CP Violation in the Decay $b \to s\gamma$ in the Two-Higgs-Doublet Model

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In the most general two-Higgs-doublet model with an approximate family symmetry and CP violation originating solely from the relative phase of two vacuum expectation values, CP asymmetry in the decay $b \to s\gamma$ may arise from the CP violation of the charged Higgs boson interactions with fermions. This asymmetry may be larger than in the standard model and can lie between 10^{-2} and 10^{-1}. The decay rate of $b \to s\gamma$ is allowed to be smaller or larger than in the standard model.

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The decay of the form $b \to s\gamma$ represents the first observation [1] of a process in B decay clearly involving a loop in the standard model. The rate of the decay is roughly that expected [2] from a loop involving a t quark and $W^{\pm}$. In the simplest extension of the standard model, the two-Higgs-doublet model (2HDM), there may be a significant contribution from the loop in which $W^{\pm}$ is replaced by the charged Higgs boson $H^{\pm}$. Here we discuss $b \to s\gamma$ in the general 2HDM with an emphasis on possible CP-violating efects.

The Yukawa interaction in general in the 2HDM is

$$L_Y = \bar{q}_L(\Gamma_1^D \phi_1 + \Gamma_2^D \phi_2)D_R + \bar{q}_L(\Gamma_1^U \tilde{\phi}_1 + \Gamma_2^U \tilde{\phi}_2)U_R\,, \tag{1}$$

where q_L is the quark doublet and D_R and U_R are the right-handed quark singlets. Γ_1^F and Γ_2^F are matrices in flavor space. The presence of both Γ_1^F and Γ_2^F in general lead to flavor-changing neutral Higgs boson exchange (FCNE) processes. To avoid these it is customary to choose either

$$\text{model 1:}\quad \Gamma_1^U = \Gamma_1^D = 0;$$

$$\text{model 2:}\quad \Gamma_1^U = 0, \qquad \Gamma_2^D = 0\,.$$

It has been pointed out by Cheng and Sher [3] and others and reemphasized by Hall and Weinberg [4] that FCNE may be suppressed by an approximate flavor symmetry, in which case both Γ_1^F and Γ_2^F may be significant. The point is that the constraints on FCNE are not so severe given the small size of Higgs boson couplings; all that is required is to suppress the off-diagonal terms in Γ_1^F and Γ_2^F by factors of 10 or 100 as occurs in elements of the quark mixing matrix. The consequences of an assumption of approximate global U(1) family symmetries (AGUFS) (i.e., one for each family) have been worked out in detail in [5] and emphasized recently in [6]. AGUFS are sufficient for a natural suppression of family-changing currents (for both charged and neutral currents). In particular, as we have pointed out [5,6], this has important consequences for the charged Higgs boson couplings allowing significant new CP-, T-, and P-violating effects on both indirect CP violation (ϵ) and direct CP violation (ϵ'/ϵ) in kaon decay and electric dipole moments of the neutron (D_n) and electron (D_e).

Our other assumption is that all significant CP violation arises from the Higgs vacuum expectation values

$$\langle\phi_1\rangle = \frac{v}{\sqrt{2}}\cos\beta\, e^{i\delta}, \qquad \langle\phi_2\rangle = \frac{v}{\sqrt{2}}\sin\beta\,. \tag{2}$$

This leads to a phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix as in the standard model but provides a variety of new sources of CP violation.

The important $H^{\pm}$ couplings can be obtained neglecting FCNE and only considering the standard CKM quark mixing. Neglecting the small off-diagonal terms, one finds for the top mass

$$m_t e^{-i\delta_t} = (h_1 \cos\beta\, e^{-i\delta} + h_2 \sin\beta)v\,, \tag{3}$$

where h_1 (h_2) are the 33 diagonal elements of Γ_1^U (Γ_2^U). The phase δ_t must be removed by rephasing t_R. The coupling of $H^{\pm}$ to t_R then has the form

