Laser Cooling to a Single Quantum State in a Trap

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(Received 6 May 1994)

Laser cooling in a trap is investigated for configurations which allow the existence of "dark states" of the combined atom-plus-trap system, i.e., states which are decoupled from the laser light by quantum interference. Two examples of approximate dark states in a 1D flat bottom and 2D harmonic trap for angular momentum 1 to 1 transitions are discussed. A wave function simulation of the quantum master equation predicts that a significant fraction of the atoms are transferred to a single trap state.

PACS numbers: 32.80.Pj, 42.50.Vk

The ultimate goal of laser cooling of neutral atoms [1] in a trapping potential is the cooling to the quantum mechanical ground state, or the selective population of a single excited vibrational state of the trap. The question of ground state cooling is particularly timely in view of current experimental efforts to observe many particle and quantum statistics effects with cold atoms, in particular Bose-Einstein condensation corresponding to a macroscopic occupation of the trap ground state [2]. We note that preparation of a single quantum state in a trap leads to a large atomic coherence length of the cold atoms of the order of the size of the trap state. Alternatively, we can view a trap as a cavity for atoms [3] and interpret the preparation of a single quantum state as a mode selection scheme in an atomic resonator—an important step toward the realization of a coherent source of atoms.

This Letter discusses novel schemes for ground state cooling and state selection in a trap based on laser and trapping configurations that allow the existence of "dark states" [4] of the combined atom-plus-trap system. We define a dark state as a superposition of internal atomic ground states times trap states which, by quantum mechanical interference of the atomic dipole excitations, is decoupled from the laser light. An atom that makes a transition to a dark state by optical pumping will remain in this state. Cooling schemes based on dark states of free atoms have been discussed recently in the context of obtaining subrecoil energies $E < E_R$ in optical molasses [4–7], where the recoil energy is defined by $E_R = \hbar\omega_R = \hbar^2k^2/2M$ with $k = 2\pi/\lambda$ the wave number of the atomic transition and M the mass of the atom. For free atoms there exist (exact) velocity selective dark states [4,5]. We note, however, that these exact dark states, being eigenstates of a free particle Hamiltonian, are delocalized improper eigenstates, whereas our discussion below will focus on finding approximate but normalizable dark eigenstates in the trap. The essential idea behind the present work is to find trap and laser cooling configurations with approximate "dark" eigenstates which are a good *local* approximation to the free particle dark state.

The quantum motion of an atom in a trap in the presence of a cooling laser is described by the master equation for the atomic density matrix,

$$\frac{d}{dt}\rho = -i(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}{}^{\dagger}) + \gamma_0\mathcal{R}_{\rho}\,. \tag{1}$$

The non-Hermitian effective Hamiltonian,

$$H_{\mathrm{eff}} = \frac{\hat{\mathbf{p}}^2}{2M} + V_T(\hat{\mathbf{x}}) + (U_0 - i\tfrac{1}{2}\gamma_0)\,\hat{V}_L(\hat{\mathbf{x}})\,, \tag{2}$$

is the sum of a kinetic energy term, a conservative trapping potential V_T, an optical potential $\hat{V}_L$ due to the ac Stark shift induced by the cooling laser [8], and a term describing decay due to optical pumping in the cooling laser [8]. In writing Eq. (1) we adiabatically eliminated the excited states by assuming weak laser excitation. The last term in (1) describes the return of the electron to the atomic ground state in an optical pumping process. An operational definition of a dark state in the trap is as follows. Assume that the system is prepared in the normalized atomic state $|\psi_t\rangle$ at time t. The probability density for the emission of the *next photon* (the next optical pumping event) at time $t + \tau$ is then given by the delay function $\tilde{c}(\tau)$ [9]. This delay function is related to the decay of the norm of the wave function by $\tilde{c}(\tau) = -(d/d\tau)\,\|\exp(-iH_{\mathrm{eff}}\tau)|\psi_t\rangle\|^2$. Consider now eigenstates of H_{eff},

$$H_{\mathrm{eff}}|N\rangle_T = (E_N - i\tfrac{1}{2}\Gamma_N)|N\rangle_T\,. \tag{3}$$

We are interested in a situation where for a *single* quantum state $|D\rangle_T$ we have $\Gamma_D \to 0$. In an ideal case $\Gamma_D = 0$; in practice we require the decay rate Γ_D to be much smaller than for all other states, $\Gamma_D \ll \Gamma_N$ $(D \neq N)$. Such a state is a dark state in the sense that an atom prepared in this state at time t will be long lived, i.e., $\tilde{c}(\tau) = \Gamma_D\exp(-\Gamma_D\tau)$ with $\Gamma_D \to 0$. Accordingly, the atom will spend a large fraction of time in this state [10], provided this state is populated in the process of laser cooling.

