Fluctuation-Induced First Order Transition between the Quantum Hall Liquid and Insulator

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(Received 10 March 1994)

We study the phase transition between the quantum Hall liquid and the insulator within the framework of the Chern-Simons-Landau-Ginzburg theory of the quantum Hall effect. For the transition induced by a background periodic potential in the absence of disorder, the model is described by a single relativistic scalar field coupled to the Chern-Simons gauge field. Within the regime of parameters where perturbation expansion is valid, we show that the transition is first order, induced by the fluctuations of the gauge field, rather than second order, with statistical angle-dependent scaling exponent.

PACS numbers: 73.40.Hm, 71.28.+d, 71.30.+h

Recently the Chern-Simons-Landau-Ginzburg (CSLG) theory [1,2] of the quantum Hall effect has been applied to study the phase transition between the quantum Hall liquids and between quantum Hall liquids and insulators [3]. The central idea of this theory is to map the transition in the quantum Hall systems to the transition from superfluid to insulator. For the primary quantum Hall states, the off-diagonal long-range order of the quantum Hall liquid state [4] allows one to identify it with the superfluid state of bosons, and the insulating state in the quantum Hall systems, the Hall insulator, is identified with the insulating state of the bosons. The resulting theory describes nonrelativistic bosons coupled to the Chern-Simons gauge field. The transition between the superfluid and the insulating state of bosons is induced by either disorder or the magnetic field. Using the CSLG one can derive a set of "laws of corresponding states" [3,5] relating the disorder-induced second order phase transition in the integer quantum Hall effect to transitions in the fractional quantum Hall regime, as long as these transitions are induced by disorder as well. Kivelson, Lee, and Zhang [3] constructed a global phase diagram of the quantum Hall effect in the two-dimensional parameter space of disorder versus magnetic field and identified relations between the various interplateau transitions and transition between quantum Hall liquid and the Hall insulator. In the case of strong disorder, the integer transition is widely believed to be of the second order. From the point of view of the law of correspondence, the fractional transition in this limit should be of the second order as well [6]. This point of view is supported by experiments [7] where similar critical exponents were measured for both integer and fractional transition. However, the situation is less clear when the transition is dominated by interaction rather than disorder. There are some theoretical indications that the transition from a quantum Hall liquid to the Wigner crystal is first order [8]. In this paper we shall study the order of the transition in the absence of disorder within the framework of CSLG theory.

Within the CSLG theory, the transition in the quantum Hall effect without disorder is mapped to a boson superfluid to insulator transition driven by either interaction or some periodic potential. It is argued that the transition of boson superfluid to insulator induced by background potential belongs to the same universality class of three-dimensional *XY* model or relativistic scalar field theory with a mass term and ϕ^4 interaction [9]. Applying CSLG theory to this system naturally leads to a model of relativistic scalar field coupled with the Chern-Simons term [10]. The critical behavior is obtained by tuning the mass of the scalar field to zero. Wen and Wu [10] studied this model in the limit where N, the number of components of the scalar field, is large. They found that the Chern-Simons gauge field is a marginal perturbation to the scalar field fixed point, so that the critical exponents depend on the coefficient of the Chern-Simons term, or the statistical angle, which is defined by the filling fraction of the quantum Hall state [12]. The same phenomenon was found in the Mott transition of anyons on a lattice [11], where the scaling exponents depend on the statistics of the anyons.

While we agree with the result [10] within the large N approximation, in this paper we show that in the physically relevant case of $N = 1$, and in the regime where perturbation expansion is valid, the phenomenon of statistical angle-dependent critical exponents does not occur [12]. In fact there is no critical point in the weak-interacting theory when gauge field is present. Instead, the transition is first order, driven by the fluctuations of the gauge field. The difference with the previous large N result lies in the underestimate of the gauge fluctuations within the large N approximation.

