Internal Geometry of an Evaporating Black Hole

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We present a semiclassical model for the formation and evaporation of a four-dimensional black hole. We solve the equations numerically and obtain solutions describing the entire space-time geometry from the collapse to the end of the evaporation. The solutions satisfy the evaporation law $\dot{M} \propto -M^{-2}$, which confirms dynamically that black holes do evaporate thermally. We find that the evaporation process is in fact the shrinking of a throat that connects a macroscopic interior "universe" to the asymptotically flat exterior. It ends either by pinching off the throat leaving a closed universe and a Minkowskian exterior or by freezing up when the throat's radius approaches a Planck size. In either case the macroscopic inner universe is the region where the information lost during the evaporation process is hidden.

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Hawking's discovery [1] of the thermal emission provoked by a gravitational collapse has provided quantum mechanical grounds [2] for the Bekenstein entropy [3]. Alas, it has also led to an apparent breakdown of predictability [4] precisely because of the thermal character of the emitted flux. In addition, if the black hole completely evaporates, this breakdown of predictability results in a loss of information since one is left with uncorrelated quanta [5]. Confronted with this potential violation of the unitary evolution of quantum mechanics, three scenarios have been proposed [6]: (i) The loss of information derived from the original semiclassical treatment of Hawking is real [4,7]. (ii) The evaporation process stops, leaving a Planckian remnant [8] which detains the lost information. (iii) A fully quantum description will reveal that the information is properly recovered within the detailed structure of the emitted quanta [9]. In order to address these issues in a simpler context, a two-dimensional dilaton gravity model [10] was introduced and analyzed intensively. This model has indeed sharpened the problem [11–13] but has not settled the question yet.

The resolution of this problem clearly requires a full dynamical description of the evaporating black hole space-time. At present, a quantum mechanical description is beyond our reach. We present here a four-dimensional semiclassical model which describes the formation and evaporation of a spherically symmetric black hole. We solve the dynamical equations numerically and obtain the space-time geometry until the Planckian regime, where the approximation breaks down and the final product depends on the regularization scheme. The solution confirms the generally accepted features of black hole evaporation, which were anticipated assuming the quasistationarity character of the external geometry [14]. But it also affords an explicit description of the interior geometry and the emergence of the fluxes. In terms of the external retarded time, the interior region is static because the outgoing flux rises in the exterior part only. Hence, at the end of the evaporation one is left with a macroscopic interior region describing a closed universe wherein the curvature is weak. Within the semiclassical treatment, this closed region detains all the correlations with the asymptotic Hawking quanta as well as the quantum specification of the infalling matter.

We use the spherical symmetric metric

$$ds^2 = e^{2f(U,V)} dU\, dV - r^2(U,V) d\Omega^2 . \tag{1}$$

U and V are outgoing and ingoing null coordinates. They are specified on the boundaries $U = U_-$ for $V > 0$ and $V = 0$ for $U > U_-$ by the standard coordinate choice, $r = (V - U)/2$:

$$r(U, V = 0) = -U/2 \, ; \; r(U_-, V) = (V - U_-)/2 . \tag{2}$$

U_- is taken large and negative in order to approximate null past infinity ($\mathcal{I}^-$).

We introduce two matter sources: a classical flux and a quantum field. The first one is a radially infalling null dust $[T^{\rm Cl}_{\mu\nu} = \delta^V_\mu \delta^V_\nu F(V)/4\pi r^2]$ carrying a mass M_0. We choose $F(V)$ as a Gaussian centered around V_i well inside our V domain. Without a quantum backreaction, $T^{\rm Cl}_{\mu\nu}$ would create an eternal black hole of radius $2M_0$ if the width σ is small enough (i.e., $\sigma < M_0$) that the reflection at $r = 0$ can be neglected. The second source is the expectation value of the energy-momentum tensor $T^Q_{\mu\nu}$. We consider a situation with no quantum particles on both boundaries. In this case $T^Q_{\mu\nu}$ is completely determined by the geometry. This tensor engenders, in turn, the Hawking flux and the black hole evaporation. As a model, we take the expectation value of the tensor [15] which arises from the quantization of a 2D massless field in the 2D curved background described by $ds^2 = e^{2f(U,V)} dU\, dV$ [see Eq. (1)] divided by $4\pi r^2$ to mimic the four-dimensional radial dependence. It reads simply

