Negative Resistance Instability due to Nonlinear Damping

D. D. Caussyn,[1] M. Ball,[1] B. Brabson,[1] J. Budnick,[1] V. Derenchuk,[1] G. East,[1] M. Ellison,[1] D. Friesel,[1] B. Hamilton,[1] K. Hedblom,[2] W. P. Jones,[1] S. Y. Lee,[1] D. Li,[1] J. Y. Liu,[1] T. Lofnes,[2] K. Y. Ng,[3] A. Riabko,[1] T. Sloan,[1] and Y. Wang[1]

[1]*Indiana University Cyclotron Facility, 2401 Milo B. Sampson Lane, Bloomington, Indiana 47408*

[2]*Uppsala University, The Svedberg Laboratory, Box 533, S-75121, Uppsala, Sweden*

[3]*Fermilab, P.O. Box 500, Batavia, Illinois 60510*

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The longitudinal dynamics of a stored proton beam bunch, acted upon by a nonlinear damping force, was studied experimentally at the Indiana University Cyclotron Facility Cooler Ring. The effect of the nonlinear damping force on synchrotron motion was explored by varying the relative velocity between the cooling electron and the stored proton beams. Maintained longitudinal oscillations were observed, whose amplitude grew rapidly once a critical threshold in the relative velocity between the proton and electron beams was exceeded. We attribute this phenomenon to a negative resistance instability occurring after a Hopf bifurcation.

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The Indiana University Cyclotron Facility (IUCF) Cooler Ring is one of many storage rings which are designed and constructed specifically to employ electron cooling to produce and use high quality medium energy ion beams for nuclear and atomic physics research [1]. For a beam of 45 MeV protons, the equilibrium 95% transverse emittance is about 0.3π mm mrad with a relative momentum spread full width at half maximum (FWHM) of about 1×10^{-4}. The motion of the beam bunch with a small emittance can closely simulate single particle motion. Several experiments studying transverse motion near betatron resonances [2] have demonstrated this advantage.

Recently, the same techniques for studying transverse motion on a turn-by-turn basis were employed to study longitudinal motion, particularly the parametric resonances generated by the rf phase or voltage modulation [3]. In the course of making these measurements we often observed driven, or maintained, longitudinal oscillations when the relative velocities between the proton and electron beams were larger than a threshold value. The maintained oscillations have been previously observed [4], however, there have been neither detailed studies nor a credible explanation to this date.

Since such an instability can heat a stored proton beam, it may have important implications for injection schemes in which the electron cooling is used to cool the newly injected beam into a previously stored stack. This instability may also be important for determining the method by which electron cooling is changed during the proton beam acceleration. More broadly, this effect is of interest in understanding any pendulumlike system with nonlinear damping.

In this Letter we report results of a series of experimental studies on the beam motion as the energy of the synchronous proton is varied while holding the electron energy constant. We compare the experimental data with results from computer simulations. The threshold behavior of the instability is also studied semianalytically.

Since its discovery in 1945 by McMillan and Veksler [5], synchrotron motion has come to be relatively well understood. However, synchrotron motion in a system with electron cooling has received relatively little attention. Electron cooling adds to the system a damping force similar to that of a damped pendulum. Longitudinal motion in a synchrotron is characterized by ϕ, the phase of a particle relative to the rf wave in the rf cavity, and its conjugate momentum variable δ, which is the fractional momentum deviation of the particle from that of the synchronous particle. The difference equations relating successive revolutions for the longitudinal motion are

$$\delta_{n+1} = \delta_n - \frac{2\pi\nu_s^2}{h\eta}(\sin\phi_n - \sin\phi_s) - f(\delta_n), \quad (1)$$

$$\phi_{n+1} = \phi_n + 2\pi h\eta\delta_{n+1}, \quad (2)$$

where η is the phase slip factor, ϕ_s is the synchronous phase angle, which for a stored beam is 0°, h is the harmonic number, $f(\delta)$ is the damping drag force, provided by electron cooling, ν_s is the small amplitude synchrotron tune, which is the ratio of the angular synchrotron frequency ω_s to the angular revolution frequency ω_0, and the subscript n refers to the revolution number. The region of phase space around the stable fixed point at $\phi = \phi_s$ and $\delta = 0$ within the separatrix is called the rf bucket, and its phase space area is called the bucket area.

