Experimental Evidence for Critical Dynamics in Microfracturing Processes

A. Petri,[1,2] G. Paparo,[1] A. Vespignani,[2] A. Alippi,[1,3] and M. Costantini[1,2]
[1]*Istituto di Acustica del Consiglio Nazionale delle Ricerche "O.M. Corbino," Via Cassia 1216, 00189-Roma, Italy*
[2]*Dipartimento di Fisica, Università di Roma "La Sapienza," P.le A. Moro 2, 00185-Roma, Italy*
[3]*Dipartimento di Energetica, Università di Roma "La Sapienza," Via Scarpa 14, 00161-Roma, Italy*
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We performed the statistical analysis of acoustic emission time series in the ultrasonic frequency range, obtained experimentally from laboratory samples subjected to external uniaxial elastic stress. We found a power law scaling behavior in both the acoustic emission amplitude distribution and time correlation function, with exponents very close to those found in fracturing processes occurring at different time and space scales. These facts strongly suggest the existence of a critical dynamics underlying the process, which might be related to the idea of a self-organized critical state based on the energy dissipation through all the length scales.

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Power law behavior in physical phenomena is usually the fingerprint of temporal and spatial critical fluctuations of which well known examples are Ising-like systems, fractal growth phenomena, turbulence, etc. Unlike the usual second order phase transitions, some of the previous examples exhibit a critical behavior without the need to fine tune any control parameter; i.e., the critical state is an attractor of the dynamics. A few years ago Bak, Tang, and Wiesenfeld [1] termed this kind of situation "self-organized criticality" (SOC) and introduced a simple model of a dynamically driven system, inspired by the dynamics of sandpiles, that evolves spontaneously to a stationary critical state. This model is an example of SOC phenomenon in which a system with short range coupling self-stabilizes in a stationary state characterized by avalanches (activity) with power law distribution functions. Hence, the system has no characteristic length (and is therefore self-similar) and is in this sense critical. The SOC concept has been proposed also as a possible mechanism for the generation of the so-called $1/f$ noise; however, it has been shown successively [2,3] that the spatialtemporal scaling in the SOC state does not necessarily manifest itself in nontrivial exponents for the power spectrum.

Because of the importance of the SOC concept as a possible unifying framework for a wide range of physical phenomena, a lot of work has been devoted to studying these systems through computer simulations, theoretical approaches, and experimental findings [4]. In particular, the SOC framework has been proposed as a possible interpretation for the empirical observation of the energy release in earthquakes [5]. In fact, existence of statistical self-similarity in seismic processes is a well established fact, which has its strongest evidence in the power law behavior of the well known Gutenberg's [6] and Omori's [7] empirical laws. Power law behavior was observed by Mogi [8] in the distribution of the maximum trace amplitude of audio signals emitted from samples subjected to various forms of stress, in analogy with the Ishimoto Iida empirical relation [9]. Hirata [10] observed self-similarity in the time frequency distribution of aftershock signals due to fracturation of basalt under constant stress. More recently, acoustic emission (AE) signals due to volcanic activity [11] and fracturing processes in metal-hydrogen systems during hydrogen precipitation were proposed to be reconducible to a SOC mechanism [12]. For both phenomena, the acoustic wave bursts show a power law distribution that seems related to an avalanche mechanism peculiar of SOC systems. This mechanism is a sort of "domino effect," giving rise to a self-similar energy release on all length scales; more precisely from the small scale that defines the microscopic relaxation processes to the large scale of the system. As earthquakes and AE phenomena are associated with fracturing processes on various length scales, it might be possible that fracture dynamics defines a class of SOC systems.