$$T^Q_{UV} = \frac{-\alpha f_{,UV}}{4\pi r^2} , \tag{3}$$

$$T^Q_{UU} = \frac{\alpha[f_{,UU} - (f_{,U})^2]}{4\pi r^2} ; \; T^Q_{VV} = \frac{\alpha[f_{,VV} - (f_{,V})^2]}{4\pi r^2} . \tag{4}$$

The normalization constant α depends linearly on the number of massless fields and will control the rate of evaporation [see Eq. (10) below]. In two dimensions, this expectation value (times $4\pi r^2$) gives an asymptotic flux which coincides with the flux obtained from the Bogoliubov transformation [16] relating the scattered modes to the asymptotic ones. In four dimensions, it offers a fair approximation [17] since most of the energy is carried away by s-waves quanta [18].

In the metric Eq. (1), the Einstein equations split into two dynamical equations and two constraints. The classical source $T^{\rm Cl}_{\mu\nu}$ appears in one constraint only while T^{Q}_{UV} modifies the dynamical Einstein equations. On the left of the infalling matter, for $V \ll V_i$, one has Minkowski space-time. Indeed, on $V = 0$, the first constraint gives $\partial f(U,0)/\partial U = 0$. Then by choosing $f(U,0) = 0$, one equates $(U + V)/2$ to the proper time at rest with respect to the spherical infalling shell. On $U = U_-$, the second constraint gives

$$f_{,V}(U_-, V) = 8\pi r(T^{\rm Cl}_{VV})\,. \tag{5}$$

A simple integration of Eq. (5) gives the Cauchy date for the dynamical integration, $f(U_-, V)$.

The modified dynamical equations read

$$f_{,UV}(1 - \alpha e^{2d}) = d_{,U}\, d_{,V} + e^{2(d-f)}/4\,, \tag{6}$$

$$d_{,UV}(1 - e^{2d}) = d_{,U}\, d_{,V}(2 - \alpha e^{2d}) + e^{2(d+f)}/4\,, \tag{7}$$

where we define $d \equiv -\ln(r)$. The inclusion of the quantum matter term [Eq. (3)] leads, as in dilatonic gravity [10], to singular equations at the critical radius $r_\alpha = \sqrt{1/\alpha}$. In four dimensions, this singularity is an artifact of our model which assumes the validity of expectation values [Eq. (3)] in the Planckian domain. But this method offers a reliable [19] approximation during most of the evaporation process when the prefactor $P = 1 - \alpha e^{2d}$ is still close to 1. If one wishes to carry out the integration into the Planckian domain one can eliminate this singularity by regularizing the prefactor P at small radii in the following way:

$$P_n = 1 - \frac{\alpha e^{2d}}{1 + (\alpha e^{2d})^n}\,. \tag{8}$$

The final sage of the evaporation, where the mass approaches the Plank mass, will then depend on n.

We integrate numerically the dynamical equations using the Cauchy data and obtain the geometry of the collapse and the subsequent evaporation. Figure 1(a) describes the geometry. On the left one finds the inner boundary of the trapped region (defined by $\partial r/\partial v = 0$). It is spacelike because of the positivity of the classical infalling energy [20]. This boundary starts at $r = 0$ and ends when the negative energy flux, which causes the evaporation, equals the tail of the classical flux. The outer part of the trapped region is the t horizon r_{ah}. Since the infalling quantum flux is negative, $r_{ah}(v)$ is timelike [14]. It ends upon reaching the critical radius r_α at which our dynamical equations break down. As can be seen from Fig. 1(a) the evaporation stage is contained in a tiny U lapse around $-13.4 < U < -12.8$. [Recall that the (U, V) coordinates constitute the inertial frame of an observer at rest at the origin.] We make, therefore, a coordinate transformation to (u, v), the inertial asymptotic system at rest with respect to the black hole. (u, v) are defined by $dr/du|_v \to -1/2, dr/dv|_u \to 1/2$ for $r \to \infty$. With this transformation the tiny U lapse is spread out since any inertial collapse results in a Doppler shift given by

$$\frac{dU}{du} = e^{-u/4M_0}\,, \tag{9}$$

wherein the imaginary frequency determines the initial Hawking temperature $1/8\pi M_0$.

