Fluctuations and Intrinsic Pinning in Layered Superconductors

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A flux liquid can condense into a smectic crystal in a pure layered superconductor with the magnetic field oriented nearly parallel to the layers. If the smectic order is commensurate with the layering, this crystal is *stable* to point disorder. By tilting and adjusting the magnitude of the applied field, both incommensurate and tilted smectic and crystalline phases are found. We discuss transport near the second order smectic freezing transition and show that permeation modes lead to a small nonzero resistivity and large but finite tilt modulus in the smectic crystal.

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In the past few years, both experimental [1,2] and theoretical [3,4] work has emphasized the importance of pinning in type-II superconductors. Although much of this work has focused on random defects, e.g., twin boundaries, the layered structure of the copper-oxide material itself provides a nonrandom source of pinning [5]. At low temperatures, since the c-axis coherence length $\xi_{c0} \approx 4$ Å $\lesssim s \approx 12$ Å, the lattice constant in this direction, vortex lines oriented in the a-b plane are attracted to the regions of low electron density between the CuO_2 layers. In this Letter, we discuss the phase diagram of intrinsically pinned vortices in near perfect alignment with the layers. In contrast to previous work, we focus on the behavior relatively close to T_c, where transport measurements are most easily performed and hysteretic effects are weak. Our research is motivated by the recent experimental work of Kwok *et al.* [6], who observed a continuous resistive transition in $YBa_2Cu_3O_7$ for fields very closely aligned ($\theta < 1°$) to the a-b plane.

To understand thermal fluctuations, we employ the boson mapping [7]. Consider an isolated flux line oriented along the a-b plane. For $T \gtrsim 80$ K, $\xi_c \approx \xi_{c0}(1 - T/T_c)^{-1/2} \gtrsim s$, and the intrinsic pinning barrier energy per unit length is [8]

$$U_p \approx 5 \times 10^2 \frac{\epsilon_0}{\gamma}\left(\frac{\xi_c}{s}\right)^{5/2} e^{-15.8\xi_c/s}, \qquad (1)$$

where the energy scale $\epsilon_0 = (\phi_0/4\pi\lambda_{ab})^2$, with ϕ_0 the flux quantum, λ_{ab} the a-b plane penetration depth, and where $\gamma \equiv \sqrt{m_c/m_{ab}}$ is the anisotropy. With coordinates $\hat{\mathbf{y}} \parallel \mathbf{B} \perp \hat{\mathbf{c}}, \hat{\mathbf{z}} \parallel \hat{\mathbf{c}}$, the free energy of the vortex [described by $x(y)$ and $z(y)$] is

$$F_v = \int_0^L dy \left\{ \frac{\tilde{\epsilon}_\parallel}{2}\left|\frac{dx}{dy}\right|^2 + \frac{\tilde{\epsilon}_\perp}{2}\left|\frac{dz}{dy}\right|^2 - U_p \cos 2\pi z/s \right\}, \qquad (2)$$

where the stiffness constants obtained from anisotropic Ginzburg-Landau (GL) theory are $\tilde{\epsilon}_\parallel = \epsilon_0/\gamma$ and $\tilde{\epsilon}_\perp = \epsilon_0\gamma$ (see, e.g., Ref. [9]). Upon integrating the x displacement, the remainder of Eq. (2) maps to the Euclidean action of a quantum particle in a one-dimensional periodic potential. In the quantum-mechanical analogy, the particle tunnels between adjacent minima of the pinning potential, leading to the well-known completely delocalized Bloch wave functions even for *arbitrarily* strong pinning. The "time" required for this tunneling maps to the distance $L_{\rm kink}$ in the y direction between kinks in which the vortex jumps across one CuO_2 layer. The WKB approximation gives

$$L_{\rm kink} \sim s\sqrt{\frac{\tilde{\epsilon}_\perp}{U_p}}\, e^{\sqrt{\tilde{\epsilon}_\perp U_p}\, s/k_BT}, \qquad (3)$$

where k_BT plays the role of $\hbar$. When the sample is larger than $L_{\rm kink}$ along the field axis the flux line will wander as a function of y, with

$$\langle [z(y) - z(0)]^2 \rangle \sim Dy, \qquad (4)$$

where the "diffusion constant" $D \approx s^2/L_{\rm kink}$.