The phenomenon of the first order transition induced by gauge fluctuations is known in both particle physics [13] and condensed matter physics [14]. Coleman and Weinberg [13] studied the four-dimensional Abelian Higgs model and found that even if one tunes bare mass to zero, the fluctuations of the gauge field always generate a mass dynamically. Thus there is no critical point in the theory. Independently, Halperin, Lubensky, and Ma [14] discovered the same phenomenon in the Ginzburg-Landau theory of superconductor to normal metal transition and showed that the fluctuations of the electromagnetic field induce a first order transition. Our work is basically an application of these ideas to the quantum

 0031-9007/94/73(24)/3282(4)$06.00

Hall systems: We show the phenomenon of fluctuation-induced first order phase transition also occurs here.

Since the CSLG model is constructed from some general principles, we believe that our analysis implies that the commensurate Mott transition from a quantum Hall liquid to an insulating state in the absence of disorder is generally a first order transition. This model can be useful in understanding the transition from the quantum Hall liquid state to the Wigner crystal state which is believed to be a first order transition [8]. Since there is no critical point in this model, it cannot be used to address any questions about the critical properties of the disorder-induced transition in the quantum Hall systems. In particular, the phenomenon of statistical-angle-dependent critical exponents does not occur in this model, therefore there is no theoretical basis to conjecture that similar phenomenon would occur in disorder-driven transition in the quantum Hall effect [10].

The Lagrangian of interest is the 2 + 1 dimensional relativistic scalar field coupled to the Chern-Simons gauge field. In Euclidean space it is given by

$$\mathcal{L}[a,\phi] = \sigma_{xy}\frac{i}{2}\epsilon^{ijl}a_i\partial_j a_l + \frac{1}{2}|(\partial_j + ia_j)\phi|^2 + \frac{1}{2}m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 + \frac{g}{3}|\phi|^6, \quad (1)$$

where $\sigma_{xy} = 1/\theta \equiv 1/2\pi q$ is the Hall conductivity, q is an odd integer, and m, λ, and g are the mass and interaction constants of the scalar field. We have explicitly included the sixth order term, which is a marginal operator in three dimensions. A logarithmically divergent contribution to the sixth order term occurs in the loop expansion, therefore, keeping this term is necessary to ensure renormalizability.

When the complex field ϕ develops a vacuum expectation value, the U(1) symmetry is spontaneously broken. This superfluid phase of the boson field corresponds to the quantum Hall liquid state [2]. On the other hand, the insulating state corresponds to the case when the vacuum expectation value of the field ϕ vanishes. At the level of the classical potential, this transition occurs at $m = 0$. The $\lambda\phi^4$ interaction is a relevant perturbation to this Gaussian fixed point. The associated infrared divergences are usually controlled either by introducing a large N generalization of the model and carrying out a systematic $1/N$ expansion or by staying close to the critical dimension of four and carrying out an $\epsilon = 4 - D$ expansion. In the present context, however, the $1/N$ expansion has the drawback of underestimating the effect of the gauge field fluctuations. Since there are N scalar components and only one component of the gauge field, the effect responsible for fluctuation-induced first order phase transition is not visible to first order in $1/N$. For example, in the case of a superconductor to normal state transition, the gauge fluctuations only induce a first order transition when $N < 365.9$ [14]. On the other hand, the ϵ expansion from four dimensions is not well suited for the Chern-Simons term which is naturally defined at $D = 3$. In this work we control the infrared divergences associated with the massless point $m = 0$ by staying close to the "tricritical point" [15] where $\lambda = 0$ as well and studying the effects of marginal operators σ_{xy} and g within the loop expansion.

Without the coupling to the Chern-Simons gauge field, the point $m = \lambda = 0$ is tricritical: For a finite positive g the transition is first order when $\lambda < 0$, and it is second order when $\lambda > 0$; the first and second order lines meet at the tricritical point. The critical dimension for the theory at the tricritical point is $D = 3$, one can therefore carry out a systematical perturbation expansion in the number of loops. We shall show that with the coupling to the Chern-Simons gauge field the physics is changed fundamentally: While the model at the tricritical point is massless and has no dimensionful coupling constants at the classical level, quantum fluctuations associated with the Chern-Simons gauge field induce an effective potential which has a minimum away from $\phi = 0$ and some dynamically generated mass scale. Since the physical mass scale at the point $m = \lambda = 0$ is *finite,* introducing a small finite λ could not lead to uncontrolled infrared divergences. From this argument one can show that the fluctuation-induced first order transition may occur even in the presence of a finite λ.

