

Coherent Propagation of Two Interacting Particles in a Random Potential

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It is shown that in a random potential two repulsing or attracting particles can propagate coherently on a distance l_c much larger than one-particle localization length l_1 . The enhancement factor l_c/l_1 for coherent localization length is proportional to l_1 . The connection of this problem with the superimposed band random matrices is established, and the theory developed is tested in numerical simulations with random potential models and models of quantum chaos.

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The phenomenon of Anderson localization of one particle in a random potential was studied extensively during the last years [1] and is now well understood. The theoretical predictions were confirmed not only in laboratory experiments but also in multiple numerical simulations which became possible due to modern computers and new numerical methods [2]. Recently, more and more interest is attracted to interacting particles in a random potential (see, e.g., [3–5]). The research in this direction is stimulated by experiment with mesoscopic metallic rings in which anomalously large persistent currents had been observed [6]. The magnitude of the current still does not find a definite theoretical explanation [7] that determines the necessity of a deeper understanding of the effects of particles interaction in a random potential.

In this Letter I address a simpler problem of only two interacting particles (TIP) in a random potential. I think that this problem itself contains important physical effects which will allow one to understand the properties of the interacting particles problem at finite particles density. As will be seen, the TIP problem can be reduced to some kind of band random matrix (BRM). During the last few years ensembles of such matrices were extensively studied both numerically [8] and analytically [9]. However, BRM arising from the TIP problem is of another type; namely, it can be presented as a superposition of two BRM. The properties of such superimposed BRM (SBRM) can be effectively studied by the transfer matrix technique [2] which allows one to determine the dependence of the localization length in SBRM and TIP models on parameters. This gives a striking result according to which there are states of a new type in which the particles are located a distance of one-particle localization length l_1 from each other and propagate together coherently on a much larger distance $l_c \gg l_1$. Such coherent propagation takes place even in the case of repulsive interaction. The physical reason for the appearance of such effective pairing for repulsing particles can be understood in the following way. In the random potential two repulsing particles, which were originally close to one another, cannot diverge on a distance much larger than l_1 due to exponential decrease of transition matrix elements for a

distance between particles $R \gg l_1$. In some sense the localization forces the particles to stay together. In such a coupled state the particles can move one with respect to the other and this can strongly increase the distance l_c in which they propagate together, if compared to l_1 .

To understand how the TIP problem can be reduced to SBRM let us consider two particles in the 1D Anderson model [2] interacting only on one site:

$$(E_{n_1} + E_{n_2} + U\delta_{n_1, n_2})\psi_{n_1, n_2} + V(\psi_{n_1+1, n_2} + \psi_{n_1-1, n_2} + \psi_{n_1, n_2+1} + \psi_{n_1, n_2-1}) = E\psi_{n_1, n_2}. \quad (1)$$

Here E_n are one-particle energies randomly distributed in the interval $-W, W$, and U characterizes the repulsive ($U > 0$) or attractive ($U < 0$) interaction between particles. We will consider the symmetric configurations. The physics of the asymmetric case for an interaction on a nearby site is qualitatively the same [10]. Without interaction the eigenstates are simply given by the symmetrized product of one-particle eigenstates ϕ_m . The transformation between the unperturbed lattice basis $|n\rangle$ and ϕ_m can be written as $|n\rangle = \sum_m R_{n,m} \phi_m$, where the index m marks the one-particle energies ϵ_m . Because of the one-particle localization the matrix R can be approximately represented as $R_{n,m} \approx \exp(-|n-m|/l_1 - i\theta_{n,m})/\sqrt{l_1}$, where $\theta_{n,m}$ are random phases. For $l_1 > 1$ and $E \approx 0$ one has $l_1 \approx 25(V/W)^2$ [2].

It is convenient to rewrite (1) in a one-particle eigenbasis:

$$(\epsilon_{m_1} + \epsilon_{m_2})\chi_{m_1, m_2} + U \sum_{m'_1, m'_2} Q_{m_1, m_2, m'_1, m'_2} \chi_{m'_1, m'_2} = E\chi_{m_1, m_2}, \quad (2)$$

where χ_{m_1, m_2} are eigenfunctions of the TIP problem in one-particle basis ϕ_m . The matrix of transitions produced by the interaction is $Q_{m_1, m_2, m'_1, m'_2} = \sum_n R_{n, m_1}^+ R_{n, m_2}^+ R_{n, m'_1} R_{n, m'_2}$. Its structure is determined by the properties of the matrix R . Since R is exponentially small for $|n-m| > l_1$ the values of Q are not negligible only for $|m_i - m'_j| < l_1$. In this case $R \approx 1/\sqrt{l_1}$, the sum over n contains l_1 terms with random phases, and $Q \approx 1/l_1^{3/2}$. Otherwise the value of Q is exponentially small, and no transitions due to interaction can take

place. The similar estimates for the transitions had been obtained for the problem of destruction of localization by nonlinear interaction [11].

