Steady-State Spatial Screening Solitons in Photorefractive Materials with External Applied Field

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Steady-state dark (bright) planar spatial solitons are predicted for photorefractive materials when the diffraction of an optical beam is exactly compensated by nonlinear self-defocusing (focusing), due to the screening field set up around a dark notch (or a bright beam) in a photorefractive material to which an external field is applied. These screening solitons appear in steady state and differ from previously observed spatial solitons in their properties and physical origin.

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Bright and dark solitons in Kerr media have been studied for three decades [1–6]. Recent work on spatial solitons in photorefractive media have shown theoretically and demonstrated experimentally that the application of an external field to these materials enables long-lived, but transient, solitons [7–11]. A related study has shown theoretically that photovoltaic materials (such as lithium niobate) may support bright and dark steady-state planar solitons [12].

We show theoretically that the application of an external field enables steady-state planar solitons in photorefractive materials, which result from nonuniform screening of the external field. The intensity of the light excites charges, which migrate in the presence of the external field and are captured (recombine) by deep (ionized donors or acceptors) traps. The shape of the light beam gives rise to nonuniform trapping, which results in nonuniform screening of the external field. Thus, the magnitude of the electric field is *lowered* in regions of higher optical intensity, and this field modifies the refractive index via the linear electro-optic (Pockel's) effect and traps the beam (or the dark notch in the beam, depending on the sign of the index perturbation). These steady-state "screening" solitons differ from those observed or studied previously in their physical origin, their properties, and their dependence on the light intensity.

We start with the standard set of rate, continuity, and Poisson's equations that describe the photorefractive effect in a medium in which electrons are the sole charge carriers, plus the wave equation for the slowly varying amplitude of the optical field. In the steady state and two dimensions these equations are [12,13]

$$(s|A|^2 + \beta)(N_d - N_d^i) - \gamma \hat{n} N_d^i = 0, \qquad (1)$$

$$\nabla \cdot \hat{\mathbf{J}} = \nabla \cdot (q\mu\hat{n}\hat{\mathbf{E}} + k_B T \mu \nabla \hat{n}) = 0, \qquad (2)$$

$$\nabla \cdot \hat{\mathbf{E}} + (q/\varepsilon_s)(\hat{n} + N_A - N_d^i) = 0, \qquad (3)$$

$$\left(\frac{\partial}{\partial z} - \frac{i}{2k}\frac{\partial^2}{\partial x^2}\right)A(x,z) = \frac{ik}{n_b}\Delta n(\hat{E})A(x,z), \qquad (4)$$

$$V = -\int_{-\ell/2}^{\ell/2} dx\, \hat{\mathbf{E}}, \qquad (5)$$

where $\Delta n(\hat{E}) = \frac{1}{2}n_b^3 r_{\rm eff}\hat{E}$ is the perturbation in the refractive index, and the independent variables are z the propagation axis and x the transverse coordinate; the five dependent variables are $\hat{n}$ the electron number density, N_d^i the number density of ionized donors, $\hat{\mathbf{J}}$ the current density, $\hat{\mathbf{E}}$ the space-charge field inside the crystal, and A the slowly varying amplitude of the optical field defined by $E_{\rm opt}(x,z,t) = A(x,z)\exp(ikz - i\omega t) +$ c.c. ($k = 2\pi n_b/\lambda$, where λ is the wavelength in vacuo and ω is the frequency). Relevant parameters of the crystal are N_d the total donor number density, N_A the number density of negatively charged acceptors that compensate for the ionized donors, s the photoionization cross section, β the dark generation rate, γ the recombination rate coefficient, μ the electron mobility, ε_s the low frequency dielectric constant, n_b the background refractive index, $r_{\rm eff}$ the effective electro-optic coefficient, and ℓ the width of the crystal between the electrodes; q is the charge on the electron, k_B is Boltzman's constant, and T is the absolute temperature. Finally, V is the external voltage applied to the crystal.