In this Letter, we report the measurements of AE [13] in the ultrasonic range due to the microfracture aftershock relaxation properties of laboratory samples subject to an external macroscopic perturbation. Our measurements confirm a power law behavior for the AE amplitude distribution in all the experimental conditions and for all the samples used. This experimental fact suggests that the same critical dynamics can hold from the large scale of earthquakes down to the microscopic scale of the sample rheological structure. We extend our analysis to the time correlation function and the spectral density of the AE signal time series. Noticeably, we find a nontrivial scaling in the power spectrum of the signal ($1/f^{\beta}$ noise), showing that the fracturing process has scaling properties both in space and time. *To our knowledge, such statistical analysis of laboratory AEs coming from separate aftershocks has not been done before.*

The acoustic emission signals were detected during the stressing of our samples. The samples used were cylinders of synthetic plaster (Ancorfix 709), 30 cm long, 5 to 10 cm diameter, with an iron bar of 2 cm diameter in their central

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axis, acting as a waveguide for the elastic radiation emitted within the specimen. In order to test the independence of the statistics in different experimental conditions, two different high sensitivity piezoelectric transducers have been used, which were coupled to the iron bar on one of their end sections. The maximum sensitivity of the two transducers is at frequencies $f_1 = 220$ kHz and $f_2 = 400$ kHz, while the bandwidth is $\Delta f_1 = 15$ kHz and $\Delta f_2 = 50$ kHz, respectively. The sample was periodically stressed at a small lateral region, in the radial direction, by a ball jointed piston that allows pressure increases of several hundred bars. The acoustic energy release takes place through a series of bursts which were finally detected by the piezoelectric transducers. The signals were then fed through an amplifier into a short rise time (0, 2 μs) peak detector, that produces at its output the amplitude of the maximum burst occurring within the discharge time $\Delta t = 200$ ms of the detector. The data are then recorded and numerically processed with a PC.

The externally applied stress and the following release of energy generate a series of microfracturing processes in the sample, producing AE signals. The detected signals are limited by the maximum recorded signal, that corresponds to the saturation level (8 V) of the amplifier, and the noise level ($\approx$ 80 mV or 500 mV, depending on the transducer), giving a 1.5–2 decade detection dynamics.

From the detected signals we obtain the time series of the AE burst amplitudes, an example of which is shown in Fig. 1. In the figure it is possible to distinguish the different stress applications which generate large AE events even long after the corresponding increase of stress.

We have performed a statistical analysis of these AE series. In Fig. 2 the frequency histogram of the AE amplitude of one of the recorded signals is shown. This distribution exhibits a power law behavior, limited by upper and lower cutoff, corresponding, respectively, to

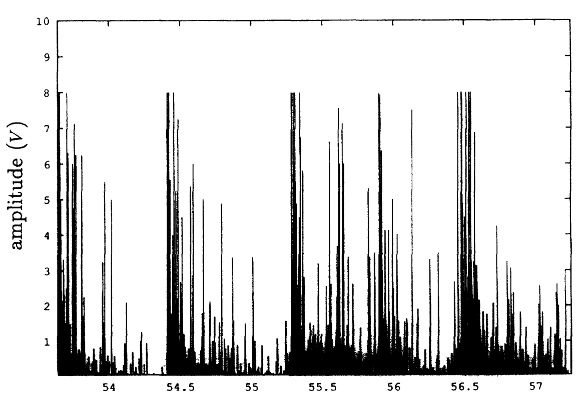

FIG. 1. Recorded signal of an acoustic emission time series. Note that the five stronger emission bursts refer to the increasing of the stress. The highest peaks correspond to the amplifier saturation level. The signal sampling rate is 200 ms.

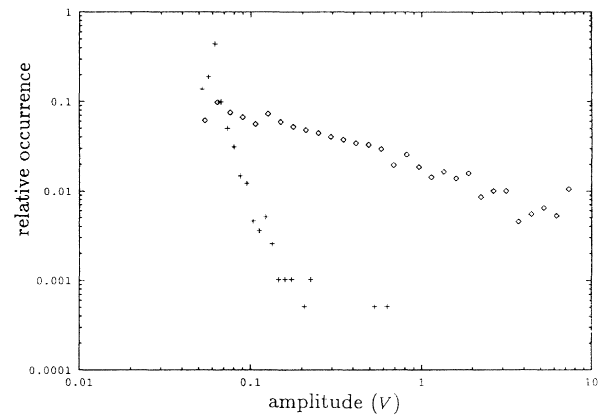

FIG. 2. Comparison between the AE burst frequency distribution in the case of stressed (diamonds) and nonstressed (crosses) samples.

the saturation threshold and the background noise level of the experimental apparatus. In Fig. 2 the same analysis for a background signal is also shown (AE in absence of external stress). In this case the distribution obtained is peaked around the noise level of the experimental apparatus and decreases sharply afterward.