The properties of the matrix Q established above mean that on the 2D lattice (m_1, m_2) the transitions couple only the states near the diagonal $m_d = m_1 = m_2$. In some sense around the diagonal we have a strip of width $\Delta m \approx l_1$, and only in this strip are the states coupled by interaction. Outside the strip the coupling is exponentially small and can be neglected. In the strip each state is coupled with roughly l_1^2 states by a matrix element $U_s \approx U/l_1^{3/2}$. We can order all levels in the strip, counting them along the serpentine line going along the diagonal. The index of such ordered sites j is connected with the index along the diagonal m_d by relation $m_d \sim j/l_1$, where m_d gives the position of the pair on the 1D lattice. After such snake ordering we obtain a band matrix G which contains $2b + 1$ diagonals with $b \approx l_1^2$. Its nonzero matrix elements are random and are of the order of U_s . However, this BRM is different from the usually studied situation [8,9], since G is the sum of BRM and a diagonal matrix, the elements of which are $\epsilon_{m_1} + \epsilon_{m_2}$. These elements vary in the interval $\pm \Delta_s$ with $\Delta_s \approx 2(2V + W)$. Dividing all the elements of G by factor $U/\sqrt{l_1}$ we come to the following SBRM G_r . It is given by the sum of BRM and diagonal matrix with random elements homogeneously distributed in the interval $\pm W_b$ where $W_b = \sqrt{l_1} \Delta_s / U$. The BRM has $2b + 1$ diagonals, and its nonzero elements are homogeneously distributed in the interval $\pm 1/\sqrt{2b + 1}$.

The reduction of the TIP problem to SBRM G_r allows one to find the localization length l_c for coherent propagation in random potential. Indeed, to find l_c it is sufficient to know the dependence of localization length l_{sb} in SBRM on parameters of the system and then to use the relation $l_c \approx l_{sb}/l_1$. The localization length l_{sb} can be found numerically by the transfer matrix technique [2,12]. This method allows one to determine the minimal positive Lyapunov exponent in the strip. The inverse value of the exponent gives the localization length. Such a numerical approach is very effective since it allows one to determine the localization length on a lattice with a length of a few 10^6 sites. The results of the numerical simulations are presented in Fig. 1 for the parameters in the following intervals: $3 \leq 2b + 1 \leq 321$, $0 \leq W_b \leq 4096$, $1 \leq l_{sb} \leq 2 \times 10^4$, $E = 0$. The result clearly demonstrates that the ratio l_{sb}/b is a function of only one parameter which is approximately $\kappa = (1.25W_b^2 + 1)/(b + 0.5)$. For the whole range $l_{sb} > 1$ the dependence on parameters is quite satisfactorily described by

$$l_{sb} = 1.6(b + 0.5)/\ln[1 + 2.5(1.25W_b^2 + 1)/b]. \quad (3)$$

This expression also works approximately for nonzero energies if $|E| < W_b$. Probably such fitting is not optimal, and one can find a better functional shape. However, the function (3) gives the correct asymptotical

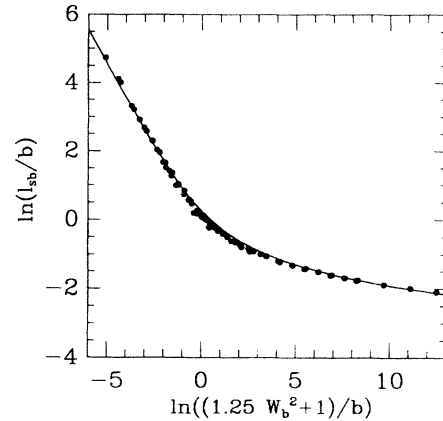


FIG. 1. Dependence of l_{sb} in SBRM model on W_b and b (about 90 points are shown). Full line is fit (3).

behavior for $W_b \ll \sqrt{b}$ and $W_b \gg \sqrt{b}$. The first case is the most interesting one, since it corresponds to the TIP problem. Indeed, we consider the case with $W < V$ and $U \sim V$ so that for large l_1 the value $W_b \sim \sqrt{l_1} \ll \sqrt{b} \approx l_1$. In this regime $l_{sb} \approx b^2/2W_b^2$. The dependence $1/W_b^2$ seems to be quite natural since in perturbation theory the probability of transition is proportional to $1/W_b^2$.