Exact dark states for *free* particles ($V_T = 0$) for an angular momentum $J_g = 1$ to $J_e = 1$ transition exist, for example, in a 1D laser configuration of two counterpropagating linearly polarized laser fields with angle θ between the polarization vectors [7], and a 2D configuration of two orthogonal standing light waves with linear light polarization [11,12] (see also [4]). In the first case the positive frequency part of the electric field of the blue detuned cooling laser is given by

$$\mathbf{E}^{(+)} = \mathcal{E}[\mathbf{e}_{+1}\cos(kz + \theta/2) + \mathbf{e}_{-1}\cos(kz - \theta/2)]e^{-i\omega t}, \tag{4}$$

with amplitude $\mathcal{E}$, ω the laser frequency, and $\mathbf{e}_{\pm 1}$ spherical unit vectors [Fig. 1(a)]. In the 2D configuration we have [11]

$$\mathbf{E}^{(+)} = \mathcal{E}[\mathbf{e}_x\cos(ky + \phi_y)e^{i\Phi} + \mathbf{e}_y\cos(kx + \phi_x)]e^{-i\omega t}, \tag{5}$$

with $\phi_{x,y}$ and Φ laser phases [Fig. 1(b)]. In both cases one finds an optical potential of the form

$$\hat{V}_L(\mathbf{x}) = (a_-|g_-\rangle + a_+|g_+\rangle)(a_-^*\langle g_-| + a_+^*\langle g_+|), \tag{6}$$

where $|g_\pm\rangle$ denotes the $m = \pm 1$ Zeeman sublevels of the atomic ground states. The coefficients $a_\pm(\mathbf{x})$ are

$$a_\pm(z) = \cos(kz \pm \theta/2), \tag{7}$$

$$a_\pm(x,y) = [i\cos(ky + \phi_y)e^{-i\Phi} \mp \cos(kx + \phi_x)]/2 \tag{8}$$

for the 1D and 2D configurations, respectively. For both cases the state

$$\langle\mathbf{x}|D\rangle_F = |g_-\rangle a_+(\mathbf{x}) - |g_+\rangle a_-(\mathbf{x}) \tag{9}$$

fulfills $\hat{V}_L|D\rangle_F = 0$, is an eigenstate of kinetic energy, and is thus an improper solution of (3) with zero imaginary part.

We first consider laser cooling in the 1D configuration (4) in a trap. We assume a trapping potential in the form of a square well with a flat bottom $0 \leq z \leq L$ of length $L \gg \lambda$ and rounded edges, with an exponential decay length l of the order of a wavelength [Fig. 1(a)]. Figure 2 is a plot of the steady-state populations of the eigenstates of the trap plus cooling laser potential as a function of the principal quantum number n [Eq. (3)]. The parameters are $L = 12\lambda$, $l = \lambda/2$, $U_0 = 40\omega_R$ [13], $\gamma_0 = 4\omega_R$, and we have taken $\theta = \pi/4$, which for this laser configuration is known to give optimum polarization gradient precooling for the free particle [7]. The insets in Fig. 2 show the energy level structure. The energy levels fall into a set of bands reminiscent of the free particle band structure in the optical potential [7]. The steady-state density matrix is to a very good approximation diagonal in the energy eigenbasis. According to Fig. 2 we find more than 50% of the total population in the ground state [13]. This large ground state population reflects the survival of the free particle dark state (5) as an approximate dark state in the trap. For $L \gg \lambda$ there are two separate spatial scales for the problem, and for the few lowest lying states an approximate eigenfunction of the combined system (3) is given by the product of a *slowly varying* trap eigenfunction and a free particle dark state, $\langle z|D\rangle_T \approx \phi_n(z)\langle z|D\rangle_F$. Here $\phi_n(z)$ is the eigenfunction of the bare trap potential V_T with quantum numbers $n = 1,2,\ldots$ [in a square well $\phi_n(z) = \sin(n\pi z/L)$ for $0 \leq z \leq L$]. We have confirmed the structure of this dark state by detailed numerical calculations. We have used two different methods to solve the master equation (1) and found excellent agreement: (i) a wave function simulation of the master equation [14], and (ii) a solution of the rate equation for the populations of the eigenbasis in which the coherences in the master equation are neglected.