We look for solutions of the form

$$A(x,z) = u(x)\exp(i\Gamma z)\, I_{\rm dark}^{1/2}, \qquad (6)$$

where Γ is the propagation constant and $I_{\rm dark} = \beta/s$ is the equivalent dark irradiance. We limit our analysis to real $u(x)$. Since $|A|^2$ depends on x alone, we look for solutions in which the dependent variables n, N_d^i, $\hat{\mathbf{J}}$, and $\hat{\mathbf{E}}$ depend solely on x, and the only component of $\hat{\mathbf{E}}$ and $\hat{\mathbf{J}}$ is in the x direction. We transform the equations to dimensionless form by the substitutions $n = \hat{n}/N_A$, $E = |\hat{\mathbf{E}}|qL_D/k_BT$, $N = N_d^i/N_A$, $r = N_d/N_A$, $J = |\hat{\mathbf{J}}|L_D/\mu N_A k_B T$, and $\xi = x/L_S$, where

$L_D = (k_B T \varepsilon_s / q^2 N_A)^{1/2}$ is the Debye length, L_S is the soliton length scale defined by $L_S = 1/(\pm 2kb)^{1/2}$, where $b = (k/n_b)\frac{1}{2} n_b^3 r_{\text{eff}} k_B T / q L_D$ is the parameter that characterizes the strength and the sign of the optical nonlinearity. However, only the relative sign between the space charge field $\hat{\mathbf{E}}$ and the crystalline axes is meaningful. In strontium barium niobate (SBN) [7–11], the largest electro-optic coefficient is r_{33}, and x is, therefore, parallel to the crystalline c axis. The sign of r_{eff}, with respect to the polarity of the externally applied field which is used in a given observation, determines the sign (positive or negative) of the perturbation in the refractive index Δn. Since b now can be either positive or negative (corresponding to $\Delta n > 0$ or $\Delta n < 0$), we introduce the dual-sign ($\pm$) notation in the definition of L_S, where the upper and the lower signs apply to positive and negative values, respectively, of b (and, consequently, of Δn). The dimensionless equations are

$$u'' = \pm[\Gamma/b + E]u, \tag{7}$$

$$n - a(1 + |u|^2)(r - N)/N = 0, \tag{8}$$

$$J = nE + \varepsilon n' = \text{const}, \tag{9}$$

$$N - 1 - n - \varepsilon E' = 0, \tag{10}$$

$$C + \int_{-\ell/2L_S}^{\ell/2L_S} d\xi \, E = 0. \tag{11}$$

The prime stands for the derivative with respect to the variable ξ, $C = qVL_D/k_BTL_S$, and a and ε are small parameters defined by $a = \hat{n}_{\text{dark}}/(N_d - N_A)$, and $\hat{n}_{\text{dark}}$ is the dark electron density and $\varepsilon = L_D/L_S$, the ratio of the Debye length to the soliton length. For typical photorefractive materials, $\hat{n}_{\text{dark}}$ is 4–10 orders of magnitude smaller than $N_d - N_A$, while the Debye length L_D is about 1 μm and the soliton length L_S is about 20 μm.

Next, eliminate J by solving Eq. (9) for E and substituting into Eq. (11)

$$J = -C \Big/ \int_{-\ell/2L_S}^{\ell/2L_S} \frac{d\xi}{n}, \tag{12}$$

where we use the boundary condition that the electron number densities at each electrode or crystal face are equal. Next we expand the dependent variables u, n, N, and E is series in ε, for example,

$$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + O(\varepsilon^3). \tag{13}$$

In the limit $a \ll \varepsilon$, we may omit n compared to l in Eq. (10) and obtain, to the lowest order in ε,

$$u_0'' \pm \left[\frac{C\eta}{1 + u_0^2} - \frac{\Gamma}{b} \right] u_0 = 0, \tag{14}$$

where $\eta = \varepsilon / \int_{-\ell/2L_S}^{\ell/2L_S} d\xi/(1 + u_0^2)$, and the upper (lower) sign in Eq. (14) corresponds to positive (negative) values of Δn.

The resulting expression for the refractive index change is of the form $\Delta n \sim 1/(1 + u^2)$, *fundamentally* different than the conventional Kerr-type [1–3] media ($\Delta n \sim u^2$), or from the saturated Kerr-nonlinearity $\Delta n \sim u^2/(1 + u^2)$ characterizing the photovoltaic solitons [12].

A first integral of Eq. (14) is obtained by quadrature and reads

$$p_0^2 = B \pm \left[\frac{\Gamma}{b} u_0^2 - C\eta \ln(1 + u_0^2) \right] = 0, \tag{15}$$

where $p_0 = du_0/d\xi$, and B is a constant.