The AE integrated distribution [i.e., the number of events $N(V)$ whose amplitude is greater than V] can be fitted by the following algebraic relation

$$N(V) \simeq k_1 V^{-\gamma} + k_2; \qquad \gamma = 1.7 \pm 0.2. \qquad (1)$$

The value of γ is obtained after repeating the same analysis on the AE signals obtained by different samples and experimental apparatus (transducer with different characteristic). Figure 3 shows an example of the resulting distributions, after subtraction of k_2. It is worthwhile to note that there is a slight departure from scaling behavior at low amplitudes and at high amplitudes. This is due to the onset of the above-mentioned upper and lower cutoff levels of the experimental apparatus. However, the plots present a well defined scaling behavior over at least one and a half decades and clearly demonstrate a nonexponential decay. Apart from shifting factors due to the different noise level and saturation threshold, we have found the same power law behavior in every set of analyzed data. This last point suggests that the microfracturing mechanism does not depend on the details of the experiment and possesses a universal statistical behavior, at least on the energy and length scales considered.

The energy involved in the process is proportional to the square of the amplitude of the AE event, and we can obtain the energy release distribution by the following relation:

$$N(E) \sim E^{-\delta}; \qquad \delta = \frac{1+\gamma}{2}. \qquad (2)$$

This relation gives, from our measurement of the value of γ, an exponent $\delta = 1.3 \pm 0.1$, which is in good agreement with the exponent found in Refs. [11,12] and

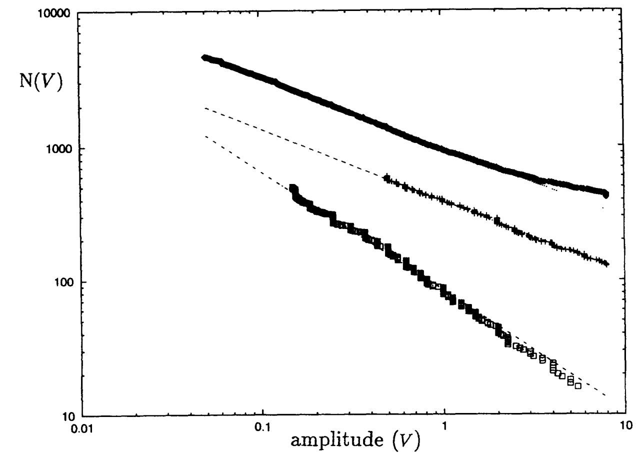

FIG. 3. Cumulate distribution of AE bursts with respect to their amplitude for different samples and experimental conditions, after subtraction of the offset. Fitting slopes are plotted for comparison ($0.52 \leq \gamma \leq 0.84$).

in Gutenberg's law [6]. This result is a clear evidence of an underlying critical dynamics, i.e., a nonexponential relation between amplitude and number of events, and thus confirms the absence of a characteristic length and the self-similarity of the microfracturing phenomenon. In addition, the quantitative coincidence of the measured exponent with the result found in phenomena ruled by a dynamics acting on a different scale, strongly suggests a general scale invariance of the fracturing processes.

One of the most interesting suggestions put forward to account for the critical behavior of the microfracturing process can be found in the SOC mechanism. By using this framework we can think that our samples can exist in very many stable states, almost all of which have different microscopic structures given by the porosity, residual stress, and microcracks, etc. By perturbing the sample, the system goes from one metastable state to another through an energy diffusion process. However, this diffusion process has a striking difference with respect to the usual diffusion, because the local microscopic rigidity of the material structure makes this a threshold process that can stop in any of the metastable states. In this sense the system is at some point of marginal stability, and it is reasonable to expect it to exhibit a self-organized critical behavior. As in usual critical phenomena, the various microscopic aspects of the material should not be crucial with respect to the critical properties. However, this aspect can have some influence on the experiment and therefore deserves a deeper analysis. For instance the content of water in the plaster could modify the microscopic rigidity of the samples (the threshold mechanism) or the dissipation, therefore affecting the critical behavior. *In addition, also the spatial distribution of aftershocks may be one clue for the microscopic dynamics of the phenomenon. To address this question* we are performing further experiments whose results will be reported elsewhere. Nevertheless, our experimental finding is also in good agreement with the numerical and theoretical results for the original SOC model of Bak *et al.* [4,14]. Thus, the energy dissipation could be the hallmark of an underlying SOC phenomenon determined by the fracturing mechanism.