In Fig. 3(a) the steady-state population of the ground state is plotted as a function of the trap size L for different decays l of the potential near the walls. The ground state population increases monotonically with the size of the trap and asymptotically approaches a constant. This behavior is the result of two competing effects: (i) in a large trap the ground state $|\psi_d\rangle$ becomes a better approximation of the free particle dark state, leading to a smaller decay rate Γ_D; (ii) approaching the free particle limit ($L \to \infty$) yields an increasing number of long-lived states near the dark state. We find that the *shape* of the potential is important: a "flat bottom" of the trap and a sharp cutoff ($l \ll L$) are essential to achieve a high ground state population. The reason is that a flat bottom avoids the accumulation of a phase mismatch between the dark state in the trap and the free particle dark state (9) over a significant part of the trap. For example, for the decay $l = \lambda/4\pi$ we find a ground state population of more than 50% in a trap with $L = 5\lambda$, whereas for $l = \lambda/2$ a trap with $L = 11\lambda$ is required to achieve the

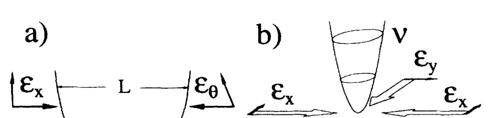

FIG. 1. Cooling and trapping configuration in (a) a 1D square well, and (b) a 2D harmonic oscillator.

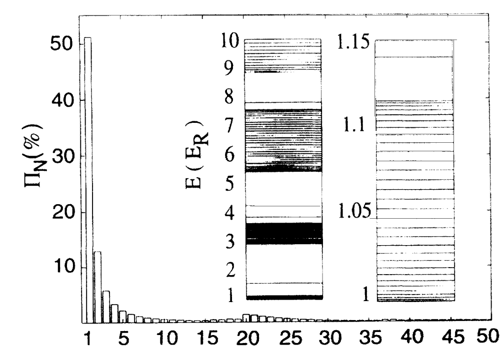

FIG. 2. Steady-state populations in the 1D trap [Fig. 1(a)] for $L = 12\lambda$, $l = \lambda/2$, $U_0 = 40\omega_R$, and $\gamma_0 = \omega_R$. Inset: lowest eigenenergies of the system. Left panel: the three lowest energy bands; right panel: the lowest band in more detail.

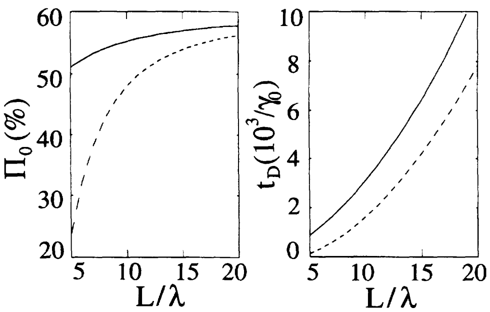

FIG. 3. Left: Steady-state populations of the ground state vs L for $l = \lambda/2\pi$ (solid line) and $l = \lambda/2$ (dashed line). Right: Cooling time into the ground state vs L from an initial thermal distribution with a mean energy of $20E_R$. Other parameters are the same as in Fig. 2.

same population in the lowest state. The cooling time into the ground state, i.e, the time necessary to reach $\frac{1}{e}$ of the stationary ground state population, is plotted in Fig. 3(b) as a function of L for the same values of l as above for a thermal initial distribution with a mean energy of $20E_R$.

