar{\Psi}\Psi|0\rangle \sim m \ln m \rightarrow 0$ as $m \rightarrow 0$. Therefore, this is a specific (2 + 1)-dimensional phenomenon.

What is the physical basis of this phenomenon? In order to answer this question, we note that the singular, $1/m$, behavior of the integral in Eq. (6) is formed at large, $s \rightarrow \infty$, distances (s is the proper time coordinate). Actually, one can see from Eq. (6) that the magnetic field effectively removes the two space dimensions in the infrared region, thus reducing the dynamics to a one-dimensional dynamics which has much more severe

infrared singularities. From this viewpoint, the action of the magnetic field in this problem is similar to that of the Fermi surface in the BCS theory [5].

This point is intimately connected with the form of the energy spectrum of fermions in a constant magnetic field. In $2 + 1$ dimensions, it is [6]

$$E_n = \pm \sqrt{m^2 + 2|eB|n}, \quad n = 0, 1, \dots \quad (7)$$

(the Landau levels). Each Landau level is degenerate. At the lowest level with $n = 0$, the density of states is $|eB|/2\pi$; at the levels with $n \geq 1$, it is $|eB|/\pi$. As $m \rightarrow 0$, the energy E_0 goes to zero, and, therefore, there is the infinite vacuum degeneracy in this case.

By using the explicit solution for the field Ψ in this problem [6], we find the charge operators $Q_i = \frac{1}{2} \int d^2x [\Psi^\dagger(x), T_i \Psi(x)]$ connected with the broken symmetry:

$$\begin{aligned} Q_1 &= i \sum_p (a_{0p}^\dagger d_{0-p}^\dagger - d_{0-p} a_{0p}) + i \sum_{n=1}^{\infty} \sum_p \\ &\quad \times [(a_{np}^\dagger c_{np} - c_{np}^\dagger a_{np}) + (b_{np}^\dagger d_{np} - d_{np}^\dagger b_{np})], \\ Q_2 &= \sum_p (a_{0p}^\dagger d_{0-p}^\dagger - d_{0-p} a_{0p}) + \sum_{n=1}^{\infty} \sum_p \\ &\quad \times [(a_{np}^\dagger c_{np} + c_{np}^\dagger a_{np}) + (b_{np}^\dagger d_{np} + d_{np}^\dagger b_{np})]. \end{aligned}$$

Here $a_{np}, c_{np} (b_{np}, d_{np})$ are annihilation operators of fermions (antifermions) from the n th Landau level [there are two types of fermions (antifermions) for the four-component field Ψ]; the momentum $k = 2\pi p/L_1$ ($p = 0, \pm 1, \dots$), L_1 is the size in the x_1 direction. A set of the degenerate vacua $|\theta_1, \theta_2\rangle$ is $|\theta_1, \theta_2\rangle = \exp(iQ_1\theta_1 + iQ_2\theta_2)|0\rangle$, where $|0\rangle$ satisfies the condition $a_{np}|0\rangle = b_{np}|0\rangle = c_{np}|0\rangle = d_{np}|0\rangle = 0$. As $L_1 \rightarrow \infty$, the vacua with different $\{\theta_1, \theta_2\}$ become orthogonal and define (as usual in the case of spontaneous symmetry breaking) nonequivalent representations of canonical commutation relations.

In each Fock space defined by a vacuum $|\theta_1, \theta_2\rangle$, there are a lot of "excitations" with nonzero momentum and zero energy created by the operators $a_{0p}^\dagger, d_{0p}^\dagger$. However, there are no genuine (i.e., with a nontrivial dispersion relation) Nambu-Goldstone (NG) modes: At the lowest Landau level, the energy $E \equiv 0$ (since the Lorentz symmetry is broken by a magnetic field in this problem, there is no contradiction with Goldstone's theorem). We shall show below that even the weakest attractive interaction between fermions and antifermions is enough to "resurrect" these modes (and a dynamical mass for fermions).

Let us consider the $(2 + 1)$ -dimensional Nambu-Jona-Lasinio (NJL) model with the $U(2)$ flavor symmetry

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} [\bar{\Psi}, (i\gamma^\mu D_\mu) \Psi] \\ &\quad + \frac{G}{2} [(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma^5\Psi)^2 + (\bar{\Psi}\gamma^3\Psi)^2], \quad (8) \end{aligned}$$

where D_μ is the covariant derivative (2) and fermion fields carry an additional, "color," index $\alpha = 1, 2, \dots, N_c$.