The damping force $f(\delta)$ produced by the electron cooling is the result of a statistical exchange of energy in collisions between the protons and relatively cold electrons as they travel together in a section in the accelerator. At IUCF, the cooling section is about 2.2 m or about 2.5% of the circumference. The damping force is generally a nonlinear function of the relative velocity $v_{\rm rel}$ between the electrons and the protons. Let δ be

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the fractional momentum deviation of a proton from the synchronous particle and δ_e be the fractional momentum deviation of a proton traveling at the same velocity as the electrons with respect to the synchronous particle. Then the $v_{\rm rel}$ in the laboratory frame is given by $v_{\rm rel}^{\rm lab} = (\delta - \delta_e)\beta c/\gamma^2$, where β and γ are the usual relativistic factors for the synchronous particle, and c is the speed of light. In the rest frame of the electrons, we have $v_{\rm rel}^{\rm rest} = \gamma_e^2 v_{\rm rel}^{\rm lab}/(1 - \gamma_e^2 \beta_e v_{\rm rel}^{\rm lab}/c)$, where γ_e and β_e are relativistic factors for the electrons. Since $|v_{\rm rel}^{\rm rest}| \ll c$ and $\gamma \approx \gamma_e$, we obtain

$$v_{\rm rel} = (\delta - \delta_e)\beta c, \tag{3}$$

where we have dropped the superscript specifying the reference frame.

The drag force, based on the nonmagnetized binary collision theory for a cooling electron beam with an isotropic phase space distribution, can be parametrized as follows [6]:

$$f(\delta) = \frac{4\pi\alpha\Delta_e}{\omega_0} g(\zeta), \tag{4}$$

where the kinematic function $g(\zeta)$ is an odd function given by $g(\zeta) = \frac{3\sqrt{\pi}}{4\zeta^2}[\mathrm{erf}(\zeta) - \frac{2}{\sqrt{\pi}}\zeta e^{-\zeta^2}]$ with $\zeta = (\delta - \delta_e)/\Delta_e$, $\Delta_e = \sigma_e/\beta c$, and α is the $1/e$ damping rate for small relative velocities in s^{-1}. Here σ_e is the rms velocity spread of the electron beam, which is related to the effective temperature. Note that $g(\zeta) \longrightarrow \zeta$ as $\zeta \longrightarrow 0$, the extrema of $g(\zeta)$ are located at $\zeta \approx \pm 0.97$, and $g(\zeta) \longrightarrow 0$ as $\zeta \longrightarrow \pm\infty$. For the case where the electron velocity is the same as the velocity of the synchronous particle, i.e., $\delta_e = 0$, the damping force $f(\delta)$ is zero for a particle at the center of the rf bucket, where the rf force is zero. On the other hand, when the electron velocity is much different from the velocity of the synchronous particle, the damping force is nonzero for a particle at the center of the rf bucket. More importantly, the slope of the damping force at the center of the bucket may change sign with respect to the relative velocity, i.e., $g'(\zeta) < 0$. As we will discuss later, this may lead to an instability at the center of the rf bucket.

In machines where the electron beam is magnetically confined by a solenoidal field, as it is in the IUCF cooler ring, the damping force can be enhanced by an effect called magnetized cooling, which becomes important when the relative velocities are small. However, for this effect to become significant there must be a rather precise alignment of the electron and proton beams. Since we made little special effort in this respect and our experiments studied effects of the nonlinear cooling force at a relatively large relative velocity, we assume that the damping force is given by the nonmagnetized theory alone in our data analysis.

This set of experiments was done with a 45 MeV proton beam injected and then stored in a 10 s cycle with electron cooling being completed within the first 5 s. The phase slip factor η was about -0.86. The rf cavity frequency was 1.031 68 MHz with harmonic number $h = 1$. The beam was a single bunch of about 3×10^8 protons with a typical length of about 60 ns (or 5.4 m) FWHM for a rf peak voltage of about 41 V. Since measurements of longitudinal motion were being made, the rf phase lock feedback loop was switched off. Damping of synchrotron oscillations while operating under these conditions occurred entirely due to the electron cooling.