Another interesting point raised by the concept of SOC is in the area of $1/f$ noise. In fact, it has been suggested that the power law behavior of many time signal spectra could be traced back to the scale invariance of SOC systems dynamics. Although this mechanism for the generation of $1/f$ noise is very interesting, it has been shown [2,3] that it is not as general as originally believed and that a SOC state does not imply a nontrivial scaling of the noise spectrum. We then proceed to compute the autocorrelation function and the power spectrum of our time signals in order to look at the occurrence of $1/f$ type spectrum. There are a number of accepted techniques to characterize the random signals $V(t)$. Both the correlation function $C_V(\tau) = \langle V(t)V(t+\tau)\rangle$ and the spectral density $S_V(f) = |\sum_{i=1}^{N} V(t_i)e^{i2\pi f t_i}|^2$ quantify correlations at the time scale $\tau \sim 1/f$. Generally, $S_V(f)$ and $C_V(\tau)$ are connected by the Wiener-Kintchine relations [15]. In the case of white noise one has $C_V(\tau) \sim \delta(\tau)$ and $S_V(f) \sim \mathrm{const}$, while in the case of the Brownian motion $S_V(f) \sim 1/f^2$.

We have analyzed both the autocorrelation function and the power spectrum of the AE signal time series, and Fig. 4 shows the autocorrelation function for two typical data sets. The behavior is an algebraic decay over almost 2 orders of magnitude, and the data fit gives the following relation:

$$C_V(t) \sim t^{-\alpha}; \qquad \alpha = 0.4 \pm 0.1. \tag{3}$$

This corresponds to a noise spectrum of the type

$$S_V(f) \sim f^{-\beta}; \qquad \beta = 1 - \alpha, \tag{4}$$

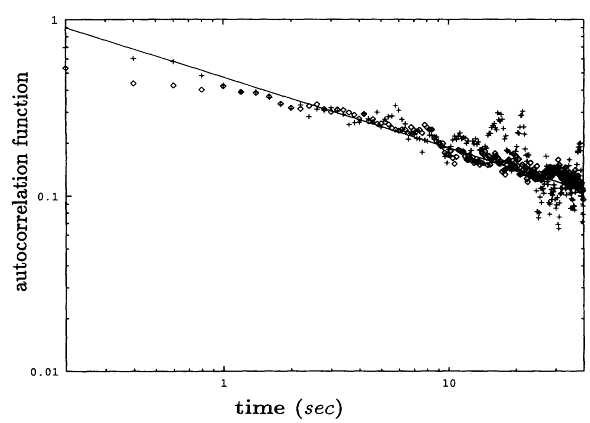

FIG. 4. Time autocorrelation functions of the AE signal time series. The scaling law with $\alpha = 0.4$ is reported for comparison.

and therefore, from our measurements, we find $\beta \simeq 0.6 \pm 0.1$.

It is important to note that the upper cutoff of the power law behavior of the correlation function is given by the scale separation between the AE activity fluctuations and the overall exponential decay of the activity of every external stress of the sample. Although the errors on the values of the α and β exponents are rather large and the numerical estimates have to be considered only as indicative, this result shows, in a striking way, the temporal self-similarity of the microfracturing process. Moreover, this behavior seems consistent with what is to be expected for SOC systems with dissipation [16], which show exponents for the power spectrum that depend on the level of energy conservation.

In conclusion, we have shown that the AE activity due to microfracturing processes in externally stressed samples follows a power law statistical behavior which is indicative of critical dynamics. This critical behavior is seen also in the time correlation of the signals ($1/f$ noise spectrum), suggesting a spatial and time scale-invariance of the phenomenon. The exponent derived here for the AE signal amplitude distribution is very close to those obtained in other fracturing processes [6,11,12] ranging from earthquakes to microscopic phenomena. This suggests that the fracturing process is a scale invariant phenomenon with critical dynamics, that might provide an example of a definite class of SOC systems with universal properties.

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