Bloch-Nordsieck Cancellations beyond Logarithms in Heavy Particle Decays

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We investigate the one-loop radiative corrections to the semileptonic decay of a charged particle at finite gauge boson mass. Extending the Bloch-Nordsieck cancellation of infrared logarithms, the two subsequent nonanalytic terms are also found to vanish after eliminating the pole mass of the decaying particle in favor of a mass defined at short distances. This observation justifies the operator product expansion for inclusive decays of heavy charged particles and implies that infrared effects are suppressed by at least three powers of the heavy mass.

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The problem of the infrared (IR) behavior of amplitudes is inherent to gauge theories since by construction they contain massless bosons. Historically, already the first studies of the problem revealed both complexities and simplicities. On the one hand, if a finite gauge boson mass λ is employed as IR regulator, the perturbative amplitudes contain logarithms in the mass $\ln\lambda^2$, which apparently do not allow for the limit $\lambda \to 0$. On the other hand, as shown first by Bloch and Nordsieck [1], these singularities cancel if one considers inclusive processes. This summation over final states is an integral part of any calculation in gauge theories.

With the advent of QCD the problem of the IR behavior of amplitudes became even more acute, since the effective coupling blows up in the infrared. Thus, if an amplitude is perturbatively sensitive to infrared momenta, it is contributed in fact by nonperturbative effects as well and cannot be evaluated reliably. The famous prediction of QCD that the total cross section of e^+e^- into hadrons measures quark charges rests on the Bloch-Nordsieck (BN) cancellation. Indeed, all the IR logarithms disappear from the total cross section and it is determined by short distance physics. Moreover, the operator product expansion (OPE) implies that IR contributions to the e^+e^- annihilation are suppressed by four powers of the large scale. The BN-type cancellations ensure that this counting is not violated by divergencies of perturbative expansions, either explicit or through its divergence in large orders. In this sense there is a correspondence between BN cancellations and the OPE.

The example of e^+e^- annihilation into hadrons is still special since there are no colored particles in the initial state. In the general situation, it is known from the work of Kinoshita and of Lee and Nauenberg (KLN) [2] that to remove all IR singularities, summation over degenerate initial states might also be needed. This summation does not necessarily reflect the actual experimental situation and in this sense some IR logarithms can survive.

The case of acute practical interest are inclusive decays of heavy hadrons. In this case there is a quark in the initial state, and though it is generally accepted that the BN theorem still ensures the absence of *explicit* soft divergencies, it is not *a priori* clear whether one can rely on *extended* BN-type cancellations in the sense of IR power counting, implied by the OPE. One might have to invoke the more general KLN cancellation which would invalidate the OPE for inclusive decays. In particular, the standard OPE states that there are no corrections to the decay width linear in the inverse mass of the heavy quark. This conclusion is not trivial in view of the recent observation [3,4] that the pole mass of a charged particle itself does receive such corrections.

In this Letter we test the counting of IR effects implied by the OPE by an explicit calculation of the IR sensitive contributions to the decay of a charged particle. The infrared contributions are singled out by nonanalytic terms in the gauge boson mass, which, if the OPE is valid, must be associated with higher dimension operators. The leading $\ln\lambda^2$ terms are the subject of the original BN cancellation [1]. We show that the BN mechanism extends to comprise the $\sqrt{\lambda^2}$ and $\lambda^2\ln\lambda^2$ terms, if the physical (or pole) mass is eliminated in favor of a short distance mass. The cancellation of linear in λ terms is in accord with the conclusions of Ref. [3], and the cancellation of $\lambda^2\ln\lambda^2$ terms will be shown to agree to the OPE as well.

Although our present calculations are not sensitive to the nonabelian nature of QCD, it is convenient to use the QCD terminology for inclusive decays of B mesons since it is well developed. Then the QCD dynamics is contained in the matrix element of the product of two weak hadronic currents between B-meson states,

$$T^{\mu\nu}(p,q) = i\int d^4x\, e^{iqx}\langle B(p)|T\{J_h^{\mu\dagger}(x)\,J_h^{\nu}(0)\}|B(p)\rangle,$$

where an average over initial state polarizations is understood. The differential inclusive width is then given by

$$d\Gamma = \frac{G_F^2|V_{qb}|^2}{2}\,\frac{d^3k_{\bar\nu}}{(2\pi)^3 2k_{\bar\nu}^0}\,\frac{d^3k_l}{(2\pi)^3 2k_l^0} \times L_{\mu\nu}2\,\mathrm{Im}\left[\frac{1}{2p^0}T^{\mu\nu}(p,q)\right]. \qquad (1)$$