The phase of the beam was determined from the relative phase between the signal from a pickup coil in the rf cavity, and a sum signal from a beam position monitor (BPM) after passing it through a 1.4 MHz low-pass filter. Our current phase detector has a range of 720°. The momentum deviation of the beam was found from changes in the beam-closed orbit $\Delta x_{\rm co}$, which was measured with a BPM in a region of high dispersion D_x. The fractional momentum deviation δ could then be determined using the relation $\delta = \Delta x_{\rm co}/D_x$ where D_x was measured to be 3.9 m. The position signal was passed through a 3 kHz low-pass filter to remove the possible effects of coherent betatron oscillations. Both the δ and phase signals were digitized using our data acquisition system [2]. As many as 16 384 points were digitized at ten-turn intervals.

To investigate the effect of the nonlinear damping force on motion, the electron velocity was displaced from the proton velocity to produce a nonzero relative velocity. This can be achieved in two ways. The most straightforward way would have been to change the electron energy. However, at IUCF the electron energy is changed by changing the high voltage power supply (HVPS) setting, which is done digitally in steps of about 4.5 V. This would result in fractional changes in the electron velocity $\Delta\beta/\beta$ in steps of about 9×10^{-5}, which proved to be too coarse. The other method, which was used in this experiment, was to change the energy of the proton beam. This was done by changing the rf frequency in steps as small as 1 Hz with a resulting change in the fractional proton velocity of about 1×10^{-6}. If the electron velocity is equal to the proton velocity when the rf cavity frequency is f_0, then the fractional momentum deviation of the electron beam δ_e from the proton beam at the new rf frequency f is given by $\delta_e = (f - f_0)/\eta f_0$.

When the rf frequency is shifted, the beam is displaced from the origin of the longitudinal phase space and begins to undergo synchrotron oscillations. The motion of the beam then damps to an attractor, which may be a fixed point or a limit cycle. A simple analog to this is a pendulum in a stiff breeze, the air resistance playing the role of the electron cooling. The motion was characterized by measuring the peak amplitudes of the oscillations, $\hat{\phi}$ and $\hat{\delta}$, after waiting for 3 s to allow the initial transient to damp out. If the damping force were linear over the entire range of $v_{\rm rel}$, the expected result is that the proton beam would damp to a new fixed point having a phase $\phi_{\rm FP}$ given approximately by

$$\phi_{\rm FP} \approx \frac{2\alpha h \eta \delta_e}{\omega_0 \nu_s^2}, \qquad (5)$$

which is normally less than 1°. This would correspond to the situation where the proton beam was continuously losing or gaining energy due to the damping force, but with it being compensated by the rf cavity. This fixed point is the synchronous phase angle needed to compensate the energy loss or gain due to the cooling electrons.

A typical result of this measurement is shown in Fig. 1 for the rf cavity voltage of 85 V. Two sets of measurements of the maximum phase amplitude $\hat{\phi}$ are shown: one using the phase detector previously described, and a second set using an oscilloscope to measure the separation in time between the extremes in the oscillation. The solid lines shown are the results of computer simulation using Eqs. (1) and (2) with the damping force of Eq. (4). The tracking was done for a time equivalent of 40τ before finding $\hat{\phi}$, where $\tau \equiv 1/\alpha \approx 25$ ms, which was measured from the small amplitude damping rate. Each solid line corresponds to a different electron temperature Δ_e, starting with $\Delta_e = 1 \times 10^{-4}$ for the pair of solid lines nearest the nominal frequency of $f_0 = 1.031\,680$ MHz and increasing in steps of 1×10^{-4} for each pair of lines as one moves away from f_0. While the agreement of the tracking results with the data is not particularly good for large amplitude oscillations, for small amplitudes the best agreement is obtained when $\Delta_e \approx 3 \times 10^{-4}$.

It is clear that a unique feature of the motion is a threshold for $v_{\rm rel}$ (or $\delta_p - \delta_e$) beyond which the steady-state motion is not a fixed point attractor, but a limit cycle [7]. The measurement of the steady-state motion was repeated for five different rf cavity voltages, at various times. Although the electron cooling may not have