For $\sqrt{\tilde{\epsilon}_\perp U_p}\, s \lesssim k_BT$, the pinning is extremely weak, and the WKB approximation is no longer valid. Instead, the diffusion constant $D \approx k_BT/\tilde{\epsilon}_\perp$, as obtained from Eq. (2) with $U_p = 0$. At much lower temperatures, when $\xi_c \ll s$, the energy in Eq. (1) must be replaced by the cost of creating a "pancake" vortex [10] between the CuO_2 planes. In this regime, $L_{\rm kink} \sim \xi_{ab}(s/\xi_c)^{\epsilon_0 s/k_BT}$.

For $T \approx 90$ K, as in the experiments of Kwok *et al.* [6], $\xi_c/s \approx 2.3$, and Eq. (1) gives $\sqrt{\tilde{\epsilon}_\perp U_p}\, s/k_BT \ll 1$, indicative of weak pinning and highly entangled vortices in the liquid state. The transverse wandering in this *anisotropic* liquid is described by a boson "wave function" with support over an elliptical region of area $k_BTL_y/\sqrt{\tilde{\epsilon}_\parallel\tilde{\epsilon}_\perp}$ with aspect ratio $\Delta x/\Delta z = \gamma \approx 5$ for $YBa_2Cu_3O_7$ ($L_y \approx 1$ mm is the sample dimension along $\hat{\mathbf{y}}$) [11].

To explain the observed transition, the interplay between intervortex interactions [5] and thermal fluctuations must be taken into account in an essential way. The experiments of Ref. [6] rule out conventional freezing, which is first order in all known three-dimensional cases. A vortex/Bose glass (VG/BG) transition also appears untenable, due to the purity of the sample (only six twin planes are present, and resistance measurements are consistent with first order melting for $\theta > 1°$). The dynamical scaling exponents are also inconsistent with VG/BG

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values. Instead, we postulate freezing into an intermediate "smectic" phase between the flux liquid (high temperature) and crystal/glass (low temperature). Such smectic freezing, as first discussed by de Gennes for the nematic–smectic-A transition [12], can occur via a continuous transition in three dimensions. Changes in the applied magnetic field tune the commensurability of the vortex system with the underlying periodic structure [13].

Our analysis leads to the phase diagram shown in Fig. 1. Upon lowering the temperature for $H_c = 0$ and a commensurate value of H_b, the vortex liquid (L) freezes first at T_s into the pinned smectic (S) state, followed by a second freezing transition at lower temperatures into the true vortex crystal (X). When $H_c \neq 0$, tilted smectic (TS) and crystal (TX) phases also exist. The TS-L and TX-TL transitions are XY-like, while the TS-S and TX-X phase boundaries are commensurate-incommensurate transitions (CITs) [13]. At larger tilts, the TX-TS and TS-L lines merge into a single first order melting line. As H_b is changed, incommensurate smectic (IS) and crystal (IX) phases appear, again separated by CITs from the pinned phases and an XY transition between the IS and L states. Note that the commensurate smectic order along the c axis is *stable* to point disorder because phonon excitations are massive (see below). This stability should *increase* the range of smectic behavior relative to the (unstable) crystalline phases when strong point disorder is present.

We now proceed with the derivation of these results; further details will be given in Ref. [11]. The a, b, and c crystallographic axes are parametrized by the coordinates x, y, and z, respectively. The CuO_2 layers are thus perpendicular to the $z = c$ axis. The magnetic field is primarily along the $y = b$ direction. When $H_c = 0$ and H_b is tuned to commensuration, de Gennes' freezing theory applies near T_s. Smectic order appears as a density wave,

$$n(\mathbf{r}) \approx n_0 \,\mathrm{Re}\{1 + \Phi(\mathbf{r}) e^{iqz}\}, \tag{5}$$

where n_0 is the background density and $q = 2\pi/a$ is the wave vector of the smectic layering (with wavelength a). The complex translational order parameter $\Phi(\mathbf{r})$ is assumed to vary slowly in space. The superconductor is invariant under translations and inversions in x and y and has a discrete translational symmetry under $z \to z + s$, where s is the CuO_2 double-layer spacing. From Eq. (5), these periodic translations correspond to the phase shifts $\Phi \to \Phi e^{iqs}$. In the commensurate limit, $a = ms$, with m an arbitrary integer. The most general free energy consistent with these symmetries is

$$F = \int d^3r \left\{ \frac{K}{2} |(\nabla - i\mathbf{A})\Phi|^2 + \frac{r}{2}|\Phi|^2 + \frac{v}{4}|\Phi|^4 - \frac{g}{2}(\Phi^m + \Phi^{*m}) + \cdots \right\}, \tag{6}$$

where the coordinates have been rescaled to obtain an isotropic gradient term. The "vector potential" $\mathbf{A}$ represents changes in the applied field $\delta\mathbf{H} = \delta H_b \hat{\mathbf{y}} + H_c \hat{\mathbf{z}}$, with $A_x = 0$, $A_y = qH_c/H_b$, and $A_z q \delta H_b/H_b$. The form of this coupling follows from the transformation properties of Φ [11,12]. Additional interactions with long wavelength fluctuations in the density and tangent fields [14] are irrelevant to the critical behavior [11].