This theory is equivalent to a theory with the Lagrangian density

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} [\bar{\Psi}, (i\gamma^\mu D_\mu) \Psi] - \bar{\Psi}(\sigma + \gamma^3\tau + i\gamma^5\pi)\Psi \\ &\quad - \frac{1}{2G} (\sigma^2 + \pi^2 + \tau^2) \quad (9) \end{aligned}$$

($\sigma = -G\bar{\Psi}\Psi$, $\tau = -G\bar{\Psi}\gamma^3\Psi$, $\pi = -G\bar{\Psi}i\gamma^5\Psi$). The effective action for these composite fields is

$$\Gamma(\sigma, \tau, \pi) = -\frac{1}{2G} \int d^3x (\sigma^2 + \tau^2 + \pi^2) + \tilde{\Gamma}(\sigma, \tau, \pi), \quad (10)$$

where

$$\tilde{\Gamma}(\sigma, \tau, \pi) = -i \text{Tr} \text{Ln} [i\gamma^\mu D_\mu - (\sigma + \gamma^3\tau + i\gamma^5\pi)]. \quad (11)$$

As $N_c \rightarrow \infty$, the path integral over the composite fields is dominated by stationary points of their action: $\delta\Gamma/\delta\sigma = \delta\Gamma/\delta\tau = \delta\Gamma/\delta\pi = 0$. We will analyze the dynamics in this limit by using the expansion of the action Γ in powers of derivatives of the composite fields.

We begin by calculating the effective potential V . Since V depends only on the $U(2)$ invariant $\rho^2 = \sigma^2 + \tau^2 + \pi^2$, it is sufficient to consider a configuration with $\tau = \pi = 0$ and σ independent of x . Then, using the proper-time method [4], we find the potential from Eqs. (4), (10), and (11):

$$\begin{aligned} V(\sigma) &= \frac{\sigma^2}{2G} + \tilde{V}(\sigma) = \frac{\sigma^2}{2G} \\ &\quad + \frac{N_c}{4\pi^{3/2}} \int_{1/\Lambda^2}^{\infty} \frac{ds}{s^{5/2}} e^{-s\sigma^2} eBs \coth(eBs). \quad (12) \end{aligned}$$

The gap equation $dV/d\sigma = 0$ is

$$\begin{aligned} 2\Lambda l \left(\frac{1}{g} - \frac{1}{g_c} \right) \sigma &= \frac{\sigma}{|\sigma|l} + \sqrt{2}\sigma \zeta \left(\frac{1}{2}, 1 + \frac{(\sigma l)^2}{2} \right) \\ &\quad + O(1/\Lambda), \quad (13) \end{aligned}$$

where the dimensionless coupling constant $g \equiv N_c(\Lambda/\pi)G$, $g_c = \sqrt{\pi}$, $l \equiv 1/|eB|^{1/2}$ is the magnetic length, and ζ is the generalized Riemann zeta function [7]. As $B \rightarrow 0$ ($l \rightarrow \infty$), we recover the known gap equation [8]

$$\sigma|\sigma| = \sigma \Lambda \left(\frac{1}{g_c} - \frac{1}{g} \right), \quad (14)$$

which admits a nontrivial solution only if g is supercritical, $g > g_c = \sqrt{\pi}$ [as Eq. (9) implies, a solution to Eq. (14), $\sigma = \bar{\sigma}$, coincides with the fermion dynamical mass $\bar{\sigma} = m_{\text{dyn}}$, and the dispersion relation for fermions is Eq. (7) with m replaced by $\bar{\sigma}$]. We will show that a magnetic field changes the situation dramatically: At $B \neq 0$, a nontrivial solution exists at all $g > 0$.

We shall first consider the case of subcritical g , $g < g_c$, which in turn can be divided into two subcases: (a)

$g \ll g_c$ and (b) $g \rightarrow g_c - 0$ (near-critical g). Assuming that $|\bar{\sigma}l| \ll 1$ at $g \ll g_c$, we find from Eq. (13)

$$m_{\text{dyn}} \equiv \bar{\sigma} \simeq \frac{|eB|g\sqrt{\pi}}{2\Lambda(g_c - g)}. \quad (15)$$

Since this equation implies that the condition $|\bar{\sigma}l| \ll 1$ fulfills at all g , satisfying $(g_c - g) \gg |eB|^{1/2}/\Lambda$, the relation (15) is actually valid in that whole region.