$$-H^- \bar{b}_{Lj} V_{ji}^{\dagger} t_{Ri}(h_1 \sin\beta\, e^{-i\delta} - h_2\cos\beta) v e^{i\delta_t} \equiv -\xi_t m_t H^- \bar{b}_{Lj} V_{ji}^{\dagger} t_{Ri}\,, \tag{4}$$

where V is the CKM matrix. A similar equation holds for the b_R couplings. It is easy to show that the coefficients ξ_t, ξ_b can be written

$$\xi_f = \frac{\sin\delta_f}{\sin\beta\cos\beta\sin\delta} e^{i\sigma_f(\delta-\delta_f)} - \cot\beta\,, \tag{5}$$

where $\sigma_f = +$ for b and $\sigma_f = -$ for t and δ_f serves to parametrize the ratio h_2/h_1. In the limiting case of model 1, $\delta_t = \delta_b = 0$, i.e., $\xi_t = \xi_b = -\cot\beta$. In the case of model 2, $\delta_b = \delta$, $\delta_t = 0$, i.e., $\xi_b = \tan\beta$, $\xi_t = -\cot\beta$. In general Eq. (5) can be seen as interpolating between the values $\tan\beta$ and $-\cot\beta$; however, outside of these limits ξ_f has a complex phase.

Considering the leading loop diagram, the decay amplitude of $b \to s\gamma$ can be written

$$\langle s\gamma|T|b\rangle \equiv \mathcal{T}_{s\gamma} = v_t(A_{s\gamma}^W + \xi_t\xi_b A_{s\gamma}^H + \xi_t\xi_t^* \tilde{A}_{s\gamma}^H)\,, \tag{6}$$

where $v_t = V_{tb}V_{ts}^*$. $A_{s\gamma}^W$, $A_{s\gamma}^H$, and $\tilde{A}_{s\gamma}^H$ can be generally expressed

$$A_{s\gamma}^{W,H}(t) = C_{s\gamma}^{W,H} O_{s\gamma}, \qquad \tilde{A}_{s\gamma}^H(t) = \tilde{C}_{s\gamma}^H O_{s\gamma}\,, \tag{7}$$

with

$$O_{s\gamma} = -\frac{G}{8\sqrt{2}\pi^2} e\bar{u}_s(p)\sigma^{\mu\nu}(1+\gamma_5)u_b(p_b)F_{\mu\nu} \tag{8}$$

and C_i the Wilson coefficient functions [2]

$$\begin{aligned} C^W_{s\gamma} &= -\eta^{16/23}[\tfrac{1}{2}A(x_t) + \tfrac{4}{3}D(x_t)(\eta^{-2/23} - 1) + \tfrac{232}{513}(\eta^{-19/23} - 1)], \\ C^H_{s\gamma} &= \eta^{16/23}[B(y_t) + \tfrac{8}{3}E(y_t)(\eta^{-2/23} - 1)], \\ \tilde{C}^H_{s\gamma} &= -\eta^{16/23}\tfrac{1}{6}[A(y_t) + \tfrac{8}{3}D(y_t)(\eta^{-2/23} - 1)], \end{aligned} \tag{9}$$

where A, B, D, and E are the integral functions of $x_t = m_t^2/m_W^2$ and $y_t = m_t^2/m_{H^+}^2$ and are defined in Ref. [2] and $\eta = \alpha_s(m_W)/\alpha_s(m_b)$.

With these considerations, we now evaluate the decay rate of $b \to s\gamma$. To reduce the uncertainties from the CKM matrix elements and b-quark mass, one usually relates $B(b \to s\gamma)$ to the inclusive semileptonic decay rate $\Gamma(b \to ce\nu)$ [7,8]

$$B(b \to s\gamma) = \left[\frac{\Gamma(b \to s\gamma)}{\Gamma(b \to ce\nu)}\right]^{\text{th}} \times B^{\exp}(b \to ce\nu) \simeq 0.031\{[C^{WH}_{s\gamma} + \mathrm{Re}(\xi_t\xi_b)C^H_{s\gamma}]^2 + [\mathrm{Im}(\xi_t\xi_b)C^H_{s\gamma}]^2\}B^{\exp}(b \to ce\nu), \tag{10}$$

where $C^{WH}_{s\gamma} = C^W_{s\gamma} + |\xi_t|^2\tilde{C}^H_{s\gamma}$.