When $\delta\mathbf{H} = \mathbf{A} = 0$, Eq. (6) is the free energy of an XY model with an m-fold symmetry breaking term. A second order freezing transition occurs within Landau theory when $v > 0$ and $r \propto T - T_s$ changes sign from positive (in the liquid) to negative (in the smectic). The renormalization group (RG) scaling dimension λ_m of the symmetry breaking term is known *experimentally* in three dimensions as $\lambda_m \approx 3 - 0.515m - 0.152m(m-1)$ [15]. For $m > m_c \approx 3.41$, the field g is irrelevant ($\lambda_m < 0$), and the transition is in the XY universality class. The magnetic fields used by Kwok *et al.* [6] correspond to $m = 9$–11 [16], well into this regime. The static critical behavior is characterized by the correlation length exponent $\nu \approx 0.671 \pm 0.005$ and anomalous dimension $\eta \approx 0.040 \pm 0.003$ [17].

Deep in the ordered phase ($r < 0$), amplitude fluctuations of Φ are frozen out. Writing $\Phi = \sqrt{|r|/v}\, e^{2\pi i u/a}$, Eq. (6) becomes, up to an additive constant,

$$F_{\mathrm{smectic}} = \int d^3r \left\{ \frac{\kappa}{2}(\nabla u - \mathcal{A})^2 - \tilde{g}\cos 2\pi u/s \right\}, \tag{7}$$

where $\kappa = 4\pi|r|K/a^2 v$, $\tilde{g} = g(|r|/v)^{m/2}$, and the reduced vector potential is $\mathcal{A} = \mathbf{A}/q$. The displacement field u describes the deviations of the smectic layers from their uniform state. The sine-Gordon term is an effective periodic potential acting on these layers. As is well known from the study of the roughening transition [18], such a perturbation is always relevant in three dimensions.

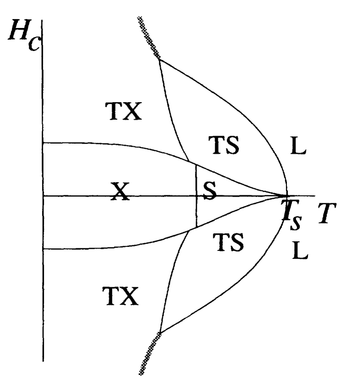

FIG. 1. Phase diagram for the pure vortex system as a function of temperature T and c axis field H_c. The fuzzy lines indicate first order transitions. A similar phase diagram holds in the δH_b-T plane with the TX and TS phases replaced by IX and IS states, respectively. The merging of the IX-IS and IS-L lines need not occur in this case.

The smectic state is thus *pinned* at long distances (i.e., the displacements u are localized in a single minima of the cosine) [19].

At lower temperatures, provided point disorder is negligible, the vortices will order along the x axis as well [20]. This is once again a freezing transition at a single wave vector and is described by a Landau theory like Eq. (6) (with $g = 0$ since the modulating effect of the underlying crystal lattice is much weaker in the x direction), with XY critical behavior.

Next, we consider the effects of $\delta\mathbf{H} \neq 0$. When the fields are small, $\mathcal{A}$ is an *irrelevant* operator in the smectic phase (as can be easily seen by replacing the periodic potential by a "mass" term $\propto u^2$). This implies that the smectic layers do not tilt under weak applied fields, i.e., $\partial\langle\partial_y u\rangle/\partial H_c|_{H_c=0} = 0$. The full susceptibility $c_{44}^{-1} \equiv \partial\langle B_c\rangle/\partial H_c|_{H_c=0}$ is obtained from the expression

$$B_c = \frac{Kq}{H_b}\,\mathrm{Im}(\Phi^*\partial_y\Phi) + (c_{44,0}^{-1} - r''|\Phi|^2)H_c\,, \qquad (8)$$