Dark solitons.—The boundary conditions for the dark soliton are $u_0(0) = 0, u_0(+\infty) = u_\infty = -u_0(-\infty)$ (u_∞ finite) while, at the same positions, all the derivatives vanish. Since $2u_0'' = dp_0^2/du_0$ (analogous identities holding for all-order derivatives), it can be shown from Eq. (15) that the vanishing of p_0 and u_0'' at any given point implies that all-order derivatives vanish at that point. This behavior assures the existence of dark solitons. By substituting $p_0(\infty) = 0, u_0''(\infty) = 0$, and $u_0(\infty) = u_\infty$ into Eq. (15) we obtain

$$p_0^2 = \pm\left\{ \frac{\Gamma}{b}(u_0^2 - u_\infty^2) - C\eta \ln\left[\frac{1 + u_0^2}{1 + u_\infty^2} \right] \right\}, \tag{16}$$

with $\Gamma/b = C\eta/[1 + u_\infty^2] > 0$. The term in braces in Eq. (16) is always positive, so that one has to choose the positive (upper) sign to obtain a real value for p_0. The consequence is rather dramatic: the *dark* screening soliton may be observed *only* for a *positive* perturbation in the refractive index, that is, if $\Delta n(\xi) > 0$ for all ξ. Intuitively, this means that the magnitude of the (positive) index perturbation *decreases* with the distance from $\xi = 0$, and the index perturbation generated by the dark screening soliton is capable of guiding a separate "probe" beam (say, at a different wavelength that cannot activate photorefractive nonlinearity on its own), in a manner similar to the guidance properties of Kerr-like dark solitons [14] (with a major difference in the sign of the index perturbation). To illustrate this important point we sketch, in the inset in Fig. 1, the characteristic index profile responsible for a dark screening soliton.

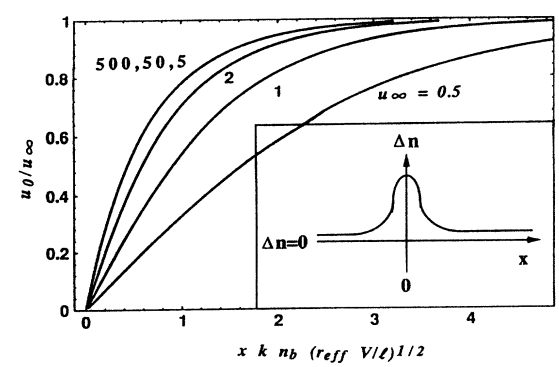

FIG. 1. Amplitude of the dark soliton u_0 divided by the amplitude at infinity u_∞, as a function of dimensionless length for $u_\infty = 0.5$, 1, 2, 5, 50, and 500. Note that the profiles are odd and that, for large u_∞, they converge to a single curve that closely resembles the conventional hyperbolic tangent soliton.

Equation (16) can be integrated numerically to obtain the profiles for the dark soliton shown in Fig. 1, as a function of $xkn_b(r_{\text{eff}}V/\ell)^{1/2}$. The propagation constant Γ can be evaluated once the solution u_0 is found. We follow this procedure in the Appendix by substituting the solution of Eq. (16) into Γ/b and performing the integral numerically in terms of the function $f[u_\infty^2]$, which is shown in Fig. 2 (dashed curve), thus obtaining

$$(\Gamma/b)^{1/2} = [CL_S/\ell + (fL_S/\ell)^2]^{1/2} - fL_S/\ell. \tag{17}$$

Note that unless u_∞^2 is large compared to 1, the first term on the right-hand side of Eq. (17) dominates, $\Gamma/b = L_S/\ell$, *independent* of u_∞^2, and the relation between soliton width and u_∞^2 is obtained directly from Fig. 1. In the regime where $u_\infty^2 \gg 1$, the relation between soliton width and u_∞^2 is obtained from Eq. (17), since the solutions shown in Fig. 1 converge to a single curve.