In general ξ_t is expected to be of order unity or less if the Yukawa couplings of the top quark are reasonable. In this case, the term proportional to $|\xi_t|^2$ makes only a small contribution. In our numerical examples and figures we let $|\xi_t|^2 = 0$, but results with $|\xi_t|^2 = 1$ are very similar. On the other hand, ξ_b may have a magnitude considerably larger, as in the limiting case of model 2 with large $\tan\beta$. In Fig. 1 we show the ratio of the 2HDM result for the branching ratio to that for the standard model as a function of $\mathrm{Re}(\xi_b\xi_t)$ and $\mathrm{Im}(\xi_b\xi_t)$ for $m_{H^+} = m_t$. For quite reasonable values the result can be either greater or smaller than in the standard model. This is in contrast to model 2 in which the rate is always greater than in the standard model. The possibility of a smaller value has also been noted in model 1 [7].

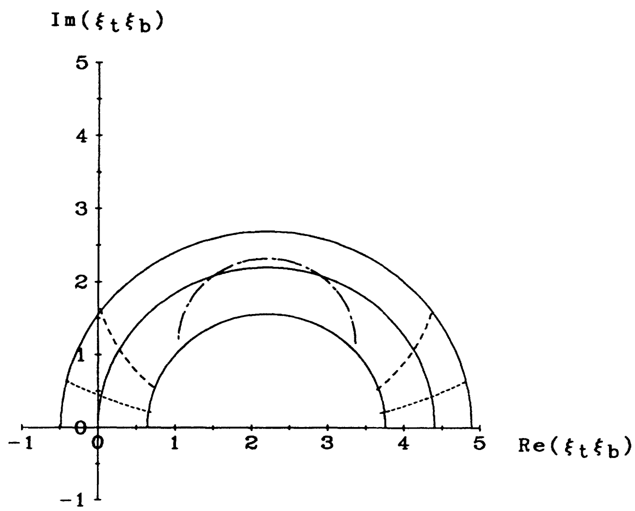

FIG. 1. The solid lines show the ratio $r = \Gamma_{2\mathrm{HDM}}(b \to s\gamma)/\Gamma_{SM}(b \to s\gamma)$ as a function of $\mathrm{Im}(\xi_t\xi_b)$ and $\mathrm{Re}(\xi_t\xi_b)$. The three solid curves are for $r = 0.5, 1, 1.5$, which correspond to radii $R = 1.56, 2.2, 2.69$, respectively, with the central point at $\mathrm{Im}(\xi_t\xi_b) = 0$ and $\mathrm{Re}(\xi_t\xi_b) = 2.2$, and $m_{H^+} = m_t$. The dotted lines are for CP asymmetry $A^{CP}_{s\gamma} = 2\%$, the dashed lines are for $A^{CP}_{s\gamma} = 5\%$, and the dot-dashed line is for $A^{CP}_{s\gamma} = 10\%$ with $m_{H^+} = m_t$.

If it is established that the rate for $b \to s + \gamma$ differs from that in the standard model, this could be the first indication of the existence of a charged Higgs boson; at the present, however, there is considerable uncertainty [9] in the standard model rate. In the 2HDM discussed here, however, there is the possibility of a more distinct signature of the charged Higgs contribution due to CP violation. To calculate the CP-violating rate difference between B and $\bar{B}$ decays, it is necessary to include final-state-interaction effects. Using the general formalism in [10], the decay amplitude of $b \to s\gamma$ in Eq. (6) is modified by including the corresponding absorbtive terms via