The expression found for l_{sb} gives the coherent localization length $l_c \approx l_{sb}/l_1$:

$$\frac{l_c}{l_1} \sim l_1 \frac{U^2}{32V^2}. \quad (4)$$

As we see for large l_1 , the length of coherent propagation is strongly enhanced. This signifies the appearance of effective pairing even in the case of repulsion. Let us, however, mention that even if large, l_c is nevertheless always finite so that eventually eigenfunctions decay exponentially both for attracting and repulsive interaction. It is interesting to note that the enhancement factor for the localization length is proportional to $U\sqrt{l_1}$ which can be regarded as the enhanced one-particle interaction. A similar statistical enhancement for weak interaction and parity nonconservation was intensively studied in neutron-nucleus reactions [13].

The estimate for l_c can also be obtained on the basis of the analogy with photonic localization in molecular quasicontinuum [12]. Indeed, there the localization length measured in units which are given by the size of transition (photon frequency) is $l_\phi \sim \Gamma\rho$, where Γ is the transition rate in units of time and ρ is the density of states. For TIP ρ is determined by the number of unperturbed components $\rho \sim l_1^2/V$, and the rate is $\Gamma \sim U_s^2\rho$. Here, the size of transition in the basis of unperturbed eigenstates is l_1 so that $l_\phi \approx l_c/l_1 \sim U_s^2\rho^2$ in agreement with (4). The similar estimate for SBRM G_r gives as in (3) $l_{sb}/b \sim b/W_b^2$ since $\rho \sim b/W_b$, effective $U_s \sim b^{-1/2}$ and $\Gamma \sim 1/W_b$.

Let us now discuss the conditions of applicability of (4). At first, the enhancement factor should be large

$l_c/l_1 \gg 1$ which corresponds to the condition $W_b \ll \sqrt{b}$ in SBRM. Second, U should be less or comparable with V , since for $U \gg V$ interaction can significantly modify the spectrum. Also (4) was derived for the lattice (1) with intersite distance $a = 1$, energy $E \sim V \gg W$, and particle wave vector $k_F \sim 1$. Separate investigations are required for continuous limit with $a/l_1 \ll k_F a \ll 1$ [14].

The result (4) was obtained for the one-channel lattice. However, the same reduction to the SBRM model can also be done for a thick wire with M transverse channels ($M < l_1$). In this case the enhancement of localization length along the wire will be $l_c/l_1 \sim M l_1 (U/V)^2 / 32$ times. Since the localization length in the infinite wire is proportional to the diffusion rate and conductance, it is possible to assume that in the metallic regime with the size of the sample $L < l_1$, the enhancement of the one-electron conductance is proportional to the factor $M/32$, which would give the correct order of magnitude. However, formally the case $L < l_1$ cannot be analyzed by the above method of reduction to the SBRM problem, and therefore it should be studied separately in more detail.

Up to now the result (4) was derived by the reduction of the initial TIP problem to the SBRM model. Being physically correct this reduction nevertheless contains few assumptions (e.g., complete statistical independence of the elements in SBRM), and therefore the direct verification of the theoretical prediction (4) is of principal importance. Since, according to (4), the enhancement starts to work only from $l_1 \sim 30$ ($U < V$), it is quite difficult to make direct matrix diagonalization of the initial problem (1). Therefore, I investigated the wave packet dynamics on the lattice (1). To simplify the numerical difficulties the random potential E_n was taken symmetric with respect to the point $n = 0$. Initially both particles were located at $n = 0$ with fixed total energy $E = 0.2$. The dynamics was characterized by the two second moments of probability distribution over unperturbed levels $\sigma_+ = \langle (|n_1| + |n_2|)^2 \rangle / 4$ and $\sigma_- = \langle (|n_1| - |n_2|)^2 \rangle$. In the case of coherent propagation σ_+ should be larger than σ_- . The dependence of the moments on time is presented in Fig. 2. The localization length l_c is enhanced approximately 2.5 times with respect to the noninteracting case that is in satisfactory agreement with the estimate (4).

The existence of coherent propagation was also checked for another standard model of quantum chaos, namely for the kicked rotator [12]. This model describes two particles on a ring perturbed by kicks periodic in time. The evolution of ψ function is given by the unitary operator

$$\hat{S} = \exp\{-i[T(\hat{n}_1^2 + \hat{n}_2^2)/2 + U\delta_{n_1, n_2}]\} \times \exp[-ik(\cos \theta_1 + \cos \theta_2)], \quad (5)$$

with $\hat{n}_{1,2} = -i\partial/\partial\theta_{1,2}$. For $U = 0$ the classical dynamics is chaotic and diffusive if $kT > 1$. Quantum interference leads to suppression of this diffusion and dynamical

