Comment on "Critical Behavior of the Coulomb Glass"

In a recent Letter [1] Grannan and Yu reported results from a Monte Carlo simulation of the so-called Coulomb glass, a model of disordered interacting localized electrons. They claimed a finite-temperature equilibrium phase transition to a spin-glass- (SG-) like low-temperature phase, characterized by an Edwards-Anderson (EA) order parameter and occurring at a temperature far below the typical interaction energy.

The Hamiltonian most often studied for the Coulomb glass is the lattice model of Efros and Shklovskii (ES) [2]

$$H = \sum_{i<j} \frac{\left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right)}{r_{ij}} + \sum_i \phi_i n_i , \qquad (1)$$

where ϕ_i is a random potential of site i and n_i its occupation number. r_{ij} is the distance between the sites which form a regular cubic lattice. In contrast, Grannan and Yu placed the sites at random into the system, a random potential is not present. In their Hamiltonian

$$H = \sum_{i<j} \frac{\left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right)}{r_{ij}} , \qquad (2)$$

the disorder is therefore contained only in the interaction. Although (2) has another symmetry than the ES model (1), they claimed their results qualitatively valid also for the ES Coulomb glass.

We argue that the results of [1], namely the existence of a phase transition and the SG-like character of the low-temperature phase, are restricted to (2) and not generally valid in a Coulomb glass. In particular, the results are not valid for the ES model. In the following we discuss that the critical behavior of models with random potential is totally different from that of models with random interactions and that the low-temperature phases also differ, namely antiferromagneticlike versus SG-like.

The EA order parameter $q = [\langle n_i - \frac{1}{2}\rangle^2]$, where $\langle\cdots\rangle$ denotes the thermal average and $[\cdots]$ the disorder average, is only an order parameter for model (2). In the presence of a random potential the average occupation, and therefore q, is not zero even for high temperatures. Grannan and Yu discussed this failure of the EA order parameter in the ES model as a numerical difficulty. However, in our point of view, it represents the physical situation. But if q which would be the natural order parameter to characterize a SG-like phase transition is always nonzero, then there is no SG-like phase transition.

To get a deeper understanding, it is useful to look at systems with short-range interactions. A short-range analogon of (2) is the EA model [3]. Although this is a bond-disordered rather than a site-disordered model, its symmetry is the same as in (2). The EA model is known to have a finite-temperature phase transition to a SG phase in spatial dimensions $D \geq 3$ [4]. In contrast, the short-range analogon of the ES model (1) is the random-field Ising model (RFIM) [5]. For strong random potential the RFIM does not have any phase transition, the paramagnetic phase is stable down to zero temperature. In the case of weak disorder the RFIM undergoes a phase transition for $D \geq 3$. However, the *low-temperature* phase is not SG-like with frozen disorder but simply an ordered ferro- or antiferromagnetic phase [6], although the existence of a SG-like phase at *intermediate* temperatures has recently been proposed [7].

In a recent paper [8] it was rigorously shown for the spherical version of the ES model that the phase diagram consists of a disordered ("paramagnetic") and an ordered ("antiferromagnetic") phase, which is present only for weak random potential. A SG-like phase with frozen order is not present in any case.

In conclusion, there is strong evidence that the critical behavior and the character of the low-temperature phase of the Hamiltonian (2) which contains only random interactions but no random potential is totally different from that of the ES model. We have no doubt that the results of Letter [1] are valid for the special case (2), and we note in passing that the observed low transition temperature is in accordance with a similar result for the site-disordered RKKY model [9]. But we also have no doubt that the results of [1] cannot be generalized to arbitrary Coulomb glasses. In particular the results are not valid for the ES model (1), which does not have an *equilibrium* phase transition to a SG-like low-temperature phase. However, preliminary results [10] suggest that the ES model features a *dynamic* glass transition corresponding to a divergence of equilibration times.

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