where $c_{44,0}$ is the tilt modulus obtained from anisotropic GL theory (without accounting for the discreteness of the layers) and $r'' \equiv \partial^2 r/\partial H_c^2|_{H_c=0}$. Equation (8) has a simple physical interpretation. The first term is the contribution to B_c from tilting of the layers (described by a phase rotation of Φ). Transverse field penetration at fixed layer orientation contributes via the second term. Such motion arises microscopically from a nonzero equilibrium concentration of vortices with large kinks extending between neighboring smectic layers. Equation (8) predicts a nondivergent singularity $c_{44}(T) - c_{44}(T_s) \sim |T - T_s|^{1-\alpha}$ at the critical point, where α is the specific heat exponent. At low temperatures in the smectic phase this crosses over to the much larger value $c_{44} \approx [(\tilde{\epsilon}_\perp/U_p)^{1/2}k_BT/B\phi_0 ms]\exp(E_{lk}/k_BT)$, where the energy of a large kink is $E_{lk} \approx \sqrt{\tilde{\epsilon}_\perp U_p}\,ms$.

As is well known from the study of the sine-Gordon model [13], a larger incommensurability can be compensated for by energetically favorable "solitons," or walls across which $u \rightarrow u + s$. Solitons begin to proliferate when their field energy per unit area $\sigma_{\text{field}} \sim -\kappa\mathcal{A}s$ exceeds their cost at zero field, $\sigma_0 \sim \sqrt{\kappa\tilde{g}}\,s$ [estimated from Eq. (7)].

Physically, these solitons correspond to extra and/or missing flux line layers and walls of aligned "jogs" for $\delta\mathbf{H}$ along the b and c axes, respectively. In the former case, this leads to an incommensurate smectic (IS) phase, whose periodicity is no longer a simple multiple of s. For $\delta\mathbf{H} \parallel \hat{\mathbf{z}}$, the solitons induce an additional periodicity along the y axis. This tilted smectic (TS) phase has long range translational order in two directions [21]. The analogous tilted *crystal* (TX) phase is qualitatively similar, but has long range order in three directions.

Unlike the corresponding CIT in two-dimensional adsorbed monolayers [13], we find that energetic interactions between widely separated solitons dominate over entropic contributions [11]. The free energy density in the incommensurate phases is thus

$$f_{\text{soliton}} \sim -\frac{|\sigma|}{l} + \frac{\Delta}{l}\,e^{-l/w}, \qquad (9)$$

where $\sigma \equiv \sigma_{\text{field}} + \sigma_0 < 0$ is the total areal free energy of the soliton; Δ and w set the energy and length scales of the soliton interactions. Minimizing Eq. (9) gives a soliton separation $l \sim w\ln(\Delta/|\sigma|)$ near the CIT.

As the temperature is increased within the IS or TS phases, the system melts into the liquid. The IS-L and TS-L transitions are described by Eq. (6) with $g = 0$ and are thus XY-like.

The shape of the CIT phase boundary is of particular experimental interest. In the mean field regime, this is obtained from the condition $\sigma = 0$ as $\delta H \sim |r|^{\Upsilon}$, with $\Upsilon_{\text{MF}} = (m - 2)/4$. By the usual Ginzburg criterion, mean field theory breaks down for $|r| \lesssim (k_BTv/K^{3/2})^2$. To determine the shape of the phase boundary in this critical regime, we follow the RG flows until $|r|$ is order 1. Then $\delta H_R \sim \xi^{\lambda_H}\delta H$ and $g_R \sim \xi^{\lambda_m}g$, with $\xi \sim |r|^{-\nu}$. Rotational invariance at the rescaled fixed point ($g = 0$) implies that the field exponent is *exactly* $\lambda_H = 1$ [11]. Using these renormalized quantities, we find $\Upsilon_{\text{crit.}} = (|\lambda_m| + 2)\nu/2 \approx 4.9$–$7.2$ for the fields used in Ref. [6]. The IS-L and TX-L phase boundaries are nonsingular and are determined locally by the smooth $\delta\mathbf{H}$ dependence of r.

We next discuss transport measurements using a coarse-grained approach similar to flux line hydrodynamics [14]. The time evolution of the fluxon density $n(\mathbf{r},t)$ is determined by the continuity equation

$$\partial_t n + \nabla_\perp \cdot \mathbf{j}_v = 0\,, \qquad (10)$$

where $\nabla_\perp = (\partial_x, \partial_z)$, and $\mathbf{j}_v$ is the vortex current, determined by the constitutive equation [14]

$$\Gamma\mathbf{j}_v = -n\nabla_\perp\frac{\delta F}{\delta n} + n\partial_y\frac{\delta F}{\delta\tau} - \tau_\alpha\nabla_\perp\frac{\delta F}{\delta\tau_\alpha} + n\mathbf{f}\,, \qquad (11)$$

where τ is the coarse-grained tangent density, Γ is related to the Bardeen-Stephen friction coefficient $\gamma_{\text{BS}} = n_0\Gamma$, and the driving force is

$$\mathbf{f} = \frac{\phi_0}{c}\mathbf{J} \wedge \hat{\mathbf{y}} + \boldsymbol{\eta}\,. \qquad (12)$$