At $g_c - g \lesssim |eB|^{1/2}/\Lambda$, introducing the scale $m^* \equiv 2\Lambda(1/g - 1/g_c)$, we find the equation

$$m^*l = \frac{1}{|\sigma|l} + \sqrt{2}\zeta\left(\frac{1}{2}, \frac{(\sigma l)^2}{2} + 1\right), \quad (16)$$

which implies that in the near-critical region m_{dyn} is

$$m_{\text{dyn}} = \bar{\sigma} \sim |eB|^{1/2}. \quad (17)$$

Thus in the scaling region, with $g_c - g \lesssim |eB|^{1/2}/\Lambda$, the cutoff disappears from the observable m_{dyn} . This agrees with the well-known fact that the critical value $g_c = \sqrt{\pi}$ is an ultraviolet stable fixed point at leading order in $1/N_c$ [8]. The relation (17) can be considered as a scaling law in the scaling region.

At $g > g_c$, the analytic expression for m_{dyn} can be obtained at weak $|eB|$, satisfying the condition

$|eB|^{1/2}/m_{\text{dyn}}^{(0)} \ll 1$, where $m_{\text{dyn}}^{(0)}$ is the solution of the gap equation (14) with $B = 0$. Then, using the asymptotic formula [7]

$$\zeta(z, q) \xrightarrow{q \rightarrow \infty} \frac{1}{(z-1)q^{z-1}} \left[1 + \frac{z-1}{2q} + \dots \right],$$

we find

$$m_{\text{dyn}} = \bar{\sigma} = m_{\text{dyn}}^{(0)} \left[1 + \frac{(eB)^2}{12(m_{\text{dyn}}^{(0)})^4} \right], \quad (18)$$

i.e., m_{dyn} increases with B [9].

Let us now turn to calculating the kinetic term $\mathcal{L}_k$ in the action. The U(2) symmetry implies that the general form of $\mathcal{L}_k$ is

$$\mathcal{L}_k = N_c \frac{F_1^{\mu\nu}}{2} (\partial_\mu \rho_j \partial_\nu \rho_j) + N_c \frac{F_2^{\mu\nu}}{\rho^2} (\rho_j \partial_\mu \rho_j) (\rho_i \partial_\nu \rho_i), \quad (19)$$

where $\rho = (\sigma, \tau, \pi)$ and $F_1^{\mu\nu}, F_2^{\mu\nu}$ are functions of $\rho^2 = \sigma^2 + \tau^2 + \pi^2$. To find the functions $F_1^{\mu\nu}, F_2^{\mu\nu}$, one can use different methods. We used the method of Ref. [10]. The result is $F_1^{\mu\nu} = g^{\mu\nu} F_1^{\mu\mu}$, $F_2^{\mu\nu} = g^{\mu\nu} F_2^{\mu\mu}$ with

$$\begin{aligned} F_1^{00} &= \frac{l}{8\pi} \left[\frac{1}{\sqrt{2}} \zeta\left(\frac{3}{2}, \frac{(\rho l)^2}{2} + 1\right) + (\rho l)^{-3} \right], \\ F_1^{11} &= F_1^{22} = \frac{1}{4\pi\rho}, \\ F_2^{00} &= -\frac{1}{16\pi} \left[\frac{(\rho l)^2}{2\sqrt{2}} \zeta\left(\frac{5}{2}, \frac{(\rho l)^2}{2} + 1\right) + (\rho l)^{-3} \right], \\ F_2^{11} &= F_2^{22} = \frac{l}{8\pi} \left[\frac{(\rho l)^4}{\sqrt{2}} \zeta\left(\frac{3}{2}, \frac{(\rho l)^2}{2} + 1\right) + \sqrt{2}(\rho l)^2 \zeta\left(\frac{1}{2}, \frac{(\rho l)^2}{2} + 1\right) + 2\rho l - (\rho l)^{-1} \right]. \end{aligned} \quad (20)$$

By using the asymptotic formulas for zeta functions [7], we find the following spectrum for the excitations σ, τ , and π from Eqs. (12), (19), and (20).

(a) Subcritical, $g < g_c$, region:

At $g_c - g \gg |eB|^{1/2}/\Lambda$ [where $|\bar{\sigma}l| \ll 1$, see Eq. (15)] we find

$$E_{\tau, \pi} \simeq \sqrt{2}(\bar{\sigma}l)(\mathbf{k}^2)^{1/2} = \frac{g|eB|^{1/2}}{\sqrt{2}\Lambda(1 - \frac{g}{g_c})}(\mathbf{k}^2)^{1/2}, \quad (21)$$

$$M_\sigma^2 \simeq \frac{8\sqrt{2}(1 - \frac{g}{g_c})}{\zeta(\frac{3}{2})g} \Lambda |eB|^{1/2} \quad (22)$$

[see Eq. (15)]. Thus τ and π are gapless NG models. As the interaction is switched off, $g \rightarrow 0$, their energy becomes identically zero, and we return to the dynamics with spontaneous flavor symmetry breaking but without genuine NG modes discussed above. Also, as $g \rightarrow 0$, the σ mode decouples ($M_\sigma \rightarrow \infty$).