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The lepton tensor $L_{\mu\nu}$ contains the lepton momenta $k_l, k_{\bar{\nu}}$ only and the imaginary part is taken in $p \cdot q$, where $q = k_l + k_{\bar{\nu}}$. Assuming validity of the OPE, a systematic expansion in powers of the heavy quark mass results in the prediction [5]

$$\Gamma_B = \Gamma_0 \left[1 + \sum_{n=0}^{\infty} r_n \alpha(m_b)^{n+1} + \frac{\mu_K - 3\mu_G}{2m_b^2} + O\left(\frac{1}{m_b^3}\right)\right], \quad (2)$$

for the total width. Here m_b is the *pole* quark mass and $\Gamma_0 = (G_F^2 |V_{qb}|^2 m_b^5)/192\pi^3$ the tree decay width for the free quark. For simplicity, the final quark q is taken massless.

A striking feature of Eq. (2) is that corrections to the free quark decay are suppressed by *two* powers of the quark mass. They are parametrized by

$$\mu_K = \frac{1}{2m_B} \langle B|\bar{b}(iD_\perp)^2 b|B\rangle,$$

$$\mu_G = \frac{g}{4m_B} \langle B|\bar{b} i\sigma_{\mu\nu} G^{\mu\nu} b|B\rangle, \quad (3)$$

where m_B is the meson mass and we ignore radiative corrections to these terms. The crucial assumption to arrive at this conclusion is the OPE in the kinematic region where the energy release into the hadronic final state is large and the decaying quark is almost on-shell. In addition, radiative corrections to the leading operator must be unambiguous to this accuracy. In particular, the pole mass which provides the overall normalization in Eq. (2) is intrinsically ambiguous by an amount of order $\Lambda_{\rm QCD}$ [3,4]. The perturbative series that relates the pole mass to a mass defined at short distances, e.g., the $\overline{\rm MS}$ mass (where $\overline{\rm MS}$ denotes the modified minimal subtraction scheme),

$$m_b = m_b^{\overline{\rm MS}} \left(1 + \sum_{n=0}^{\infty} c_n \alpha^{n+1} \right), \quad (4)$$

exhibits a strong divergence as n increases, which leads to an IR renormalon pole in its Borel transform (to be defined below) at $t = -1/2\beta_0$ with t the Borel parameter and β_0 the first coefficient of the β function. Truncating the series at its optimal order leaves an uncertainty of order $\Lambda_{\rm QCD}$, assuming $m_b^{\overline{\rm MS}}$ as given.

The important question is whether the OPE captures *all* IR effects *after* the effects associated with the definition of mass have been accounted for. To clarify this point, we have investigated the IR structure of the asymptotic behavior of the radiative corrections to the leading term in Eq. (2). Through the appearance of IR renormalons the asymptotic behavior signals the presence of power corrections, which should be added as explicit nonperturbative corrections. These explicit corrections may—and often do—turn out numerically larger than higher order perturbative corrections, which are neglected. Any indication of such terms of order $1/m_b$ threatens the validity of the OPE for inclusive decays.

The absence of IR renormalons can be formulated as an extension of the BN cancellations. Such a relation might be envisaged, since IR renormalons probe the IR behavior of Feynman amplitudes and can therefore be traced by an IR regulator. To establish a formal connection, we recall that the large-order behavior of radiative corrections to first approximation is generated by diagrams such as in Fig. 1, with a chain of fermion bubbles inserted in the gluon line. This procedure generates a gauge-invariant set of diagrams in the Abelian theory. Denote by $\{r_n^f\}$ the series of perturbative corrections to B decays generated in this way, and define the Borel transform of the series by $B[\{r_n^f\}](t) \equiv \sum_{n=0}^{\infty} r_n^f t^n/n!$. The fermion loop insertions are renormalized and each loop is proportional to $(-\beta_0)[\ln(-k^2/\mu^2) + C]$, where β_0 includes the fermionic contribution only, μ is a renormalization point, and C a finite subtraction constant. Next call $r_0(\lambda)$ the one-loop radiative correction calculated with a finite gluon mass. We work with the standard propagator

$$-i\delta^{AB} \frac{1}{k^2 - \lambda^2 + i\epsilon} \left[g_{\mu\nu} - (1 - \xi) \frac{k_\mu k_\nu}{k^2 - \xi\lambda^2 + i\epsilon} \right]. \quad (5)$$