$$\begin{aligned} \mathcal{T}_{s\gamma} &= v_t(A^W_{s\gamma} + iA^W_{sg}t_{sg\to s\gamma}) + \sum_q^{u,c} v_q iA^W_{sq\bar{q}}(q)t_{sq\bar{q}\to s\gamma} \\ &\quad + v_t[\xi_t\xi_b(A^H_{s\gamma} + iA^H_{sg}t_{sg\to s\gamma} + \xi_t\xi_t^*(\tilde{A}^H_{s\gamma} + i\tilde{A}^H_{sg}t_{sg\to s\gamma})], \end{aligned} \tag{11}$$

where $v_q = V_{qb}V^*_{qs}$ are products of CKM elements and $t_{i\to s\gamma}$ are the scattering amplitudes. $A^{W,H}_{sg}$ and $\tilde{A}^H_{sg}$ are expressed

$$A^{W,H}_{sg} = C^{W,H}_{sg}O_{sg}, \qquad \tilde{A}^H_{sg}(t) = \tilde{C}^H_{sg}O_{sg}, \tag{12}$$

with

$$O_{sg} = -\frac{G}{8\sqrt{2}\pi^2}g\bar{u}_s(p)\sigma^{\mu\nu}T^a(1 + \gamma_5)u_b(p_b)G^a_{\mu\nu} \tag{13}$$

and C_i the Wilson coefficient functions [2,8]

$$\begin{aligned} C^W_{sg} &= -\eta^{14/23}[\tfrac{1}{2}D(x_t) + 0.1687], \\ C^H_{sg} &= \eta^{14/23}E(y_t), \qquad \tilde{C}^H_{sg} = -\eta^{14/23}\tfrac{1}{6}D(y_t), \end{aligned} \tag{14}$$

where $\eta = \alpha_s(m_W)/\alpha_s(m_b) \simeq 0.56$ for $\Lambda_{\mathrm{QCD}} = 150$ MeV and $m_b = 4.9$ GeV.

$A_{sq\bar{q}}$ is the $b \to sq\bar{q}$ amplitude

$$A_{sq\bar{q}} = -\frac{G}{\sqrt{2}} c_1 \bar{u}_q(p_1)\gamma_\mu(1-\gamma_5)u_b(p_b)\bar{u}_s(p')\gamma^\mu(1-\gamma_5)v_{\bar{q}}(p_2), \tag{15}$$

where c_1 is the QCD correction with $c_1 = 1.1$.

The rate difference is calculated to be

$$\begin{aligned} \Delta_{s\gamma} &\equiv \Gamma(\bar{b} \to \bar{s}\gamma) - \Gamma(b \to s\gamma) \\ &= 4|v_t|^2 (C^H_{s\gamma} C^{WH}_{sg} - C^{WH}_{s\gamma} C^H_{sg}) O^\dagger_{s\gamma} O_{sg} t_{sg\to s\gamma} \mathrm{Im}(\xi_t \xi_b) \\ &\quad + \sum_q^{u,c} 4 C^{WH}_{s\gamma} O^\dagger_{s\gamma} A_{sq\bar{q}} t_{sq\bar{q}\to s\gamma} \mathrm{Im}(v_t v_q^*) + \sum_q^{u,c} 4 C^{H}_{s\gamma} O^\dagger_{s\gamma} A_{sq\bar{q}} t_{sq\bar{q}\to s\gamma} \mathrm{Im}(v_t v_q^* \xi_t \xi_b). \end{aligned} \tag{16}$$

Here we have omitted the flux factor and the phase space integrals. As shown by Soares [11] after integration over the phase space variables for $\Delta_{s\gamma}$, the absorbtive term with the $sq\bar{q}$ intermediate state is suppressed by a factor of about $\alpha_s/4$ for the up quark, and for the charm quark it has an additional suppression factor of about 0.12 from phase space. For the absorbtive term with the sg intermediate state, there is no extra phase space suppression and its magnitude after integration over the phase space variables is expected to be suppressed just by a factor of order α_s.