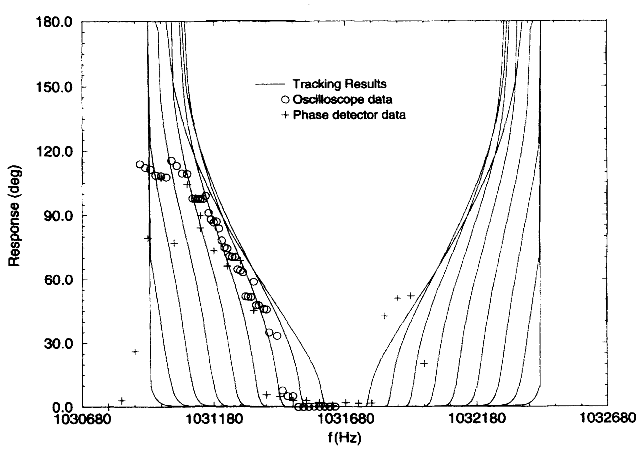

FIG. 1. A plot of the peak phase of the steady-state motion as the proton energy (rf cavity frequency) is varied. The solid lines are computer tracking results for Δ_e from 1×10^{-4} to 9×10^{-4}. The rf voltage is 85 V.

been identical for all cases, the results were qualitatively similar. While the computer tracking produces results which are consistent with the data, a more understandable description of the motion and an explanation of threshold behavior can be obtained from the equation of motion with appropriate approximations. To obtain the threshold behavior, we linearize the synchrotron Hamiltonian, i.e., $\sin\phi \approx \phi$. For simplicity, let $x = \phi / h\eta\Delta_e$, so that $\dot{x} = \delta/\Delta_e$, where the dot corresponds to the derivative with respect to the orbiting angle θ with $\dot{\phi} = (\phi_{n+1} - \phi_n)/2\pi$. From Eqs. (1) and (2), the synchrotron equation of motion becomes

$$\ddot{x} + \frac{2\alpha}{\omega_0} g(\dot{x} - \dot{x}_e) + \nu_s^2 x = 0, \qquad (6)$$

where $\dot{x}_e = \delta_e/\Delta_e$. Because of the nonlinearity in the kinematic function $g(\dot{x} - \dot{x}_e)$, Eq. (6) is a complicated nonlinear differential equation. However, the location of the attractor may be determined by using a method called harmonic linearization [8]. Let the ansatz of Eq. (6) be $x = A\sin(\nu\theta)$ and $\dot{x} = A\nu\cos(\nu\theta)$, where A and ν are parameters to be determined from the equation of motion. Substituting the solution into the damping force term in Eq. (6), the damping force becomes harmonic in time. We can expand the kinematic function in a Fourier series as

$$g(\dot{x} - \dot{x}_e) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\nu\theta) + \sum_{k=1}^{\infty} b_k \sin(k\nu\theta), \qquad (7)$$

where

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} g(\dot{x} - \dot{x}_e)d\theta, \qquad (8)$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} g(\dot{x} - \dot{x}_e)\cos(k\nu\theta)d\theta, \qquad (9)$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} g(\dot{x} - \dot{x}_e)\sin(k\nu\theta)d\theta \qquad (10)$$

are functions of $A\nu$ and δ_e/Δ_e. Now, the harmonic linearization approximation is to keep only the dominant terms of this expansion, i.e., the dc and the first-harmonic terms. The damping force becomes

$$\frac{2\alpha}{\omega_0} g(x, \dot{x}) = 2\alpha_1 \dot{x} + 2\nu_1\nu_s x + \nu^2 x_1, \qquad (11)$$

where $\alpha_1 = a_1\alpha/\omega_0\nu A$, $\nu_1 = b_1\alpha/\omega_0\nu_s A$, and $x_1 = 2\alpha a_0/\omega_0\nu^2$. Thus the equation of motion becomes

$$\ddot{x} + 2\alpha_1\dot{x} + \nu^2(x - x_1) = 0, \qquad (12)$$

where $\nu_1 = 0$ and $\nu = \nu_s$. Note that α_1 corresponds to the average value of the damping coefficient over a complete oscillation and is a function of the amplitude A of the solution. If there exists a limit cycle for $A \neq 0$, then α_1 must be zero. Therefore, A can be obtained from the zeros of the function a_1. While closed form expressions for the functions in Eqs. (8)–(10) have not been obtained, these functions and their zeros were determined numerically.