Figure 1(b) depicts the same space-time in the (u, v) coordinates. We note that the apparent horizon, in these coordinates, is almost a "static" line: $v = u + \text{const}$. This proves that the inwards flux (T^{Q}_{vv}) on this horizon is equal and opposite to the outgoing flux (T^{Q}_{uu}) at infinity. The mass loss can be calculated from either flux. In practice we measure it from dr_{ah}/dv using $r_{ah} = 2M_{ah}$. When the mass is still large (compared to the critical mass $M_\alpha = 2r_\alpha$) our dynamical model yields

$$\frac{dM_{ah}}{dv} = \frac{-\alpha}{32}\,\frac{1}{M^2_{ah}(v)}\,. \tag{10}$$

This demonstrates that "black holes do evaporate thermally" in perfect agreement with Bardeen [14] whose original derivation assumes a quasistationary character of the external geometry. Naturally, when the mass approaches M_α the mass loss depends on the regularization scheme [see Eq. (8) and Fig. 2]. For $n = 1$ we find that $\dot{M}$ increases sharply while for $n = 2$ a rapid drop in $\dot{M}$ and in the temperature appears upon approaching the critical mass.

In order to illustrate the internal geometry we have depicted, in Fig. 3, the radii of the spheres encountered by successive outgoing light rays (i.e., along $u = \text{const}$) in the Eddington-Finkelstein coordinates (r, v). When the matter is far away from its Schwarzschild radius, the evolution begins with $r_u(v) = (v - u)/2$. In the absence of backreaction it would have ended with the asymptotic line which starts at $r = 0$ and approaches asymptotically $r = 2M_0$, forming the event horizon. When backreaction is included the null rays still spread out from $r = 0$ to $r = 2M_0$ but then contract within the trapped region reaching a minimal radius at r_{ah} before spreading out again. As the black hole loses its mass r_{ah} diminishes accordingly. This geometry describes a shrinking throat that separates the inner "universe" around the infalling mass from the exterior space-time. Since the inner universe is static [see Fig. 1(b)] during the whole evaporation, it remains macroscopic. Thus, what seems to be, for an external observer, the evaporation of the black hole is in fact the shrinking of the throat that connects the internal region to the rest of the world [21].

Before analyzing what implications to the loss of information paradox this geometry might give, we recall

