Comment on "Application of Finite Size Scaling to Monte Carlo Calculations"

In a recent Letter, Kim [1] claims to have uncovered a new way of applying finite size scaling (FSS), which eliminates the need for large lattices to investigate critical behavior; he then employs this technique to study the O(3) model in two dimensions (2D) and concludes that asymptotic scaling (AS) holds for $\beta \geq 2.25$, in disagreement with our prediction [2,3] that for β sufficiently large the mass gap vanishes. We believe that this paper is based on a misunderstanding of both FSS and of our reasons for doubting the existence of AS in the O(3) model [4].

Kim's basic assumption seems to be that FSS can be applied for *any* L. This is not correct; in fact, FSS says that near a critical point quantities such as $r = \chi(L)/\chi(\infty)$ depend essentially only on the ratio $x = L/\xi(\infty)$, instead of L and β separately. But the statement is an asymptotic one meaning that in the limit $L \rightarrow \infty$ at fixed x, r approaches a finite limit. In particular, there is nothing in this statement that says how close for a given L this r is to its limiting value, which is clearly a model dependent issue. Now Kim is not following this procedure. Instead his x becomes smaller and smaller (in steps) as β increases. This is so irrespective of whether the critical point is at a finite β value or at $\beta = \infty$. Consequently, he is not studying really FSS, but the limit $x \rightarrow 0$. On his small lattices he is bound to recover the results of perturbation theory (PT), as the numbers below show.

Our prediction of a vanishing mass gap at large β necessarily will imply violations of FSS, because for small L one will see an apparent FSS with ξ_{as} (the correlation length predicted by asymptotic scaling) replacing the true correlation length, whereas on large lattices one would see the true FSS involving the true correlation length.

The next question is, how large an L one needs to use for FSS to become reasonably accurate. In O(3), as explained in [4], there are good reasons to believe that the minimal L needed for observing truly thermodynamic behavior is $O(\exp(\pi\beta))$; this scale is suggested by PT itself. Indeed, for fixed L, PT is (demonstrably) producing the correct asymptotic expansion. This is quite understandable since PT is an expansion around an ordered state, and in a finite ferromagnet, at sufficiently low temperatures, such a state is sure to exist. The subtle question is whether PT remains valid when L goes to ∞, since in 2D the Mermin-Wagner theorem guarantees the restoration of the symmetry in the infinite volume limit.

The success of PT on small lattices is illustrated in Table I, where we use the PT values of the 2-point function [5] to calculate ξ_L and χ_L and compare with Kim's Monte Carlo values. It is clear that in what Kim calls "the scaling region" ($\beta > 2.25$), the Monte Carlo results are quite accurately reproduced by PT. So the only thing Kim's data show is that for L sufficiently small, PT works. In particular, they tell nothing about the absence of a massless phase at low temperature, as predicted by our percolation arguments [2,3]. To answer that question, one could employ FSS techniques, but only on lattices of size $L \gg O(\exp(\pi\beta))$, where nonperturbative effects would have a chance of manifesting themselves. Let us emphasize that in [4] we have already explained that any sort of FSS, such as the MCRG quoted by Kim, is bound to show AS on sufficiently small lattices, since that is a true property of PT, and the latter works in that regime.

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TABLE I. Comparison of Kim's Monte Carlo data with perturbation theory (the numbers marked by an asterisk were given incorrectly in [1]).

L/ξ_{as}	β	L	ξ_L(MC)	ξ_L(PT)	ξ_L(PT)/ξ_{as}	χ_L(MC)	χ_L(PT)	χ_L(PT)/χ_{as}
5×10^{-5}*	2.246 308 598 4	30	23.42(4)	23.43	0.391×10^{-4}	432.(2)	439.0	0.615×10^{-8}
5×10^{-5}*	2.381 067 991 2	66	51.61(8)	51.16	0.388×10^{-4}	1846.(2)	1865.0	0.605×10^{-8}
5×10^{-5}*	2.509 085 359 2	140*	109.64(27)	107.56	0.384×10^{-4}	7441.(16)	7458.0	0.598×10^{-8}
5×10^{-5}*	2.628 499 188 8	283	222.05(57)	215.53	0.381×10^{-4}	27574.(68)	27409.0	0.591×10^{-8}
2×10^{-6}	2.706 998 905 0	18	17.07(3)	17.02	0.189×10^{-5}	201.9(1)	203.0	0.184×10^{-10}
2×10^{-6}	2.793 293 745 0	30	28.49(5)	28.37	0.189×10^{-5}	524.6(2)	527.3	0.183×10^{-10}
2×10^{-6}	2.918 347 080 0	63	59.84(6)	59.40	0.189×10^{-5}	2110.9(1)	2117.0	0.182×10^{-10}

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