We first present our result of the one-loop effective potential calculated within the path integral technique. Within one-loop accuracy, we assume a space-independent configuration of the scalar field $\phi(x) = \phi_0$ and integrate out the fluctuations due to both the scalar and Chern-Simons gauge fields. The equivalent sequence of Feynman diagrams for this calculation is shown in Fig. 1.

At the classical level, the Chern-Simons gauge propagator is purely off diagonal; it is proportional to $\epsilon^{ijl}\hat{p}_l/p$. At the quantum level only finite renormalization of the Chern-Simons term may happen [16]. There could also appear an additional diagonal term in the gauge propagator; we verified that it is also finite to the second-loop order. The "dressed" propagator, accounting for this correction, is given by

$$D_{ij} = \rho_{xx}\frac{p^2\delta_{ij} - p_i p_j}{p^3} - \rho_{xy}\frac{\epsilon^{ijl}p_l}{p^2}, \quad (2)$$

where, as usual, $\rho_{xx} = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2)$, $\rho_{xy} = \sigma_{xy}/(\sigma_{xx}^2 + \sigma_{xy}^2)$. At the one-loop level σ_{xy} is not renormalized, whereas a finite $\sigma_{xx} = N/24$ is obtained for the N-component scalar field. We shall use this form of the gauge propagator to keep our analysis general.

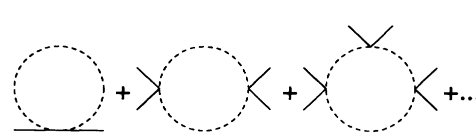

FIG. 1. Gauge loops contributing to the effective action (7).

The one-loop effective action is defined by

$$S_{\rm eff}[\phi_0] = S[0, \phi_0] + \frac{1}{2} \ln \det\left(\frac{\delta^2 S[a,\phi]}{\delta[a,\phi]^2} \bigg|_{\substack{a=0 \\ \phi=\phi_0}} \right), \tag{3}$$

where the Euclidean action is $S[a,\phi] = \int d^3x\, \mathcal{L}[a,\phi]$. We calculate this fluctuation determinant at $\phi_0 =$ const within the covariant gauge $\partial_i a_i = 0$ by the ζ-function regularization scheme. The result is given by

$$\delta V(\phi) = \frac{4\rho_{xx}(3\rho_{xy}^2 - \rho_{xx}^2)\phi^6}{3\pi^2}\left(\ln\frac{2\phi^2}{M} - \frac{11}{6}\right) - \frac{(m^2 + 3\lambda\phi^2 + 3g\phi^4)^{3/2} + (m^2 + \lambda\phi^2 + g\phi^4)^{3/2}}{12\pi}, \tag{4}$$

where M is a mass scale introduced by the regularization procedure. Note that within the first-loop approximation regularized corrections due to self-interactions contain no logarithmic divergences.

Next we introduce normalization conditions to define values of the renormalized coupling constants. Because of the logarithmic dependence of our effective potential on ϕ, the coupling constant g has to be defined at some finite scale μ

$$V'''_{\phi^2}(\phi^2 = \mu) = 2g\,. \tag{5}$$

Also, in the tricritical point

$$V''_{\phi^2}(\phi^2 = 0) = \lambda = 0, \quad V'_{\phi^2}(\phi^2 = 0) = m^2 = 0\,. \tag{6}$$