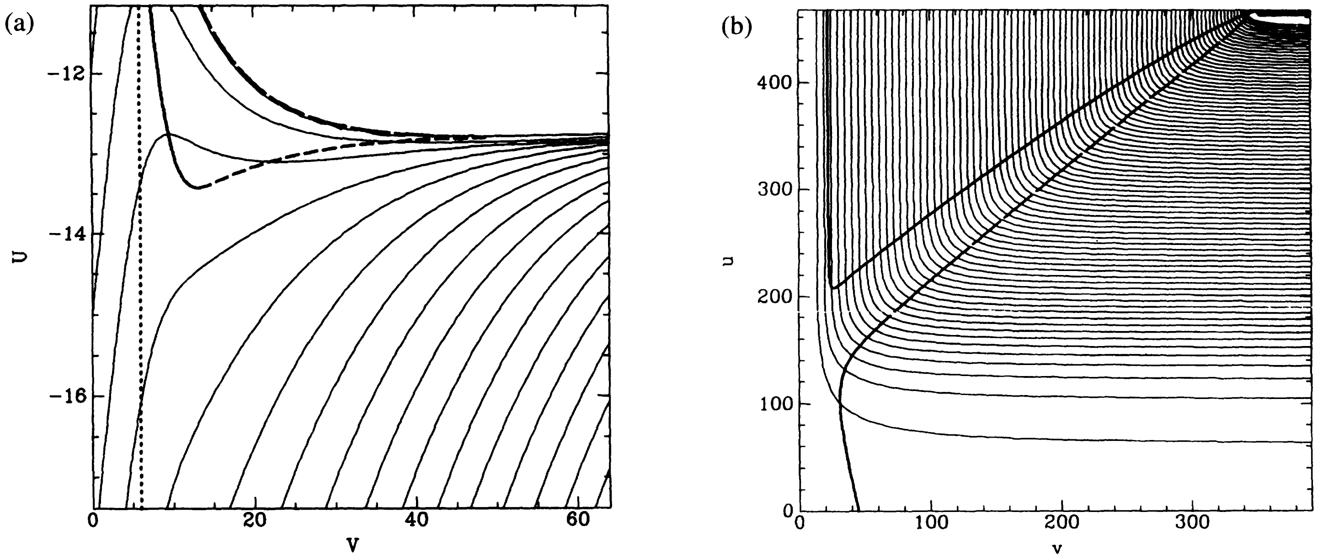

FIG. 1. (a) Contour lines of $r = \text{const}$ from $r = 6$ (upper left corner) to $r = 28.5$ (lower right corner) in the (U, V) coordinate system. The heavy solid line depicts the inner boundary of the trapped region and the short dashed line depicts the outer boundary: the apparent horizon, r_{ah}. The critical radius r_α is depicted by a heavy short dashed line. The infalling classical matter is centered at $V_i = 6$ which is shown by a heavy dotted line and its width is $\sigma = 3$. For $V \ll V_i$ the space-time is Minkowski, as can be seen from the fact that the $r = \text{const}$ lines have a unit slope $(dU/dV = 1)$. For $V \gg V_i$ the space-time has a Schwarzchild geometry characterized by the time dependent residual black hole mass. The $r = 6$, $r = 7.5$, and $r = 9$ curves cross the inner boundary of the trapped region when the black hole forms and the apparent horizon when it evaporates. The other $r = \text{const}$ curves do not intersect the black hole. The semiclassical approximation breaks down around $V = 47$ and $U = -12.8$ where the curves of the critical radius and the apparent horizon intersect. (b) Contour lines of $f = \text{const}$ of the same evaporating geometry in the (u, V) coordinates. Note that in these coordinates the region around $U \approx -12.8$ [of Fig. 1(a)] is stretched to $u \to \infty$. The heavy dashed line designates the locus where T^Q_{uu} reaches half of its asymptotic value. It coincides with the places where $f_{,VU}$ is maximum, and it is always outside the trapped region. The fact that $f_{,VU}$ vanishes inside the trapped region indicates that T^Q_{vv} is constant and T^Q_{uu} is zero. This implies that the infalling matter keeps collapsing under its own gravitational field ignoring completely the Hawking flux which develops outside its past light cone. The critical radius is at $u = \infty$ in these coordinates.

that the geometry was obtained using a semiclassical approximation. Thus, the forthcoming analysis is meaningful only if the *mean* geometry is correctly approximated by the semiclassical method (beware that, for instance, the "trans-Planckian" fluctuations [19] might completely ruin it). Then, in order to address the problem of information loss, which deals with quantum correlations, one has to analyze the quantum evolution on the resulting background.

Before the pinching off, the Schrödingerian evolution of the quantum matter state can be used on all our slices [22] to evaluate the correlations between the field configurations inside and outside the throat. As the evaporation approaches the Planckian regime, a region of high curvature appears *only* at the throat, separating "practically" the left unaffected interior region from the exterior space-time. Hence, whether or not the evaporation stops does not change significantly the situation. If the black hole completely disappears, one finds two disconnected macroscopic regions: a quite big "baby universe" [21] and a Minkowskian exterior. On the other hand, if a remnant characterized by a Planckian size throat is left, there is still a tiny connection with the macroscopic internal region. In either case, the quantum matter field configurations in the internal region are still correlated to external configurations as they were just before the pinching off. The loss of information in the exterior part of the space-time is analogous [7] to the loss of the quantum correlations which occur in any subsystem upon tracing over the quantum states belonging to its complementary subsystem. This loss of information is now inevitable to an outside observer because the dynamics of the evaporation force quantum mechanics to operate in the realm of space-time with a varying topology.

