Transition Toward Developed Turbulence

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Using an axisymmetric jet with helium at low temperature we have studied the velocity intermittency on a large range of Taylor scale based Reynolds number R_λ. The results show, for the first time, evidence for a transition between two types of turbulence in a jet. Above the transition ($R_\lambda > 180$) a clear separation occurs between small and large scales. In the intermediate range, the existence of a Reynolds dependent scaling law is confirmed. It illustrates the way in which viscosity can have some influence up close to the integral scale in developed turbulent flows.

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One central question of developed turbulence is whether there is a unique universal behavior or whether there are transitions between different kinds of high Reynolds number turbulence. The major difficulty in studying this question is that usually one experimental system gives access only to a limited range of Reynolds numbers. Physicists are forced to compare results from different experiments like grid, jets, and tunnel flows with the hope that the Taylor scale based Reynolds number R_λ is the unique and universal parameter for turbulence [1].

Cryogenic experiments with gaseous ^{4}He [2] offer the possibility of covering a large range of R_λ with a fixed geometry. Helium gas has the smallest known kinematic viscosity, $\nu \simeq 2 \times 10^{-8}$ m^2/s, close to the critical point (2.2 bars, 5.2 K) and can be an appreciably viscous fluid at low pressure, P ($\nu P \simeq$ const) [3]. We have constructed the first open flow working in such conditions with an axisymmetric jet. We have developed the corresponding anemometry technics [4]. In the following, we present briefly the experimental setup and the anemometer. Then we discuss the first results which concern the velocity intermittency at inertial scales in developed turbulence.

The experimental system consists of a chamber (Fig. 1) immersed in a liquid ^{4}He bath which also acts as a ballast for the working gas. After laminarization, this gas enters the chamber through a contracting nozzle of section ratio 100 ($\phi = 2$ mm). The jet develops downward in the chamber, which is pumped to ensure the circulation. The return flow is isolated from the jet by a jacket.

An on-scale reproduction of the chamber has been made of glass to work with dyed water at room temperature. It showed that a grid was necessary downstream before the jet developed up to the walls: In the absence of the grid, the jet randomly attached to the walls. We have visually checked to see that the grid introduces no perturbation upstream and that no secondary flow is visible.

Our anemometer is an adaptation of the hot wire technics using superconducting materials. For more details see Ref. [4]. Its typical space resolution is about 10 μm.

The wire is operated at constant resistance ("constant temperature") through a homemade 10 MHz lock-in amplifier, and the total response time to a perturbation can be less than 1 μs depending on the loop gain [4]. The voltage signal is recorded through a 12 bit HP5183 driven

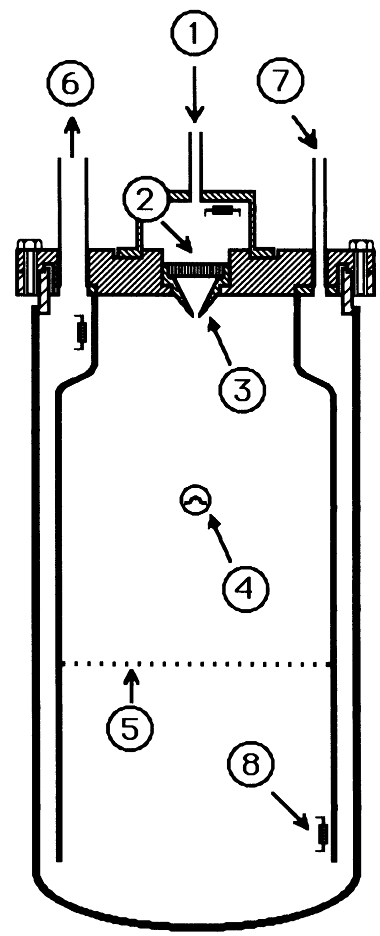

FIG. 1. Experimental setup: (1) gas injection, (2) laminarization honeycomb, (3) nozzle, (4) detector, (5) stabilization grid, (6) pumping exit, (7) pressure measurement, and (8) symbol for thermometers.

by a McQuadra 700 computer. The average signal is about 3.5 to 5.5 V, and the instrumental noise is about 10^{-7} V^2/Hz. The calibration of the wire has been made in the potential cone of the jet as described in Ref. [5].