With these normalization conditions, we arrive at the one-loop effective potential

$$V_{\rm eff}(\phi) = \left[g + \frac{4\rho_{xx}(3\rho_{xy}^2 - \rho_{xx}^2)}{\pi^2}\left(\ln\frac{\phi^2}{\mu} - \frac{11}{6}\right)\right]\frac{\phi^6}{3}\,. \tag{7}$$

The qualitative features of this effective potential are very similar to those obtained from the scalar electrodynamics in four dimensions [13]. In four dimensions, both coupling constants, the electric charge e, and λ are dimensionless, and the resulting effective potential can be viewed as logarithmic corrections to the $\lambda\phi^4$ coupling. Here in three dimensions, both the gauge couplings ρ_{xx} and ρ_{xy} and the scalar coupling g are dimensionless; the resulting effective potential can be viewed as logarithmic corrections due to these marginal operators. The effective potential is bounded when $3\rho_{xy}^2 > \rho_{xx}^2$, which is so in the clean fractional quantum Hall system we are interested in. In the opposite case, the effective potential is unbounded at the one-loop level; however, we believe that higher order corrections will cure this unphysical feature [17]. In the case of $3\rho_{xy}^2 > \rho_{xx}^2$ one can easily see that (7) has a global minimum at $\phi_{\rm min}^2 = \mu \exp(3/2 - g/\alpha)$, away from the origin, and the value of the effective potential at this minimum $V_{\rm eff}(\phi_{\rm min}) = -(5\alpha\mu^3/6)\exp(9/2 - 3g/\alpha)$ is negative. Here α is the coefficient in front of the logarithmic term in (7). Furthermore, the effective mass at this new minimum is given by $m_{\rm eff}^2 = V''_{\rm eff}(\phi_{\rm min})/2 = 2\alpha\mu^2 \exp(3 - 2g/\alpha)$. This is the general feature of the dimensional transmutation: Although the classical model at the tricritical point $m = \lambda = 0$ is free of any dimensionful parameters, a mass scale is generated dynamically by the quantum fluctuations. The classical tricritical point is, in fact, not a critical point at all.

Our analysis is carried out at the point $m = \lambda = 0$. Without the coupling to the Chern-Simons gauge field, this is a Gaussian fixed point if $g = 0$ and a tricritical point if $g > 0$. In both cases λ is a relevant perturbation in three dimensions. Conclusions reached for the case $\lambda = 0$ certainly do not carry over to the case of finite λ because of the strong infrared divergence associated with this relevant perturbation. In the present case, however, since the effective mass at the point $m = \lambda = 0$ is finite, there is a finite "Ginzburg region" defined by

$$\lambda \ll m_{\rm eff}, \quad m^2 \ll m_{\rm eff}^2\,. \tag{8}$$

As long as λ and m lie inside this region, their effects can be treated classically. By studying the structure of the minimum of the total potential, $\frac{1}{2}m^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4 + V_{\rm eff}$, we obtain the phase diagram shown in Fig. 2. Note that while the position of the tricritical point is shifted, it is still far away from the edge of the applicability of the theory which in the dimensionless variables is given by $\lambda_* \ll \sqrt{2}e^{3/2}, m_*^2 \ll 2e^3$; this implies that the fluctuations due to λ and m do not change the qualitative features of the phase diagram. From Fig. 2 we see explicitly that the transition is first order for small enough λ. For large enough λ the transition from the insulator to quantum Hall liquid may be second order, and there is also a first

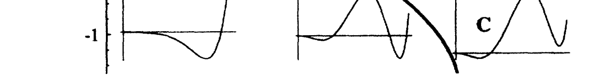

FIG. 2. The phase diagram of the one-loop effective action (7) in the scaled coordinates $m_*^2 = m^2 e^{2g/\alpha}/\alpha\mu^2$ versus $\lambda_* = \lambda e^{g/\alpha}/\alpha\mu$. The solid line represents the first order phase transition between the nonsymmetric and symmetric phases A and B for $m_*^2 > 0$; for $m_*^2 < 0$ it is the first order transition between two nonsymmetric phases A and C with different expectation values of the order parameter ϕ. The second order transition between B and C is shown with the dashed line. Captions draft the effective potential $V_{\rm eff}(\phi)$ in appropriate regions. Dots show the phase boundary without the coupling to the gauge field.

order transition between two quantum Hall liquid states. However, these phenomena all occur in the regions of parameter space which are not perturbatively connected to the noninteracting theory; one cannot make any definitive statement about the character of these transitions.