This analysis of the quantum correlations on the shrinking geometry requires, as well, a drastic modification of the interpretation of the Bekenstein entropy. The area of the black hole does not count the (logarithm of the) number of microscopic states contained in the internal region but only the number of them which is still coupled to the external world. (For instance, this entropy reflects the potential increase in the external entropy due to a complete evaporation of the black hole and has to be used upon enclosing the black hole in a cavity and searching for thermodynamical equilibrium since only the states coupled to the external world states could participate to the thermalization.) The universality of this area entropy now comes from the universality of the characteristics of the throat (the throat has no hair) that connects the interior region to the rest of the world. Within the interior region

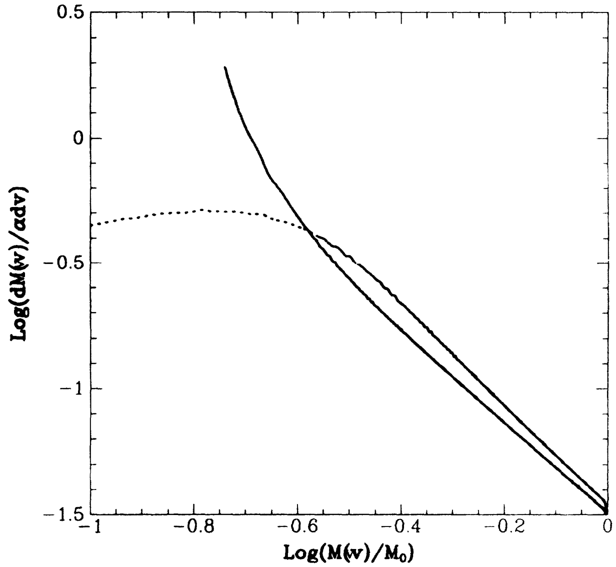

FIG. 2. The logarithm of the rate of evaporation vs the logarithm of the residual mass for different regularization schemes [see Eq. (8)]: $n = 1$ (solid line) with an exploding solution and $n = 2$ (dashed line) with a decreasing evaporation rate. The nonregularized solution ($n = 0$) follows the $n = 2$ curve and stops at the critical mass around the intersection of the curves. Note that the slope being -2 gives Eq. (10).

the number of available microstates depends explicitly on the history of the collapse, since the characteristic size of this region has nothing to do with the actual throat radius.

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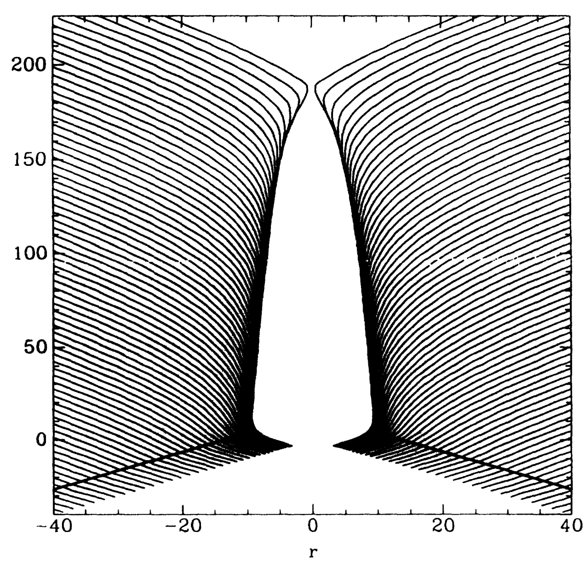

FIG. 3. The radii of spheres $[r_u(v)]$ on successive $u =$ const slices in advanced Eddington-Finkelstein coordinates (r, v). The bold line denotes the center of the infalling matter $v = v_i$.

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