With these considerations, we then obtain the CP asymmetry observable

$$\begin{aligned} A^{CP}_{s\gamma} = \frac{\Delta_{s\gamma}}{2\Gamma(b\to s\gamma)} &\simeq \frac{(C^H_{s\gamma}C^{WH}_{sg} - C^{WH}_{s\gamma}C^H_{sg})\alpha_s}{C^2_{s\gamma}} \mathrm{Im}(\xi_t\xi_b) \\ &- \frac{0.12\mathrm{Re}(v_t v_c^*) + \mathrm{Re}(v_t v_u^*)}{|v_t|^2} \frac{C^H_{s\gamma} c_1 \alpha_s}{2C^2_{s\gamma}} \mathrm{Im}(\xi_t\xi_b) \\ &+ \frac{\mathrm{Im}(v_t v_u^*)}{|v_t|^2} \frac{[C^{WH}_{s\gamma} + \mathrm{Re}(\xi_t\xi_b) C^H_{s\gamma}] c_1 \alpha_s}{2C^2_{s\gamma}}, \end{aligned} \tag{17}$$

where the first two terms arise from the new source of CP violation for the charged Higgs boson interactions of the fermions with intermediate states sg and $sq\bar{q}$ ($q = u, c$), respectively, and the last term arises from the CKM phase. This last term has been analyzed in detail by Soares [11] in the standard model with the resulting asymmetry between 10^{-2} and 10^{-3}. Our major interest lies in the first two terms which can result in a much larger asymmetry than in the standard model. Values of the asymmetry considering these terms alone for various values of $\mathrm{Re}(\xi_t\xi_b)$ and $\mathrm{Im}(\xi_t\xi_b)$ are illustrated in Fig. 1. It is seen that asymmetries between 2% and 10% are quite reasonable. These results, as those in the standard model, are necessarily uncertain because of the use of quark diagrams to calculate the final state interaction effect. However an asymmetry well above 1% would be a strong indication of this new physics.

It is important to look at effects of $\xi_t\xi_b$ on other observables. In the case of $\mathrm{Re}(\xi_t\xi_b)$ there is no observable that is as sensitive as the $b \to s + \gamma$ rate. On the other hand, $\mathrm{Im}(\xi_t\xi_b)$ is relevant for the neutron electric dipole moment D_n due to the Weinberg dimension-6 gluonic operator [12]. This operator can be induced from two-loop diagrams through the charged Higgs boson exchange with internal loop top and bottom quarks and three external gluons [13]. In the Weinberg three-Higgs-doublet model, the CP-violating phase arises from the mixings among the charged Higgs bosons, which is not relevant in the 2HDM. In the version of the 2HDM discussed here the CP-violating phase comes from the complex Yukawa couplings ξ_t and ξ_b, giving

$$d_n = (3.3 - 0.11) \times 10^{-25} \mathrm{Im}(\xi_t\xi_b)[12 h_C(y_b, y_t)]\, e\,\mathrm{cm}, \tag{18}$$

where the first value in the bracket is from naive dimensional analysis [12] and the second value in the bracket is from a recent reanalysis [14] for the hadronic matrix element. h_C is an integral function [13] with $h_C = 1/12$ for $m_{H^+} = m_t$. From the present experimental upper limit for the neutron electric dipole moment $d_n < 1.2 \times 10^{-25}$ e cm, it is seen that the limit on $\mathrm{Im}(\xi_t\xi_b)$ ranges from 0.3 to 10 due to the large uncertainties of the hadronic matrix element. This limit still allows the large asymmetries discussed above. On the other hand, the detection of such a CP-violating asymmetry in $b \to s\gamma$ would indicate that d_n is not too far below the present limit.

In conclusion, we have shown that in the most general two-Higgs-doublet model [5,6] with approximate global U(1) family symmetries (AGUFS) and CP violation originating solely from a single relative phase of two vacuum expectation values, the CP asymmetry in the decay $b \to s\gamma$ due to new sources of CP violation for charged Higgs boson interactions and final-state-interaction effects can lie between 10^{-2} and 10^{-1}, which is 1 order of magnitude larger than in the standard model.

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