The probe is placed on the axis of the jet at a distance of 50 times the nozzle diameter (2 mm), 7 cm above (before) the grid. At this distance, the diameter of the jet is about 3 cm, and the diameter of the jacket is 13 cm. The mean velocity U, the turbulence ratio (~25%), and the dissipation are in agreement with those commonly observed [6]. It is traditional [1] to study the inertial scales intermittency by looking at the statistics of velocity differences δv between two times on the same probe. This time lag τ is labeled as a distance $r = U\tau$, invoking the Taylor frozen turbulence hypothesis. This r-τ correspondence gives reasonable values for instance for the integral scale L ($L = 1 \pm 0.2$ cm, independent of the Reynolds number). It has only a qualitative importance in our analysis.

For every Reynolds number 10^7 data points were recorded in the form of 20 records of 5×10^5 points. Between two records a few seconds were needed for the transfer of data. A typical sampling frequency at $R_\lambda = 700$ is 6×10^4 Hz, and the whole recording time is less than 10 min.

Next, we want to present the first results of our system. We investigate the shape of the probability density function (PDF) of δv as a function of the scale r at different Reynolds numbers. We discuss it in terms of the parameter $\lambda^2(r)$ whose physical meaning is proportional to the number of energy cascade steps down to the considered scale r [7].

It is now well known that the shape of these PDF depends on the scale r. It goes from a nearly Gaussian shape at large scales L to a nearly exponential shape at the small (Kolmogorov) scale η, where viscous dissipation occurs [1]. The PDF at scale r can be written in a normalized way:

$$\frac{1}{\sigma_r} P_r \left(\frac{\delta v}{\sigma_r} \right), \tag{1}$$

which defines the function $P_r(\sigma_r^2 = \langle \delta v^2 \rangle)$. It has been shown [8] to be both useful and physically sound to represent this PDF as the superposition of large scale ones:

$$\frac{1}{\sigma_r} P_r \left(\frac{\delta v}{\sigma_r} \right) = \int G_r(\ln\sigma) \frac{1}{\sigma} P_L \left(\frac{\delta v}{\sigma} \right) d\ln\sigma \,. \tag{2}$$

This allows us to simply review the various approaches to intermittency. In the Kolmogorov-Obukhov theory [1,9], G_r is Gaussian:

$$G_r(\ln\sigma) = \frac{1}{\lambda (2\pi)^{1/2}} \exp\left[-\left\{ \frac{\ln^2(\sigma/\sigma_m)}{2\lambda^2} \right\} \right], \tag{3}$$

and λ^2 is proposed to be linear in $\ln r$. In the multifractal theory [10] σ is proportional to r^h and the distribution of σ comes from the fractal distribution of exponents h [11].

In any case the shape of P_r is mainly determined by $\langle (\delta \ln\sigma)^2 \rangle$ (i.e., by the width of G_r). As shown in previous papers [7,12], this allows one to measure this important parameter in the following way: As the shape of G_r has a poor influence, we use a Gaussian ansatz [Eq. (3)] and calculate a model distribution $P_{\lambda^2}(\delta v/\sigma_r)$ through Eq. (2) [13]. We then search for the parameter λ^2 which makes the shape of P_{λ^2} fit the shape of the experimental distribution P_r. It has been shown [7] that, to a good accuracy,

$$\lambda^2 = \langle (\delta \ln\sigma)^2 \rangle \,.$$

Figure 2 illustrates the assertion that the shape of P_r depends mainly on λ^2. PDF corresponding to various Reynolds numbers and various scales r, but having the same measured λ^2, have been plotted versus $\delta v/\sigma_r$. The difference between them is small. Also shown, by a solid line, is the model distribution P_{λ^2}.

Next we are interested in characterizing the changing shape of the PDF with the chosen scale r, i.e., we investigate the r dependence of λ^2. Within the energy cascade picture, the measured λ^2 [that is, $\langle (\delta \ln\sigma)^2 \rangle$] can be interpreted as proportional to the number of cascade steps [7,11]. It has been shown in several flows [12,14] that the λ^2 behavior vs r might be closer to a power law $\lambda^2 \propto r^{-\beta}$ than to the Kolmogorov-Obukhov law $\lambda^2 \propto \ln(L/r)$. In order to firmly establish the validity of the former law on pure experimental grounds we differentiate $\ln\lambda^2$ vs $\ln r$. This is possible thanks to the quality of our determination of λ by systematically minimizing a χ^2 quantity [7].