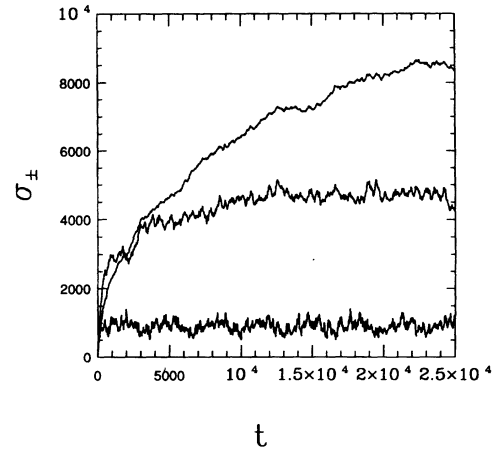


FIG. 2. Time dependence of second moments in (1) with $W = 0.7, V = 1$: upper (σ_+) and middle (σ_-) curves are for $U = 1$; lower curve (σ_+) is for $U = 0$. At $t = 0$ both particles are at $n = 0$, basis is $-501 \leq n \leq 501$.

localization with the length $l_1 \approx k^2/2$ for $k > 1$ [12]. For $U \neq 0$ the situation is analogous to (1) but there are two important differences. First, this is a dynamical model without external randomness. Second, many states ($2k \gg 1$) are coupled by one-particle perturbation. Also, interparticle interaction is neither attractive nor repulsive. The results in Fig. 3 for σ_{\pm} show obvious enhancement of coherent propagation which can be estimated as $l_c/l_1 = \sqrt{\sigma_+(U=2)/\sigma_+(U=0)} \approx 5 \approx l_1/4$.

Another study of the enhancement was done for a bag model of two particles in the 1D random potential with interaction being zero if the distance between particles $\Delta n < B$ and infinity if $\Delta n > B$ (B is the bag size). The wave function is zero at the bag edges, and transfer

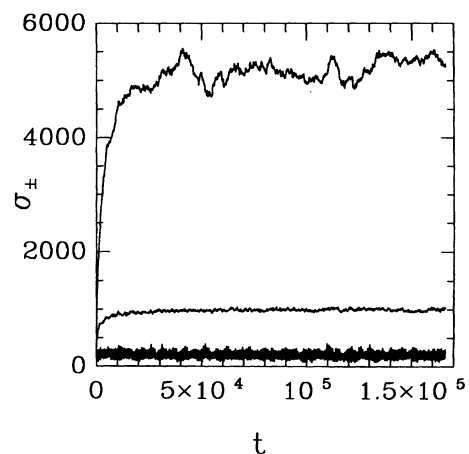


FIG. 3. Same as in Fig. 2 for model (5) with $k = 5.7, kT = 5$: upper (σ_+) and middle (σ_-) curves are for $U = 2$; lower curve (σ_+) is for $U = 0$. At $t = 0$ both particles are at $n = 0$, basis is $-301 \leq n \leq 301$, t is given in number of kicks.

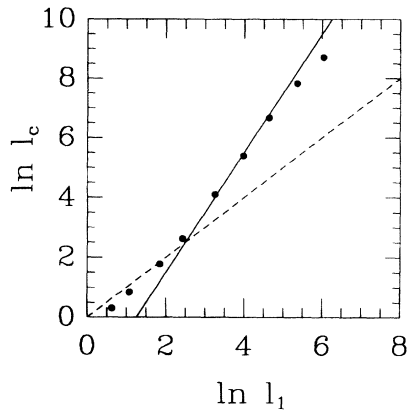


FIG. 4. Dependence of l_c on l_1 in the bag model (points), $E = 0$. Dashed line shows $l_c = l_1$; full line shows slope 2 corresponding to $l_c \sim l_1^2$ with $l_1 \approx 25V^2/W^2$.

matrix technique is very efficient for calculation of the maximal localization length. The simulations were done for the symmetric configuration of particles in the bag of size $B = 323 > l_1$. The dependence of the coherent localization length l_c in this model on the one-particle localization length l_1 is shown in Fig. 4. For disorder $W \geq 1.5V$ the length l_c is close to l_1 , while for the smaller values of W the length l_c starts to grow approximately as l_1^2 in agreement with (4). The introduction of interaction between particles does not lead to a significant change of l_c . As a result of the bag model it is possible to give another qualitative explanation of the enhancement. The coupled pair feels only the potential averaged over the size of the pair l_1 . Such averaging decreases the disorder to $W_{\text{eff}} \approx W/\sqrt{l_1}$, giving a new effective lattice with the distance between sites l_1 in which the localization length $l_\phi \approx l_c/l_1 \sim (U/W_{\text{eff}})^2 \sim l_1 U^2/V^2$.

For a finite particles density ρ_e such that $l_1 \ll 1/\rho_e \ll l_c$ there are two interesting possibilities. In one case particles on the lattice are distributed by pairs of size l_1 ; pairs collide and destroy coherent localization leading to appearance of finite nonzero conductivity in the infinite quasi-one-dimensional system. In another case particles are distributed one by one, the distance between them is much larger l_1 , and then conductivity should be exponentially small. It is possible

that a similar situation can take place for quasiparticles at high density $\rho_e l_1 \gg 1$.

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