Then, in the Landau gauge, $\xi = 0$, we find the identity

$$r_0(\lambda) = \frac{1}{2\pi i} \int_{-1/2-i\infty}^{-1/2+i\infty} ds\, \Gamma(-s)\Gamma(1+s) \times \left(\frac{\lambda^2}{\mu^2} e^C\right)^s B[\{r_n^f\}](s), \quad (6)$$

with $s = -\beta_0 t$. To obtain this identity, one uses the effective Borel-transformed propagator of the massless gauge boson which is proportional to $1/(k^2)^{1+s}$ after summation over the fermion loops [6], whereas $(k^2 - \lambda^2)^{-1}$ in Eq. (5) can be written in an (inverse) Mellin

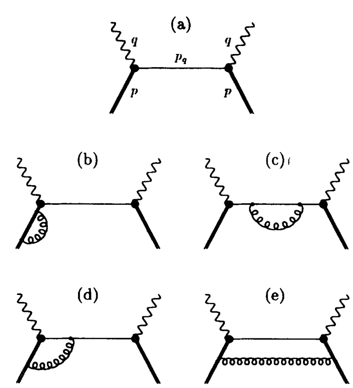

FIG. 1. (a) Tree diagram and (b),(c) radiative corrections to the free quark decay. Diagrams (b) and (d) are counted twice.

representation. The significance of Eq. (6) rests on the observation that the coefficients in front of the $(\sqrt{\lambda^2})^{2n+1}$ and $\lambda^{2n}\ln\lambda^2$ terms of the expansion of $r_0(\lambda)$ in the small mass determine the residues of the IR renormalon poles of $B[\{r_n^f\}](t)$ at half-integer and integer multiples of $-1/\beta_0$, which in turn fixes the overall normalization of the large-order behavior of the series r_n^f. In particular, cancellations of terms in the expansion of $r_0(\lambda)$ imply the absence of the corresponding renormalons. In this unified framework, the BN cancellations of $\ln\lambda^2$ in physical processes appear simply as the absence of an IR renormalon pole at $s = 0$ [7]. Since the Borel transform is gauge-independent for physical processes and the S-matrix elements of the Abelian gauge theory are independent of ξ even in the presence of a mass term, Eq. (6) holds in fact in any gauge.

Using the relation between IR renormalons and nonanalytic in λ^2 terms, we can rewrite the relation between the pole and a short-distance mass as [8]

$$m_b \sim m_{\rm SD}(1-2\tilde{\lambda}), \qquad \tilde{\lambda} \equiv \frac{C_F\alpha}{4\pi}\times\frac{\pi\lambda}{m_b}. \tag{7}$$

The presence of linear terms in this place is equivalent to the asymptotic behavior $c_n \overset{n\to\infty}{=} (C_F\mu e^{5/6})/(\pi m_b^{\overline{\rm MS}}) \times (-2\beta_0)^n n!$ for the series in Eq. (4) obtained in [3,4].

We have calculated the radiative corrections to the total width and the lepton spectrum keeping the gluon mass finite (and the final quark massless). The results for the diagrams of Fig. 1 in $4-2\epsilon$ dimensions are ("$\sim$" means the $1/\epsilon$, $\ln\lambda^2$, $\sqrt{\lambda^2}$, and $\lambda^2\ln\lambda^2$ terms of the left-hand side equal the right-hand side)