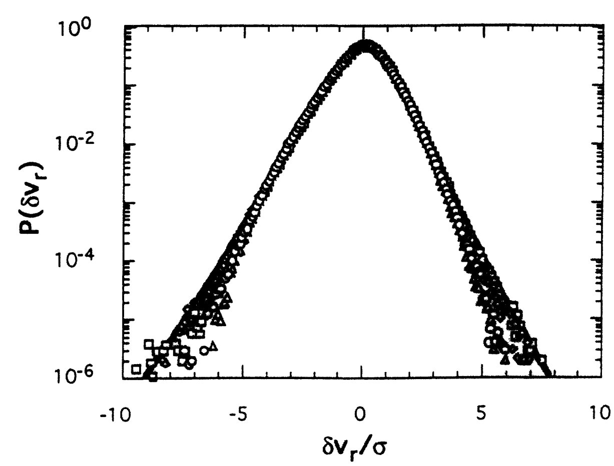

FIG. 2. Probability density functions of δv_r (normalized to its standard deviation) for different R_λ and the same λ^2 parameter. The open symbols correspond to the following parameters (η is the Kolmogorov dissipative scale): triangle, $R_\lambda = 90, r = 20\eta$; circle, $R_\lambda = 161, r = 24\eta$; diamond, $R_\lambda = 328, r = 24\eta$; square, $R_\lambda = 598, r = 62\eta$. The solid line presents the calculated PDF with $\lambda^2 = 0.09$.

We estimated the error bar on the derivative in the following way: One of our 10^7 measurements recording was cut into 10 samples of 10^6 measurements each. The derivative $d\ln\lambda^2/d\ln r$ was estimated for each of these samples, and the distribution of the obtained values gave an estimate of the random error. This error can be written as

$$\Delta\left(\frac{d\ln\lambda^2}{d\ln r}\right) \simeq \frac{A(10^6)}{\lambda\delta\ln r},$$

where $\delta\ln r$ is the step taken in the calculation of the derivative and $A(10^6) \simeq 6\times10^{-3}$ is measured. We consider that using the whole 10^7 measurement recording is equivalent to averaging, and we estimate $A(10^7) = 2\times10^{-3}$.

Figure 3(a) compares a Kolmogorov-Obukhov analysis and a power law analysis for λ^2 at $R_\lambda = 328$. We see, as in previous studies [12,14], that a power law allows one to fit a wider range than a logarithmic law. The implications of this result are lengthily discussed in Refs. [12] and [14], and we focus here on the experimental evidence.

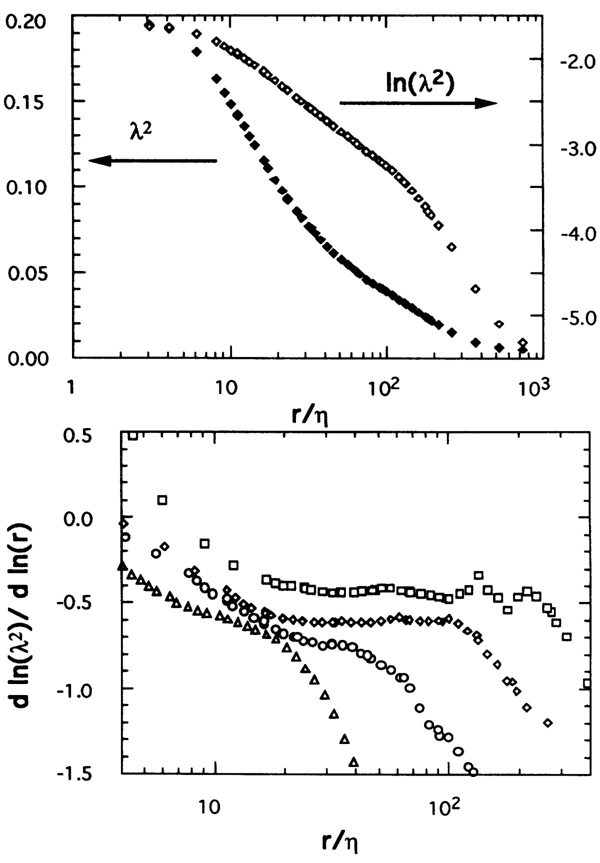

FIG. 3. (a) Dependence of λ^2 on r for $R_\lambda = 328$. The solid diamonds correspond to a logarithmic linear presentation, whereas the open diamonds correspond to a double logarithmic presentation. (b) $d\ln\lambda^2/d\ln r$ as a function of r for different R_λ values. (The different symbols present the same parameters as in Fig. 1.)

The derivative of $\ln\lambda^2$ vs $\ln r$ for several representative Reynolds numbers is shown in Fig. 3(b). This new analysis demonstrates the existence of plateaus for the largest R_λ, which experimentally proves the validity of the power law. For instance, for $R_\lambda = 328$, the plateau extends from $r = 15\eta = 320\ \mu$m to $r = 100\eta = 2.2$ mm that is close to the largest eddy size (η is the Kolmogorov length, the integral scale is 1 cm).