In the near-critical region, with $g_c - g \lesssim |eB|^{1/2}/\Lambda$, we find from Eqs. (19) and (20)

$$E_{\tau, \pi} = f(\bar{\sigma}l)(\mathbf{k}^2)^{1/2}, \quad (23)$$

where

$$f(\bar{\sigma}l) = \left(\frac{2}{\bar{\sigma}l} \right)^{1/2} \left[\frac{1}{\sqrt{2}} \zeta\left(\frac{3}{2}, \frac{(\bar{\sigma}l)^2}{2} + 1\right) + (\bar{\sigma}l)^{-3} \right]^{-1/2}.$$

Since in the near-critical (scaling) region the parameter $\bar{\sigma}$ is $\bar{\sigma} \sim |eB|^{1/2} = l^{-1}$, we see that the cutoff Λ disappears from the observables E_τ and E_π in the scaling region.

(b) Supercritical g , $g > g_c$, and weak $B(l \rightarrow \infty)$:

$$E_{\tau, \pi} = \left(1 - \frac{1}{4(\bar{\sigma}l)^2} \right) (\mathbf{k}^2)^{1/2}, \quad (24)$$

$$M_\sigma^2 = 6\bar{\sigma}^2 \left(1 - \frac{3}{4} \frac{1}{(\bar{\sigma}l)^2} \right), \quad (25)$$

where $\bar{\sigma}$ is given in Eq. (18). These relations show that the magnetic field leads to decreasing both the velocity of the NG modes (it becomes less than 1) and the mass (energy gap) of the σ mode.

As is known, in this model, an interacting continuum ($\Lambda \rightarrow \infty$) theory appears only at the critical value $g = g_c$

(the continuum theory is trivial at $g < g_c$) [8]. At $B = 0$, the continuum theory is in the symmetric phase at $g \rightarrow g_c - 0$ and in the broken phase at $g \rightarrow g_c + 0$. On the other hand, as follows from our analysis, in a magnetic field, it is in the broken phase both at $g \rightarrow g_c - 0$ and $g \rightarrow g_c + 0$ (though the dispersion relations for fermions and collective excitations ρ are different at $g \rightarrow g_c - 0$ and $g \rightarrow g_c + 0$).

The present model illustrates the general phenomenon in $2 + 1$ dimensions: In the infrared region, an external magnetic field reduces the dynamics of fermion pairing to one-dimensional dynamics (at the lowest Landau level), thus catalyzing the generation of a dynamical mass for fermions. A concrete sample of dynamical symmetry breaking is, of course, different in different models.

We also studied the thermodynamic properties of the NJL model (8) at finite temperature T and finite fermion density n . In leading order in $1/N_c$, we found that the effective (thermodynamic) potential is

$$V_{\beta,\mu}(\sigma) = \frac{\sigma^2}{2G} + \frac{N_c}{4\pi^{3/2}l^3} \int_0^\infty \frac{dt}{t^{3/2}} e^{-(t^2\sigma^2)} \coth t \\ \times \Theta_4\left(\frac{i}{2}\mu\beta \left| \frac{i}{4\pi t} \left(\frac{\beta}{l}\right)^2\right.\right), \quad (26)$$

where $\beta = 1/T$, μ is a chemical potential, and Θ_4 is the fourth Jacobian theta function [7]. The analysis of Eq. (26) shows that there is a symmetry restoring (second order) phase transition both at high temperature and high density. The critical temperature T_c is $T_c \sim m_{\text{dyn}}$, and the critical density n_c is $n_c = |eB|/2\pi$ at $g \leq g_c = \sqrt{\pi}$ (corresponding to the filling of the lowest fermion Landau level), and $n_c > |eB|/2\pi$ at $g > g_c$. We recall that there cannot be spontaneous breakdown of a continuous symmetry at finite ($T > 0$) temperature in $2 + 1$ dimensions (the Mermin-Wagner-Coleman (MWC) theorem [11,12]). In the NJL model, the MWC theorem manifests itself only beyond the leading order in $1/N_c$. A plausible hypothesis, we believe, is that beyond this order, the phase transition at $T = T_c \sim m_{\text{dyn}}$ will become the Berezinsky-Kosterlitz-Thouless (BKT) type phase transition [13], and, in particular, at $0 < T < T_c$, the NG modes τ and π will transform into BKT collective modes.

In this Letter, we considered the dynamics in the presence of a constant magnetic field only. It would be

interesting to extend this analysis to the case of inhomogeneous electromagnetic fields in $2 + 1$ dimensions (for a recent consideration of this problem see Ref. [14]).

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