$$\begin{aligned}
T^{\mu\nu}_{(b)}(p,q) &\sim T^{\mu\nu}_{(a)}(p,q)\frac{C_F\alpha}{4\pi}\left[-\frac{\xi}{\epsilon}-(3-\xi)\ln\frac{\lambda^2}{m_b^2}+3\pi\frac{\lambda}{m_b}\right],\\
T^{\mu\nu}_{(c)}(p,q) &\sim T^{\mu\nu}_{(a)}(p,q)\frac{C_F\alpha}{4\pi}\left[-\frac{\xi}{\epsilon}+\left(2-\frac{1-\xi^2}{2}\right)\frac{\lambda^2}{p_q^2+i\epsilon}\ln\left(-\frac{\lambda^2}{p_q^2}\right)\right],\\
T^{\mu\nu}_{(d)}(p,q) &\sim T^{\mu\nu}_{(a)}(p,q)\frac{C_F\alpha}{4\pi}\left[\frac{2\xi}{\epsilon}-4\pi\omega\frac{\lambda}{m_b}\right]-\frac{1}{p_q^2+i\epsilon}\mathrm{tr}(\not{p}\Gamma^\mu\not{p}\Gamma^\nu)\frac{C_F\alpha}{4\pi}\left[2\pi\frac{\lambda}{m_b}\right]\\
&\quad+T^{\mu\nu}_{(a)}(p,q)\frac{C_F\alpha}{4\pi}[1-\xi^2]\frac{\lambda^2}{p_q^2+i\epsilon}\ln\left(-\frac{\lambda^2}{p_q^2}\right)\\
&\quad-\frac{1}{p_q^2+i\epsilon}\frac{C_F\alpha}{4\pi}\left[\frac{m_b^2}{p_q^2+i\epsilon}\mathrm{tr}(\not{p}_q\Gamma^\mu\not{p}_q\Gamma^\nu)-\frac{1}{2}m_b^2\,\mathrm{tr}(\gamma_\tau\Gamma^\mu\gamma^\tau\Gamma^\nu)\right.\\
&\qquad\left.+\left(\omega^2-2\omega-\frac{m_b^2}{p_q^2+i\epsilon}\right)\mathrm{tr}(\not{p}\Gamma^\mu\not{p}_q\Gamma^\nu)-(\omega-2)\,\mathrm{tr}(\not{p}\Gamma^\mu\not{p}\Gamma^\nu)\right]\frac{\lambda^2}{m_b^2}\ln\frac{\lambda^2}{m_b^2},\\
T^{\mu\nu}_{(e)}(p,q) &\sim T^{\mu\nu}_{(a)}(p,q)\frac{C_F\alpha}{4\pi}\left[(3-\xi)\ln\frac{\lambda^2}{m_b^2}+\pi(2\omega-3)\frac{\lambda}{m_b}\right]\\
&\quad-\frac{1}{p_q^2+i\epsilon}\mathrm{tr}(\not{p}\Gamma^\mu\not{p}\Gamma^\nu)\frac{C_F\alpha}{4\pi}\left[-\pi\frac{\lambda}{m_b}\right]+T^{\mu\nu}_{(a)}(p,q)\frac{C_F\alpha}{4\pi}\left[-\frac{1-\xi^2}{2}\right]\frac{\lambda^2}{p_q^2+i\epsilon}\ln\left(-\frac{\lambda^2}{p_q^2}\right)\\
&\quad-\frac{1}{p_q^2+i\epsilon}\frac{C_F\alpha}{4\pi}\left[-\frac{m_b^2}{p_q^2+i\epsilon}\mathrm{tr}(\not{p}_q\Gamma^\mu\not{p}_q\Gamma^\nu)+\frac{1}{2}m_b^2\,\mathrm{tr}(\gamma_\tau\Gamma^\mu\gamma^\tau\Gamma^\nu)\right.\\
&\qquad\left.-(\omega^2-2\omega)\,\mathrm{tr}(\not{p}\Gamma^\mu\not{p}_q\Gamma^\nu)+(\omega-2)\,\mathrm{tr}(\not{p}\Gamma^\mu\not{p}\Gamma^\nu)\right]\frac{\lambda^2}{m_b^2}\ln\frac{\lambda^2}{m_b^2}.
\end{aligned}\tag{8}$$

Here $\Gamma^\mu = \gamma_\mu(1-\gamma_5)$, $C_F = 4/3$, $p_q = p - q$, $\omega = 2(p\cdot p_q)/(p_q^2+i\epsilon)$, and the tree diagram is given by

$$T^{\mu\nu}_{(a)}(p,q) = -\frac{1}{2}\frac{1}{p_q^2+i\epsilon}\mathrm{tr}(\not{p}\Gamma^\mu\not{p}_q\Gamma^\nu).$$

Diagram (b) is to be understood as an on-shell wave function renormalization and m_b is the pole mass. The ultraviolet (UV) divergent terms vanish in the sum of all diagrams as well as the logarithms in λ as implied by the BN theorem. In addition, the $\lambda^2\ln\lambda^2$ terms add to zero.