Based on the established power law scaling of λ^2 with r, it is now possible to characterize with only one exponent β the whole statistics of the turbulence within the scaling range [14]. The exponent β, which is the value of the derivative $-d\ln\lambda^2/d\ln r$ in the plateau region, has been measured for a whole range of Reynolds numbers and compared to previous measurements. Following the proposed scaling of β [14], we plot, in Fig. 4, $1/\beta$ vs $\ln R_\lambda$. Good agreement is found with other jet experiments.

Coming back to Fig. 3(b), it is obvious that the plateau disappears for the lowest Reynolds numbers. This shows the existence of a transition by which the flow goes from a "hard turbulence" [15] in which the inertial range could be assimilated to the plateau region to a "soft turbulence" state in which no inertial range is evidenced. This transition occurs for $R_\lambda = 180 \pm 40$, which explains why we report no value of β under this threshold (Fig. 4), while we made experiments down to $R_\lambda = 90$. Curiously the grid experiments referred to in Fig. 4 give a value of β for Reynolds numbers under the transition. A possibility thus exists for the position of this transition being flow dependent.

As often, the nature of the transition gives a hint of characterizing the turbulent states above and below. At low R_λ, the evolution is continuous from small scales to large ones. The soft turbulence could be said to be

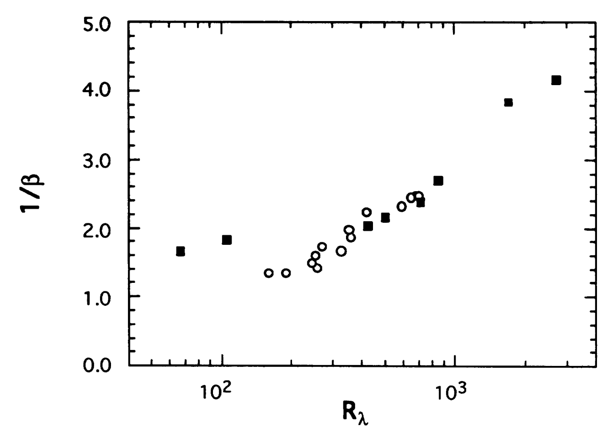

FIG. 4. Inverse of the scaling exponent β as a function of R_λ. Open circles are results from our jet experiment. The solid squares (see Ref. [14]) correspond to analog results of several different experiments, for which the four middle ones correspond to air jets, and the two smallest ones in R_λ correspond to a grid flow.

dissipative in nature. At large R_λ, the plateau introduces a clear separation between dissipative scales and large (inviscid?) scales. The plateau extends up to scales in the neighborhood of the integral one. The value of β is R_λ dependent, which shows how the viscosity can have an indirect influence far from the dissipative scales.

Looking at Fig. 3(b), we note further that it suggests an analogy with isotherms of a thermodynamic liquid-gas system. The transition would be the analog of the critical point, and $\ln(L/r)$ takes the place of the volume. λ^2 would be the equivalent of the partition function Z in statistical mechanics, and $\ln\lambda^2$ is proportional to the "free energy." If this interpretation is correct, it means that the power law behavior for λ^2 vs r results from a convexity condition for $\ln\lambda^2$ vs $\ln r$. A theoretical justification for such a convexity would result in a great step in our understanding of intermittency. We shall not develop this analogy here [16]. Let us, however, remark its coherence with what is said above: It implies indeed that an equilibrium exists between the large, turbulent scales and the small, dissipative ones. This equilibrium governs the whole organization of the intermediate scales.

In conclusion, we have built an original jet experiment allowing a broad variation of Reynolds number at constant geometry. With this apparatus we have shown for the first time that the inertial range appears in a turbulent jet flow through a well defined "second order" transition (analogous to a critical point). Two important characteristics of our experiment allowed for the discovery of this transition. First, this is due to the precision of our λ^2 measurements. This quantity (which is intuitively proportional to the number of energy cascade steps) appears as one of the finest in characterizing intermittency. Second, the possibility to span a large range of R_λ at constant geometry was essential to understand the nature of this transition. It seems to be that by which an inertial range appears. Above the transition, viscosity has some influence much beyond the Kolmogorov scale, up to the neighborhood of the integral scale. This is of prime importance for characterizing flows when the small scale resolution cannot be reached, for instance in the large eddy simulations.

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$$P_L(x) = A\exp\left\{-\frac{x^2}{2}\left[1 + a_s\frac{x}{(1+x^2)^{1/2}}\right]\right\},$$

where $a_s = 0.18$, and A is normalization constant..

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