Bright solitons.—One requires $u_0(+\infty) = u_0(-\infty) = 0$ while, at the same positions, all the derivatives vanish. Furthermore, we require $p_0(0) = 0$ and $[u_0''(0)/u_0(0)] < 0$ to assure a local *maximum* at $\xi = 0$. Accordingly, we get

$$p_0^2 = \pm\left[\frac{\Gamma}{b}u_0^2 - C\eta\ln(1 + u_0^2)\right], \tag{18}$$

with $\Gamma/b = C\eta\ln[1 + u_0^2(0)]/u_0^2(0)$. The term inside the square brackets in Eq. (18) is always negative, so the negative (lower) sign yields a real value for p_0. This corresponds to a *negative* perturbation in the refractive index $\Delta n(\xi)$ for all ξ. Intuitively, this means that the magnitude of the index perturbation *increases* with distance from $\xi = 0$, but, since the perturbation is negative, the material behaves overall like a self-focusing medium, as sketched in the inset in Fig. 3. Equation (18) can be integrated numerically thus getting the soliton profiles shown in Fig. 3. Note again that for large $u_0(0)$, the solutions in units of $(\Gamma/b)^{1/2}$ tend towards a universal profile. We can solve as before for the propagation constant

$$\left(\frac{\Gamma}{b}\right)^{1/2} = \left[\frac{CL_S}{\ell}\frac{\ln[1 + u_0(0)^2]}{u_0(0)^2} + \left(\frac{gL_S}{\ell}\right)^2\right]^{1/2} - \frac{gL_S}{\ell}, \tag{19}$$

where the function

$$g[u_0(0)] = \lim_{\delta\to 0}\int_\delta^{u_0(0)(1-\delta)} \frac{du\,u^2}{\{-u^2 + u_0^2(0)\ln((1 + u^2)/[1 + u_0^2(0)])\}^{1/2}(1 + u^2)}$$

is plotted as a function of $u_0(0)$ in Fig. 2 (solid curve).

Both bright and dark screening solitons can be observed at modest external fields with the commonly used photorefractive crystals. For example, for SBN:75 $r_{33} = 1340 \times 10^{-12}$ m/V, $n_b = 2.3$ and at $\lambda = 0.457$ μm, and maximum intensity at least 100 times larger than the dark irradiance (which is less than 1 mW/cm^2), bright and dark solitons of 10 μm diameter require $V/\ell \sim 670$ V/cm. We note also recent observations of self-focusing effects that may lead to soliton formation under steady-state conditions in bismuth titanate with an applied field [15].

To examine convergence of the soliton expansion, we solved for the first order correction to the soliton amplitude u_1 and found that this term is small. The correction u_1 is proportional to the first derivative of the irradiance and hence tends to break up the soliton, because it represents an asymmetrical tilt, i.e., $\Delta n(\xi) = -\Delta n(-\xi)$. The requirements that these first derivative terms be small can be physically expressed in terms of the diffusion field at the soliton length scale, $E_D = k_BT/qL_S$, the limiting space charge field at the soliton scale $E_q = qN_aL_S/\varepsilon_s$ and the field across the soliton $\hat{E}$. Basically, one requires $E_D \ll \hat{E}$ and $\hat{E} \ll E_q$. If the applied field V/ℓ is taken as $(E_DE_q)^{1/2}$ and the effective trap density is about 10^{17} cm^{-3}, then for solitons larger than10 μm the small parameters of the convergence scheme are less than $\frac{1}{100}$.

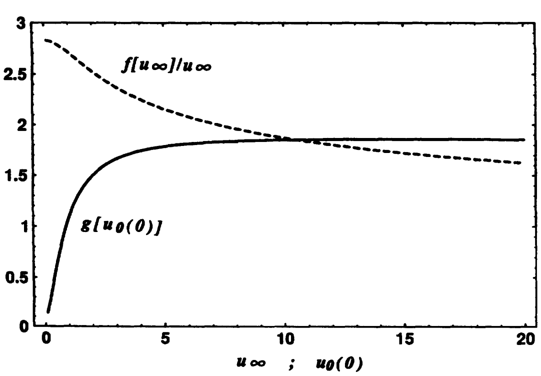

FIG. 2. Dimensionless function $f[u_\infty]/u_\infty$ as a function of u_∞ (dashed curve) and dimensionless function $g[u_0(0)]$ as a function of $u_0(0)$ (solid curve).