The terms linear in λ do not yet cancel. The reason is the use of the pole mass which contains terms linear in λ; see Eq. (7). One may now eliminate the pole mass from the differential width with the result ($p_{\rm SD}^2 = m_{\rm SD}^2$):

$$\frac{1}{2p^0}T^{\mu\nu}(p,q) \sim \frac{1}{2p_{\rm SD}^0}T^{\mu\nu}_{(a)}(p_{\rm SD},q),$$

with *no* corrections linear in λ to first order in α. Since all dependence on m_b has been eliminated in Eq. (1), it is never reintroduced through the subsequent phase space integrations. Integrating the differential width with the radiative corrections, Eq. (8) gives

$$\Gamma_B \sim \frac{G_F^2|V_{qb}|^2 m_b^5}{192\pi^3}(1 + \tilde{\lambda}\{3_b + 20_d - 13_e\}). \quad (9)$$

The subscript indicates the diagram, from which the respective terms originate. This form makes the cancellation of linear terms, when m_b is replaced through Eq. (7), more explicit. Let us rephrase the meaning of Eq. (9) in the language of renormalons: The presence of linear terms in the gluon mass implies an IR renormalon at $t = -1/2\beta_0$ in the asymptotic behavior of the radiative corrections to the free quark decay in Eq. (2). This divergent behavior cancels exactly with the IR renormalon in the series, Eq. (4), when the pole mass in the overall normalization of the width is replaced by a short-distance mass. This cancellation supports the validity of the OPE in heavy particle decays. Even though the initial state is nontrivial, to eliminate the nonanalyticity one does not have to invoke a summation over degenerate initial states, which would go beyond an OPE treatment. Note that the disappearance of linear terms in the total width has already been concluded in Ref. [3] on the basis of the OPE [9].

As already stated above, the subsequent nonanalytic term $\lambda^2 \ln\lambda^2$, which is related to an IR renormalon at $t = -1/\beta_0$, adds to zero in the sum of all contributions in Eq. (8). Note that this nonanalyticity is not reintroduced when the pole mass is eliminated, since the mass shift, Eq. (7), does not contain such terms, when $m_{\rm SD}$ is a MS-like mass. Thus, IR effects associated with the summation of radiative corrections to the decay of a charged particle (or of a free quark, if the results generalize to the QCD case [8]) are suppressed by at least 3 powers of the heavy quark mass. This may be anticipated, since explicit nonperturbative corrections of order $1/m_b^2$ are present only due the spin interaction and the kinetic energy of the heavy quark [cf. Eq. (3)]. These matrix elements between B states are not affected by an IR region in the Feynman diagrams for the free quark decay. This is clear for μ_G, the spin energy, which can be related to an observable, the mass difference of the vector and pseudoscalar mesons, to leading order. Protection of the kinetic energy from ambiguities due to renormalons is a consequence of Lorentz symmetry. When combined with the heavy quark expansion for the meson mass, it is related to the absence of an IR renormalon in the pole mass at $t = -1/\beta_0$, observed in Ref. [4].

Thus we have demonstrated that the suppression of IR effects implied by the OPE is indeed valid for heavy particle decays in QED. Phenomenologically, the subtleties associated with the definition of mass are only of secondary importance as long as one does not attempt a determination of the (unphysical) mass parameter. One may always sacrifice one measurement to predict another quantity and use any mass parameter in intermediate steps.

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[7] Some care is required to disentangle the IR pole at $s = 0$ from an UV pole at the same position, related to renormalization. Note also that analytic terms λ^{2n} in the expansion of $r_0(\lambda)$ originate from modification of high-momentum region in Feynman diagrams. They are of purely kinematic origin and not related to renormalons and IR effects in general.

[8] Note that in the original QED with a massless photon, $\tilde{\lambda}$ represents a nonperturbative term arising from the Borel summation of the IR effects in the perturbative expansion, cf. Eq. (7). Moreover, the divergence of the perturbative expansion in large orders is the only source of nonperturbative effects in QED. The cancellation of $\tilde{\lambda}$ from the final result implies insensitivity to the IR region to this accuracy. Although nonperturbative terms in case of QCD are more complicated, the latter conclusion could well be true in that case as well since the structure of divergences of the perturbative expansions is similar.

[9] The arguments of Ref. [3] tacitly assume that the Coulomb gauge is used. In this gauge, diagram (e) and the wave function renormalization (b), not considered in Ref. [3], do not contribute to linear terms [A. Vainshtein (private communication)], leaving only diagram (d) to cancel the mass shift. Note that in covariant gauges the linear terms are gauge independent for each diagram individually.