In the second example corresponding to configuration (5), we illustrate the existence of dark states in a 2D isotropic harmonic trapping potential [Fig. 1(b)]. For the laser phases we choose $\phi_{x,y} = \pi/2$ so that the minimum of the potential $\hat{V}_L = 0$ coincides with the center of the harmonic trap at $x = y = 0$; the choice of the phase Φ is not essential for the following, so we take $\Phi = \pi/2$. The eigenstates of (3) can be classified according to a complete set of commuting operators including reflections with respect to the x and y axes. In particular we find there is a manifold of laser-plus-trap eigenstates that have the same symmetry properties as the 2D free particle dark state (9). We will show below that the lowest energy state of this symmetry is an approximate dark state in the trap. Our starting point is degenerate perturbation theory in $\hat{V}_L$ for the harmonic trap levels. The first excited (bare) oscillator states $\{\phi_{n_x=1}(x)\,\phi_{n_y=0}(y)|g_\pm\rangle,\ \phi_{n_x=0}(x)\,\phi_{n_y=1}|g_\pm\rangle\}$ comprise the lowest manifold containing states of the given symmetry. Here $\phi_{n_x n_y}(x,y) = \phi_{n_x}(x)\,\phi_{n_y}(y)$ denotes the harmonic oscillator wave functions with energy $E = \hbar\nu(n_x + n_y + 1)$, and $n_x, n_y = 0, 1, \ldots$ are vibrational quantum numbers for motion in the x and y directions, respectively. Diagonalizing the Hamiltonian in this subspace yields for the state of lowest energy

$$\langle x,y|D\rangle_T = \tfrac{1}{2}(\phi_{01} - \phi_{10})|g_+\rangle + \tfrac{1}{2}(\phi_{10} + \phi_{01})|g_-\rangle, \tag{10}$$

with eigenvalue

$$E_D - i\tfrac{1}{2}\Gamma_D = 2\nu + (U_0 - i\tfrac{1}{2}\gamma_0)\,{}_T\langle D|\hat{V}_L|D\rangle_T\,. \tag{11}$$

We find that ${}_T\langle D|\hat{V}_L|D\rangle_T \ll 1$, i.e., for *this state* $\hat{V}_L$ is a small perturbation with a long lifetime. To understand the physical meaning of the state (10), we compare in Fig. 4

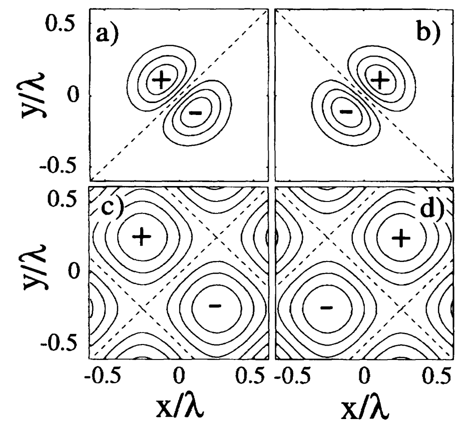

FIG. 4. (a), (b) Contour plots of the 2D trap dark state $\langle x,y;g_\mp|D\rangle_T$, and (c), (d) free dark state $\langle x,y;g_\mp|D\rangle_F$.

its wave function with the free 2D dark state (8) and (9). In the region of space where the wave function is nonzero, the dark state in the trap is a good *local approximation* of the dark state (9) (which fulfills $\hat{V}_L|D\rangle_F = 0$). In the present example the choice of a harmonic trapping potential is not essential for the existence of dark states. The only requirement is that the optical potential and the trapping potential be aligned so that a linear combination of degenerate trap eigenstates is "mode matched" to the dark state (9). We have confirmed the existence of the dark state (10) and the validity of the above argument by numerical solution of the full eigenvalue Eq. (3). We find that the imaginary part Γ_D of the exact eigenvalue is typically smaller than the width estimated in perturbation theory. For example, for $U_0 = 30\omega_R$ and $\gamma_0 = 6\omega_R$ we find $\Gamma_D = 3 \times 10^{-4}\ \gamma_0$ while the ration of the decay width of the neighboring state is $\Gamma_N/\Gamma_D \approx 40$.