Here $\mathbf{J}$ is the applied transport current density and $\eta(\mathbf{r})$ is a random thermal noise. Upon projecting the critical smectic density modes near $\pm q\hat{\mathbf{z}}$ out of Eq. (10), we find [22]

$$\gamma_{\text{BS}}\partial_t\Phi = -4q^2\frac{\delta F_{\text{crit.}}}{\delta\Phi^*} - i\mu J_x\Phi - \tilde{\eta}\,, \qquad (13)$$

where $\mu = q\phi_0 n_0/c$ and $\tilde{\eta}(\mathbf{k}) = in_0 q\eta_z(q\hat{\mathbf{z}} + \mathbf{k})$. Remarkably, Eq. (13) has the same form as the model E dynamics [23] for the complex "superfluid" order parameter Φ, where now J_x plays the role of the "electric field" in the Josephson coupling. The actual electric field is $\mathcal{E}_x = j_{v,z}\phi_0/c$, leading via Eq. (11) to

$$\mathcal{E}_x \approx -\frac{n_0\phi_0}{2qc}\,\mathrm{Im}(\Phi^*\partial_t\Phi) + (1 - |\Phi|^2/2)\left(\frac{B}{H_{c2}}\right)\rho_{xx,n}J_x\,, \tag{14}$$

where $\rho_{xx,n}$ is the normal state resistivity in the x direction, whose appearance in the last term follows from the relation $(n_0\phi_0/c)^2/\gamma_{\mathrm{BS}} \approx (B/H_{c2})\,\rho_{xx,n}$. Equation (14) is interpreted in close analogy with Eq. (8). The first term is the contribution to vortex flow from translation of the layers, while motion of equilibrium vortex kinks yields the second term. Such flow at "constant structure" is analogous to the permeation mode in smectic liquid crystals [12].

The presence of this defective motion implies a small but nonzero resistivity at the L-S transition. Near T_s, Eq. (14) predicts a singular decrease of the form $\rho_{xx}(T) - \rho_{xx}(T_s) \sim |T_s - T|^{1-\alpha}$. At lower temperatures (but still within the S phase) transport occurs via two channels. The permeation mode gives an exponentially small linear resistivity $\rho_{xx} \sim \exp(-E_{lk}/k_BT)$ (above T_s, single layer kinks give $\rho_{xx} \sim \exp(-E_k/k_BT)$, with $E_k \approx E_{lk}/m$). Nonlinear transport occurs via thermally activated liberation of vortex droplets, inside which u (or u_z in the crystal plane) is shifted by s. Balancing the soliton energy on the boundary with the Lorentz energy in the interior gives $\mathcal{E}_{nl} \sim e^{-(J_c/J)^2}$, where $J_c \sim (c/B)(\kappa\tilde{g})^{3/4}(s/k_BT)^{1/2}$. A more detailed discussion of the full scaling form of $\mathcal{E}(J)$ will be given in Ref. [11].

In the TS phase, net vortex motion along the c axis occurs by sliding soliton walls along the b direction. The resulting electric field is proportional to J and the soliton density, leading to an additional contribution to the resistivity which vanishes at the CIT like $\rho_{xx}^{\mathrm{soliton}} \sim \rho_0^{\mathrm{soliton}}/\ln(\Delta/|\sigma|)$. The situation in the *IS* phase is more complex, due to the possibility of a roughening transition for a single soliton wall and will be deferred to Ref. [11].

Lastly, we consider the effects of weak point disorder, which enters the free energy as a random field $F_d = \int d^3r\, n_0 V_d(\mathbf{r})\,\mathrm{Re}\{\Phi(\mathbf{r})e^{iqz}\}$, where $V_d(\mathbf{r})$ is a quenched random white noise potential. Such a perturbation alters the universality classes of the phase transitions in Fig. 1, and renders all but the L and S phases glassy [11]. The stability of the S phase to randomness [which takes the form $F_d = \int d^3r\, 2n_0\sqrt{|r|/v}\, V_d(\mathbf{r})\cos 2\pi(u + z)/a$ in this regime] is a consequence of the phonon "mass" $\tilde{g}/s^2$. The nature of the glassy phases and altered critical behavior will be discussed in Ref. [11].

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