A drawback of the one-loop approximation is that logarithmic corrections to the potential $g\phi^6$ vanish when $\rho_{xx} = 0$; therefore, if we strictly use the one-loop approximation with the pure Chern-Simons propagator rather than Eq. (2), the regularized effective potentials do not change. It turns out that at the second-loop level the finite renormalization to the gauge propagator is automatically included. Therefore, to make our analysis complete, we have carried out the two-loop calculation of the effective potential. We have checked that infinite corrections for ρ_{xx} and ρ_{xy} vanish in the second-loop order (this result for the pure Chern-Simons propagator has been obtained earlier [16]). The effective potential is obtained from the solution of the renormalization-group (RG) equation:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + 2\gamma(g)\phi^2\frac{\partial}{\partial\phi^2}\right)V = 0, \quad (9)$$

subject to initial conditions (5) and (6). To the second-loop order the coefficients are

$$2\gamma = -\left(\frac{4\rho_{xx}}{3\pi^2} + \frac{7\rho_{xy}^2}{24\pi^2}\right), \quad (10)$$

$$\beta = -\frac{12}{\pi^2}\rho_{xx}\rho_{xy}^2 - \frac{33}{4\pi^2}\rho_{xy}^4 + g\left(\frac{37}{8\pi^2}\rho_{xy}^2 + \frac{4}{\pi^2}\rho_{xx}\right) - \frac{7}{12\pi^2}g^2. \quad (11)$$

From (11) we see explicitly that g flows to negative values if its bare value is small enough,

$$g < g_* \approx 3\rho_{xy}^2\frac{\rho_{xx} + 11\rho_{xy}^2/16}{\rho_{xx} + 37\rho_{xy}^2/32}.$$

This "runaway" trajectory is usually taken as an indication of the first order phase transition [14]. For small g, we can keep only the first two terms in the expansion of the β function in g, $\beta(g) = \beta_0 + \beta_1 g$. Integrating the RG equation within this approximation we obtain the effective potential

$$V(\phi) = 2\phi^6\left(\frac{g - g_*}{(3+\delta)(2+\delta)(1+\delta)}\left(\frac{\phi^2}{\mu}\right)^{+\delta} + \frac{g_*}{6}\right), \quad (12)$$

where $\delta = \beta_1/(1 - 2\gamma) \approx \beta_1$. This potential has a global minimum away from the origin, indicating the first order transition. In our approximation it is bounded for all physical values of ρ_{xx} and ρ_{xy}, and the first order transition occurs even when $\rho_{xx} = 0$.

In conclusion, we have investigated the order of the commensurate Mott transition from a quantum Hall liquid state to an insulating state *in the absence of disorder.* Within the CSLG theory, and in the regime of parameter space where perturbation expansion is valid, we show that this transition in generally of first order, induced by the fluctuations of the Chern-Simons gauge field. Because there is no critical point in this theory, the phenomenon of the statistical-angle-dependent critical exponents does not occur. We believe that our theory is generally applicable to quantum Hall phase transitions dominated by interaction rather than disorder, which is the opposite limit compared to that studied in the work on the global phase diagram [2]. In particular, this theory could provide a framework to study the transition from a quantum Hall liquid to a Wigner crystal in the limit of pure samples.

We would like to thank S. Kivelson for a stimulating discussion that motivated this work and R. Kallosh for discussion on some aspects of perturbative expansions in the quantum field theory. S. C. Z. is supported in part by the Center for Materials Research at Stanford University, and L. P. would like to thank the IBM Corporation for support through the IBM graduate fellowship program.

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