We have performed wave function simulations of the master equation [14] for the 2D configuration (5). Figure 5(a) is a histogram of the stationary population as a function of the principal quantum number n of

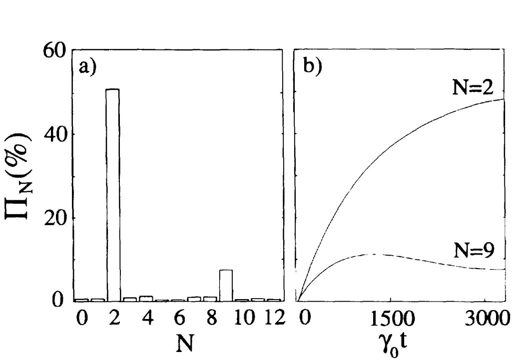

FIG. 5. (a) Trap population for the $n = 0, \ldots, 11$ eigenstates in the 2D trap; (b) time dependence of population of states 2 and 9; $\nu = 2\omega_R$, $U_0 = 30\omega_R$, and $\gamma_0 = 6\omega_R$.

the combined atom-plus-trap system for $\nu = 2\omega_R$, $U_0 = 30\omega_R$, and $\gamma_0 = 6\omega_R$. This corresponds to a Lamb-Dicke parameter $\eta \equiv 2\pi a_0/\lambda = \sqrt{\omega_R/\nu} = 1/\sqrt{2}$ where a_0 is the size of the harmonic oscillator ground state. In Fig. 5(a) the dark state (10) is the state $n = 2$ which has a population of 52%. We note the appearance of a second approximate dark state $n = 9$ ($\Gamma_9/\Gamma_D \approx 10$) in the same symmetry class as the state $n = 2$ with a steady-state population of 8%. As in the 1D case we find that the steady-state density matrix elements are essentially diagonal in the eigenbasis of the combined atom-plus-trap system. Fig. 5(b) illustrates the time evolution of the population of the states $n = 2$ and 9 for the same parameters, and for a thermal initial momentum distribution with mean energy $20E_R$.

In this Letter we have shown the survival of a free particle dark state as an approximate dark ground state of a mesoscopic trap $L \gg \lambda$, and we have shown the existence of dark states in a harmonic oscillator potential. The present schemes allow the preparation of a significant fraction of the atoms in the ground state of the trap, although the energy level separation between the trap states is only a fraction of the recoil energy and/or much smaller than the optical pumping rate. This is in contrast to sideband cooling in ion traps [15] where ions can be pumped to the vibrational ground state provided the frequency separation between the trap states is larger than the atomic decay width—a condition that can only be satisfied experimentally for very tight binding when the trap ground state is much smaller than a wavelength. The trap configurations and cooling schemes discussed above are experimentally feasible. A harmonic trapping potential for neutral atoms is realized in a dipole trap [16] where an intense focused laser beam that is strongly detuned to the red side of the atomic resonance provides, via the ac Stark shift, a conservative potential. "Boxes of light" a few wavelengths in size are obtained, for example, with evanescent light waves [3] or with "sheets of light" using cylindrical lenses to form a dipole trap with blue laser detuning. The present scheme can also be applied to ion traps. For $J_g = 1$ to $J_e = 1$ transitions, the cooling and trapping schemes we have described can be generalized to three-dimensional configurations. Dark state cooling has been shown to exist in 2D and 3D (see, e.g., Refs. [5,11,12]); adopting the arguments presented for our 1D model, these free particle dark states will survive as approximate dark eigenstates of a 2D/3D flat bottom trap. Furthermore, we have been able to show the existence of an approximate dark state in a harmonic 3D trap for a laser configuration consisting of three standing light waves with orthogonal laser polarizations [12].

We thank E. Cornell, M. Kasevich, S. Rolston, W. Phillips, and C. Wieman for discussions. Work at JILA is supported in part by the NSF.

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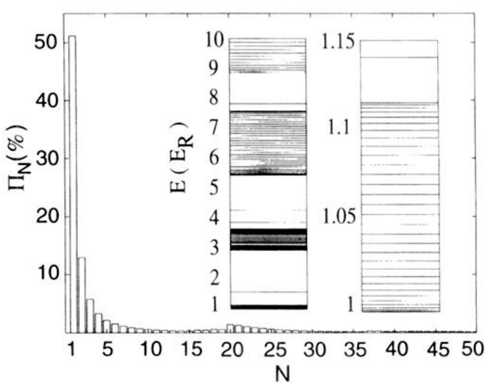

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