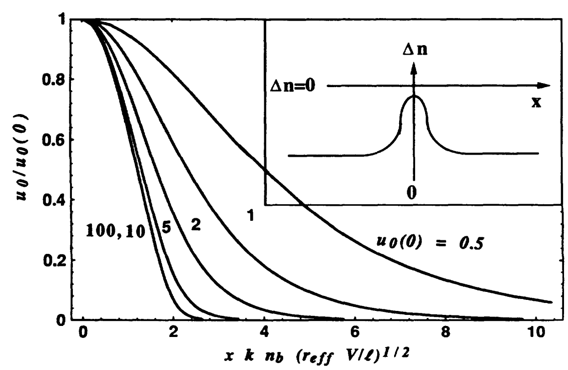

FIG. 3. Amplitude of the bright soliton u_0 divided by the amplitude at 0, $u_0(0)$ as a function of dimensionless length for $u_0(0) = 0.5$, 1, 2, 5, 10, and 100. Note that the profiles are even and that, for large $u_0(0)$, they converge to a single curve that resembles the conventional hyperbolic secant soliton.

We point out that the quasisteady photorefractive solitons [7–11] result from the terms of the order ε^2, since during the time window of their existence, the zero order terms do not depend on x (the external field is not screened yet) and the terms of order ε are very small for sufficiently large V/ℓ.

Finally, we discuss the distinct properties of these steady-state screening solitons. Although the photorefractive effect is nonlocal, the nonuniform screening effect that drives these solitons is a local effect, as manifested by the dependence of the nonlinear term in Eq. (14) on the local irradiance $|u_0(\xi)|^2$. While the photovoltaic soliton [12] is local as well (although it stems from a different physical origin), the quasisteady photorefractive soliton is nonlocal [7–11], and the space-charge field in that case is dependent on the derivatives of the optical intensity distribution. A second major difference between the screening solitons and the quasisteady photorefractive solitons is the absolute sign of the nonlinearity: In the former case *bright* solitons are generated by *negative* perturbations in the index, and *dark* solitons are driven by *positive* index perturbations, while the quasisteady photorefractive bright solitons are driven by positive index perturbations. This leads to the third difference that appears within the time window of observation: The screening solitons appear in steady state only, while the quasisteady solitons are transient by nature and are observed after the gratings have formed but before the field is screened [9–11], that is, prior to all the effects that are responsible for the screening solitons. Another property of the screening solitons is the existence of a unique solution; that is, in a given material at a specific applied voltage and a given maximal irradiance, the solitons possess a unique shape. On the other hand, the quasisteady solitons are independent of the light intensity [7–11], and they may possess different sizes, depending on the light intensity distribution at the crystal input face [11]. Control over the width of the steady-state solitons may be provided by artificially produced dark irradiance obtained through additional incoherent uniform illumination.

In conclusion, we have found a new type of spatial solitons which rely on the nonuniform screening of externally applied voltages in photorefractive media.

Appendix.—We need to evaluate the integral

$$A = \int_{-\ell/2L_S}^{\ell/2L_S} \frac{d\xi}{1+u_0^2} = 2\int_0^{\ell/2L_S} \frac{d\xi}{1+u_0^2} = 2\int_0^{u_\infty} \frac{du_0}{(du_0/d\xi)(1+u_0^2)}.$$

We separate this integral into two parts

$$A = A_1 + A_2 \equiv 2\int_0^{u_\infty(1-\delta)} \frac{du_0}{(du_0/d\xi)(1+u_0^2)} + 2\int_{u_\infty(1-\delta)}^{u_\infty} \frac{du_0}{(du_0/d\xi)(1+u_0^2)},$$

where $u_\infty = u(\ell/2L_S)$, and δ is taken to be sufficiently small so that we can approximate

$$A_2 \cong \frac{2}{1+u_\infty^2}\int_{\xi[u_\infty(1-\delta)]}^{\ell/2L_S} d\xi = \frac{\ell/L_S - 2\int_0^{u_\infty(1-\delta)} du_0/(du_0/d\xi)}{1+u_\infty^2}$$

We now add A_1 and A_2 back together and substitute for $du_0/d\xi$ to obtain

$$A = \{2(b/\Gamma)^{1/2} f[u_\infty] + \ell/L_S\}/(1+u_\infty^2),$$

where

$$f(u_\infty) = \lim_{\delta\to 0}\int_0^{u_\infty(1-\delta)} \frac{du_0(u_\infty^2 - u_0^2)}{\{(1+u_\infty^2)\ln[(1+u_\infty^2)/(1+u_0^2)] - (u_\infty^2 - u_0^2)\}^{1/2}(1+u_0^2)}.$$

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