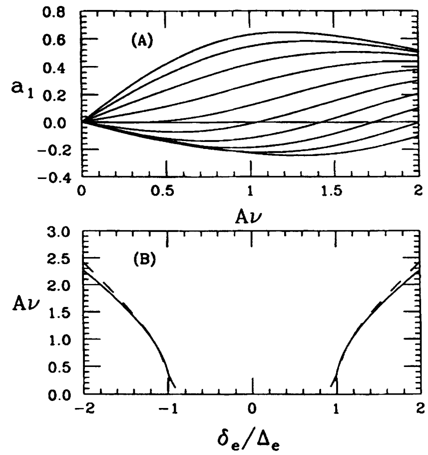

FIG. 2. In (a) a plot of the function a_1 versus $A\nu$ for $\delta_e/\Delta_e = 0.2, 0.4, \ldots$ in a step of 0.2 from the uppermost curve. In (b) the location of the zeros for the curves in (a) versus δ_e/Δ_e are plotted (solid line). The tracking result from one case is shown (dashed lines) for comparison.

Figure 2(a) shows a plot of a_1 as a function of $A\nu$ for various values of δ_e/Δ_e. The locations of the zeros of a_1 as a function of the momentum offset δ_e/Δ_e are shown in Fig. 2(b), where they are joined with a solid line. For comparison, the location of the attractor as found from computer tracking of the difference equations is shown as a dashed line. As the fractional momentum offset of the electron beam is increased, the value of α_1 for small amplitude oscillations begins to decrease and eventually changes sign. The value of δ_e for which α_1 changes sign is where a driven, or maintained, oscillation first appears. This point happens to be near Δ_e, where $f(\delta)$ is an extremum. Thus the normally damped system becomes antidamped. When this condition is reached, the dynamics of the system is similar to that of a system with negative resistance [9]. At the threshold, the fixed point attractor has bifurcated into a limit cycle, and the fixed point attractor turns into an unstable fixed point. A bifurcation of this type is called a Hopf bifurcation [7].

In conclusion, we have done a series of detailed experiments to investigate an observed instability created for an electron cooled bunched proton beam when the electron velocity differs from the proton velocity. We have shown that for an electron cooled beam, this instability can be explained as a maintained oscillation generated by the negative resistance resulting from a large relative velocity between the electron and proton beams at which the damping force *decreases* with increasing relative velocities.

Since the transition from motion which damps to a fixed point to a maintained oscillation is quite sharp and strongly dependent on Δ_e, this effect may be useful in determining the effective electron temperature Δ_e. This phenomenon should also be an important consideration in injection schemes in which electron cooling is used to cool a proton beam at an energy different from the injected beam, as it is done at IUCF, since quite clearly it can have the unintended effect of heating the injected beam. Another place where this phenomenon may have an impact is in the way in which the cooling electron and the proton beam energies are ramped. It may be inadvisable to ramp the electron and proton energies separately, depending on the strength of the electron cooling. The degree to which this phenomenon may affect these operations requires further study. A more sophisticated treatment of the motion which can predict the amplitude of the transient and its growth rate is needed. Along with the maintained oscillations, we have also observed coherent dipolelike oscillations with a beamlet lying on or near the limit cycle. Detailed analyses of these data and other related experimental results will be reported in a regular article.

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[1] R.E. Pollock, Annu. Rev. Nucl. Part. Sci. **41**, 357 (1991).
[2] S.Y. Lee *et al.*, Phys. Rev. Lett. **67**, 3768 (1991); D.D. Caussyn *et al.*, Phys. Rev. A **46**, 7942 (1992).
[3] M. Ellison *et al.*, Phys. Rev. Lett. **70**, 591 (1993); H. Huang *et al.*, Phys. Rev. E **48**, 4678 (1993); D. Li *et al.*, Phys. Rev. E **48**, R1638 (1993); M. Syphers *et al.*, Phys. Rev. Lett. **71**, 719 (1993); Y. Wang *et al.*, Phys. Rev. E **49**, 1610 (1994).
[4] Tim Ellison, Ph.D. dissertation, Indiana University, 1990 (unpublished).
[5] E.M. McMillan, Phys. Rev. **68**, 143 (1945); V.I. Veksler, C. R. Acad. Sci. U.S.S.R. **43**, 329 (1944); **44**, 365 (1944).
[6] H. Poth, Phys. Rep. **196**, 135 (1990).
[7] J.P. Eckmann, Rev. Mod. Phys. **53**, 643 (1981).
[8] I. Huntly and R.M. Johnson, *Linear and Nonlinear Differential Equations* (Halsted Press, New York, 1983), pp. 166–168.
[9] A.B. Pippard, *The Physics of Vibration* (Cambridge University Press, New York, 1978), pp. 306–342.